SIGNIFICANCE OF ELONGATION ON SEISMIC RESISTANCE OF CONCRETE FRAME STRUCTURES

AUCKLAND UNISERVICES LIMITED

a wholly owned company of

THE UNIVERSITY OF AUCKLAND

PRIVATE BAG 92019

AUCKLAND
Reports from Auckland UniServices Limited should only be used for the purposes for which they were commissioned. If it is proposed to use a report prepared by Auckland UniServices for a different purpose or in a different context from that intended at the time of commissioning the work, then UniServices should be consulted to verify whether the report is being correctly interpreted. In particular it is requested that, where quoted, conclusions given in UniServices Reports should be stated in full.
EQC Project 93/157

"Significance of Elongation on Seismic Resistance of Concrete Frame Structures"

Investigators
Fenwick, R.C., Davidson, B.J. and Megget, L.M.


Work on this project is progressing on two fronts. Ms. Kim Douglas, a third year PhD student, is developing an analytical model of a plastic hinge zone, which is to be incorporated into the dynamic analysis program, DRAIN2DX. Currently this model can predict the deformations associated with pure flexure. To achieve this new elements have been developed, which can represent the stress-strain characteristics of concrete and reinforcement subjected to cyclic inelastic loading. The next steps involve modifying these elements to allow for the changes in plastic hinge length, which occur during an earthquake, and incorporating shear deformation into the model. While some progress has been made on these steps a lot remains to be done.

On the experimental side a number of tests have been carried out. The most significant of these is the test of an approximately one third scale model of a three bay bent of a multi-storey frame. This test was carried out by Mr. A. McBride, a ME (Thesis) student. The beams in this bent were cast with a composite floor slab. The results of this test are currently being reduced and the planning for a second test of a similar bent is underway. It is intended that the second bent will be identical to the first except that the composite slab will be omitted. These two tests will allow direct observations to be made on the significance of the slab on both the elongation and the shear deformations which develop in the plastic hinges.

In addition to the three bay bent tests three further cantilever beams have been built and tested under cyclic inelastic loading. Two of these were rectangular beams, in which the quantity of shear reinforcement was varied. These were tested by an ME (project) student. The project report is currently being written up. A further Tee beam was tested by a final year student as part of a one paper project. The object of this test was to obtain some direct experimental data on the influence of the slab on the deformations sustained in the plastic hinge zone.

SUMMARY

The EQC research fund has been used to:-

(1) provide financial support for two research students, namely Ms. Kim Douglas and Mr. D. McBride,

(2) to pay for the materials involved in the tests described in the previous section, and

(3) to provide some support for attending a conference in San Diego. At this conference a paper describing the work carried out on elongation (completed before Feb. 1993) was presented. A copy of this paper is enclosed.
Paper Presented at the
Tom Paulay Symposium
San Diego

September 1993
ELONGATION IN DUCTILE SEISMIC RESISTANT REINFORCED CONCRETE FRAMES

by R.C. Fenwick and B.J. Davidson

Synopsis: To survive a major earthquake, current practice requires seismic resistant frames to be designed to be ductile. To achieve the required level of ductility in multi-storey frames, the majority of the potential plastic hinge zones are located in the beams. The inelastic rotation, which may develop in these zones, arises predominately from the tensile yielding of the reinforcement. The associated compressive strains are small and as a consequence elongation occurs. Test results show that elongations of the order of 2 to 4 percent of the member depth develop in plastic hinge zones of beams subjected to cyclic loading before strength degradation occurs. The factors influencing elongation are reviewed. The results of a time history analysis, in which elongation effects are modeled, shows that this action, which is neglected in current design practice, has important implications for the detailing of columns and the design of supports for precast components and external cladding.
Richard C. Fenwick is an Associate Professor at the University of Auckland in New Zealand. His teaching activities are primarily related to the structural design. Research activities are in the seismic resistance of concrete structures and the performance of concrete bridges.

Barry J. Davidson is a Senior Lecturer at the University of Auckland. His major research interest is in the dynamic behaviour of buildings under earthquake and wind forces. His primary teaching activities are in the structural analysis field.

INTRODUCTION

Current design practice requires multi-storey frame buildings to be designed to perform in a ductile manner in the event of a major earthquake (1,2,3). To achieve the required level of ductility frames are detailed so that a beam sway mode develops in preference to a column sway mode. This objective is achieved in codes of practice by requiring the sum of the flexural strengths of the columns at each beam-column joint to exceed by some margin the sum of the combined beam flexural strengths. With this arrangement, in a major earthquake the vast majority of the plastic hinges will be located in the beams. It is the behaviour of these zones that largely determines the dynamic performance of these structures after initial yielding has occurred.

In the paper, the effects of elongation due to the formation of plastic hinges in the beams of ductile frames are discussed. This aspect has received very little attention in the literature and it has been ignored in the vast majority of time history analyses that have been made to develop design procedures. The mechanisms causing elongation are described and the different factors which may influence its magnitude are investigated in beam tests. A model of a potential plastic hinge zone, which allows elongation effects to be predicted, is described, and it is used in the time history analysis of a six storey frame. By repeating the analysis with a different plastic hinge model, which neglects elongation effects, the significance of this action on the seismic performance is demonstrated.

The design concept of providing a hierarchy of strengths in a ductile frame to ensure that a beam sway mode develops in preference to a column sway mode is relatively new. As a relatively small number of structures have been designed and constructed on the basis of this concept, little practical experience has been gained from their actual performance in severe earth-
quakes, in which ductility demands comparable to the design level have been sustained. As the stock of these structures increases this experience will be gained. This lack of practical experience makes it important to examine the performance of individual structural components so that realistic models can be developed for use in time history analyses. Such work is essential in developing satisfactory code rules. Inadequacies in the modeling can lead to incorrect predictions of behaviour and design rules which may not ensure satisfactory performance in a major earthquake.

PLASTIC HINGES IN BEAMS

Plastic Hinge Types

Two different forms of plastic hinge, namely reversing and unidirectional, can develop in the beams of seismic resistant frames during a severe earthquake. These are illustrated in Figs. 1 and 2, for a beam with uniform flexural strengths along its length.

When the seismic actions on a beam increase to a critical level, two plastic hinges develop, one of which sustains a negative bending moment and the other a positive bending moment. The locations of these plastic hinges depends upon the relative magnitudes of the maximum seismic shear that can be sustained, and the shear arising from the gravity loads supported by the beam. Where the maximum seismic shear is greater than the gravity load shear, the positions of maximum positive and negative bending moment are located against the column faces and the plastic hinge zones also form in these locations, as is illustrated in Fig. 1. With a reversal in the direction of the seismic forces the sign of the bending moment acting on each plastic hinge changes. Hence, positive and negative plastic hinge rotations are imposed on the same hinge zones. These are referred to as reversing plastic hinges.

For the case where the gravity shears exceed the maximum seismic shear force, a point of zero shear force occurs in the span. This defines the location of the maximum positive bending moment and hence the location of the positive moment plastic hinge. As illustrated in Fig. 2, when the structure sways to the right, a negative moment hinge forms against the right hand column face and the positive moment hinge develops in the span on the left hand side of the beam center-line. With the reversal in the seismic actions, the negative moment plastic hinge forms against the left hand column and a further positive moment plastic hinge forms in the span on the right hand side of the center-line. Each of the four plastic hinges in the beam sustains inelastic rotations of one sign only. They are referred to as unidirectional plastic hinges. This form of plastic hinge can be expected to develop in many medium to low rise frame buildings
in severe earthquakes, where the frames provide lateral resistance and the beams support gravity loads.

In beams which develop unidirectional plastic hinges in a severe earthquake, the negative moment inelastic rotations are sustained in the plastic hinges adjacent to the column faces and the positive moment inelastic rotations are sustained in the spans. Each inelastic lateral displacement causes additional inelastic rotations to be sustained by two of the plastic hinges. As the earthquake progresses, so the rotation in each hinge progressively increases together with the beam deflection, as is illustrated in Fig. 2(d).

During the passage of an earthquake the locations of positive moment plastic hinges can be expected to vary along the span, due to variation of the vertical seismic forces acting on the beam and the magnitude of the strain hardening sustained in the negative moment plastic hinges. Furthermore, as the maximum positive bending moments occur in locations of low shear, the associated yielding may be expected to spread along an appreciable length of beam, generating only small strains in the reinforcement and relatively little strain hardening.

The situation is very different for the negative moment plastic hinges. These are confined to short lengths in high shear zones at the ends of the beam and consequently high curvatures and significant strain hardening occurs in these plastic hinges.

ELONGATION IN PLASTIC HINGES

Previous Research

The behaviour of reversing plastic hinge zones in concrete beams has been extensively studied. However, the elongation effects in these have received little attention. The possibility of unidirectional plastic hinges forming in seismic resistant frames has been largely neglected in structural testing. Frequently where models have been developed for time history analyses the possibility of unidirectional hinges forming in the beam spans has been overlooked and as a result, frame strengths have been over-estimated and plastic hinge rotations under-estimated.

Elongation effects are generally not apparent in structural tests on statically determinate units, such as beams or beam-column sub-assemblies. In these situations no reaction is induced due to the elongation and as a consequence it is easily overlooked. However, the situation changes when indeterminate sub-assemblies are tested. A clear example of this occurred in
the test of a seven storey building (4) where the lateral force resistance was provided by a combined frame-wall system. Elongation occurred in the wall due to the formation of a plastic hinge at its base. This increase in length was restrained by the surrounding columns, which went into axial tension, with an axial compression force being induced in the wall. As a result the lateral strength of the structure was very significantly increased, though there was a loss of ductility. The foundation forces were very different from those predicted by a conventional analysis, and if the structure had not been constructed on a strong floor, a premature non-ductile failure could have been anticipated in the foundations.

Zerbe and Durrani (5,6,7) found in simulated seismic lateral load tests on a number of indeterminate two bay beam-column and column-slab sub-assemblies that the behaviour was appreciably different from that which could be expected from test results obtained from individual statically determinate elements. In their tests a lateral load was applied to a stiff distribution beam, which was connected by pin joints to the tops of the columns. The bottom of each column was pinned to a rigid base. The test arrangement is shown diagramatically in Fig. 3. With this test arrangement, elongation of the beam due to plastic hinging was restrained by the flexural stiffness of the short columns. The axial forces induced in the beams improved their flexural strength together with the performance of the beam-column and column-slab joints. Such advantageous restraint can not be expected in frames, except to a very limited extent in the first floor beams where the foundation provides some restraint.

For a number of years elongation measurements have been made in structural tests carried out at Auckland and Canterbury Universities (8,9,10,11). These show that elongations of the order of 2 to 4 percent of the member depth can be expected in reversing plastic hinge zones before strength degradation occurs.

**Beam Tests at University of Auckland**

In this section elongation measurements obtained in several series of beam tests are described. In all cases the beams were tested as simple cantilevers springing from an anchorage block, which was prestressed to the floor. The testing arrangement, which is shown in Fig.4 for the beams M1 and M2U, was typical of that used in all tests. Displacement transducers, which were attached via studs to the longitudinal reinforcement, enabled shear, flexural and elongation measurements to be obtained along the length of the member.
The cross-section of all the beams was 500 mm by 200 mm, though in one case (T3T) a composite slab was added. Additional 10 or 12 mm bars were welded to all the beam longitudinal reinforcement where it entered the springing block, to ensure that the plastic hinge was confined to the beams. Details of the beams are given in Figs. 4 and 5 and in Table 1. In all cases the reinforcement details in the plastic hinge zones complied with the requirements contained in the UBC-91 and NZS 3101-82 concrete codes (1, 12).

At the start of all tests two "elastic" load cycles were applied to the beam. In these the maximum load in each direction was taken to three quarters of the value which would generate the theoretical ultimate flexural strength at the critical section of the beam. The average of the four maximum displacements was found and the ductility one displacement was defined as this value divided by three quarters. In the standard loading sequence, which was followed unless noted otherwise in Table 1, the elastic load cycles were followed by two complete load cycles, in which for each cycle a displacement ductility of 2 was applied in each direction. From here pairs of load cycles to displacement ductilities of 4 and 6, with further cycles to ductilities of 6 or 8 and 10 were applied.

The loading cycle for beam M2U was modified so that a unidirectional hinge was formed, in which inelastic deformation was sustained in one direction only. In this case the elastic load cycles were followed by pairs of load cycles, in which the downward deflections were taken to 2, 4, 6, 8, 10 and 12 displacement ductilities, but the upward load was limited so that the bending moment at the critical section just reached three quarters of the positive theoretical flexural strength.

Two of the beams were tested dynamically, so that the strain rates were comparable to those that could be expected in a major earthquake. The loading cycles used in these were a little different from the standard cycles in that after each peak displacement in each direction a few additional load cycles to smaller displacements were added before the next peak displacement was applied in the opposite direction (10).

**Strains in Unidirectional and Reversing Plastic Hinges**

Two identical beams were build and tested. The details are shown in Fig. 4 and Table 1. The first of these, M1, was tested with the standard load history to form a reversing plastic hinge, while in the second one, M2U, was tested with the modified load history, to form a unidirectional hinge (see previous section). In the beam with the reversing hinge, failure occurred in the first load cycle at a displacement ductility of 10. With the unidirectional plastic
hinge failure occurred in the first cycle of the displacement ductility 14. Some of the displacement measurements made on the longitudinal flexural reinforcement in the plastic hinge zones of these two beams are shown in Fig. 6. It can be seen that the behaviour of the two plastic hinge zones was very different. With the unidirectional plastic hinge, the strains in the bottom bars which were not yielded, were negligible compared with those in the top bars. In this case the elongation, measured at the mid-depth of the beam can be assessed in terms of the rotation, \( \theta \), the plastic hinge sustains, by the expression -

\[
elongation = \theta \frac{(d - d')}{2} \tag{Eq. 1}
\]

where \( d - d' \) is the distance between the centroids of the top and bottom reinforcement.

With the reversing plastic hinge, in the first inelastic displacement, the compression reinforcement sustained a small compressive strain. With the reversal of the loading direction, the reinforcement in the new compression zone, which had yielded in tension in the previous half cycle, did not yield back to allow the cracks to close. With each subsequent load cycle the reinforcement in the compression zone increased in length until close to the failure when the bars buckled in compression. The elongation in the reversing plastic hinge zone arises from two causes; namely the extension of the longitudinal reinforcement in the compression zone, "e", and the rotation sustained by the zone. It is given by the expression -

\[
elongation = e + \theta \frac{(d - d')}{2} \tag{Eq. 2}
\]

From the test results shown in Fig. 6 it can be seen that the elongation associated with the extension of the compression zone reinforcement was approximately twice that associated with rotation in the ductility 4, 6 and 8 load cycles.

There are two reasons why the magnitude of "e" increases with load cycling in a reversing plastic hinge zone. Firstly, when a deformed reinforcing bar yields in tension, extensive cracking occurs in the surrounding concrete. This causes the concrete to dilate. In addition aggregate particles become wedged in the cracks. To close the cracks an appreciable force has to be applied to the concrete. Secondly, as the direction of loading reverses sets of intersecting diagonal cracks develop right through the beam. The only viable shear resisting mechanism in this situation is provided by a truss like action, with the stirrups going into tension and diagonal compression forces being sustained in the web of the beam, as illustrated in Fig.7. The equilibrium requirements at a normal section show that the flexural compression force, \( C \), is always smaller than the corresponding tension force, \( T \), due to the longitudinal component of the diagonal compression forces. Both these actions
lead to the compression force in the compression zone reinforcement being less than the flexural tension force. As a result inelastic rotation occurs more by the tensile reinforcement extending, than contraction of the reinforcement in the compression zone. Thus the value of "c" increases with each cycle until buckling of the reinforcement occurs.

The elongation measurements made on the two beams, M1 and M2U, at the peak displacements in the load cycles, are shown in Fig. 8, together with the values predicted by Eq. 1. It can be seen that the elongation in the unidirectional plastic hinge is accurately predicted. However, for the reversing hinge the values are greatly underestimated by this equation. In this case approximately two thirds of the elongation arises from the extension, e, of the reinforcement in the compression zone. The shear deformation in the reversing hinge is greater than that in the unidirectional hinge. This resulted in much smaller rotations being sustained in the former beam than latter beam at comparable load stages (13).

Influence of Moment to Shear Ratio, Beam Details and Axial Load on Elongation

If the monotonic stress-strain relationships are known for the reinforcement and the concrete, conventional flexural theory can be used to predict the moment rotation and elongation characteristics of unidirectional plastic hinges. However, this does not hold for reversing plastic hinges. The cyclic yielding of the reinforcement changes its stress-strain characteristics, and the extensive cracking associated with this cyclic yielding modifies the stress-strain behaviour of the concrete. In addition the shear resisting mechanism, which is illustrated in Fig. 7, has an appreciable influence on the response of reversing hinges. In the remainder of this section the influence of different factors on elongation in reversing plastic hinge zones is investigated by reviewing the results of beam tests.

To investigate the effect of the moment over the shear force times effective depth ratio (M/Vd) on elongation in reversing plastic hinges, the results of tests on beams F1 to F4 (see Table 1) were examined. These beams had equal top and bottom steel areas. To vary the shear stress level the length of the shear span was changed between tests. Beam F1 had the highest shear stress level, with a M/Vd ratio of 2.1. The corresponding values for beams F2 to F4 were 3.0, 3.9 and 4.8 respectively. To enable the results to be compared in a consistent manner the displacement ductilities were related to the reference point, which was located 1100 mm from the springing. The ductility one displacements were 5, 6, 4.5 and 5 mm respectively for the beams F1 to F4. From the results in Fig. 9 it can be seen that the M/Vd ratio had little effect on
the elongation. The high shear stresses sustained in the shortest beam (M/Vd = 2.1) caused this member to degrade prematurely in strength and stiffness, which resulted in smaller elongations in the ductility 8 cycles.

In Fig. 10 the effect of varying the section shape and of having different longitudinal reinforcement areas on each face is investigated. Beam T,3T was a tee beam, with each outstanding flange being reinforced with 5 deformed 10 mm bars. Measurements indicated that all bars exceeded the yield strain during the test. The average elongation obtained in the upward and downward displacements was not appreciably influenced by the ratio of the longitudinal reinforcement areas on each side of the beam or the addition of the composite flanges. The elongation did increase in beams in the half loading cycles where the smaller area of reinforcement was subjected to tension.

In Fig. 11 the effect on elongation of applying axial loads to plastic hinges is illustrated. In these tests the ratio of the axial load level to gross cross-sectional area times the concrete strength (P/Asfs) varied from zero to 0.145. It can be seen that applying the highest axial load level reduced the elongation to approximately one third of the value which would be expected in a beam without axial load.

**PLASTIC HINGE MODELS**

The sub-assembly for modeling the potential plastic hinge zones in the beams is illustrated in Fig. 12(a). At the potential plastic hinge position two rigid flexural members are mounted on the beam. These members, which are 20 mm apart, are joined by "c" and "r" truss members and the "s" beam member. The r and c members represent the concrete and longitudinal reinforcement on each side of the beam. The assumed stress strain characteristics of these are shown in Fig. 12(b). When the concrete is subjected to tension a crack forms and its load carrying capacity is lost until the crack closes. The reinforcement is assumed to behave in a bi-linear relationship. The function of the s member is to transmit shear between the rigid flexural members. It is held in position at one end, with an axial load release. This particular method of modeling unidirectional plastic hinges has been found to give realistic predictions of elongation, and a reasonable prediction of the moment rotation characteristics (15). It also provides a realistic way of modeling the effect of axial load on the flexural strength of the beam for axial forces in the range of 0.05 Asfs in tension and 0.15 Asfs' in compression.

To assess the significance of elongation on the seismic performance of a structure, by means of time history analyses, it is necessary to duplicate the analyses both with and without elongation effects being modeled. The non-
elongating sub-assembly for a plastic hinge is obtained by making a number of changes to the elongating model. The first of these is to give the "s" member a high axial stiffness and replace the axial load release at one end by a pin. With this arrangement the member resists the axial load. The second change is to reduce the stiffness of the "c" truss elements so that they carry negligible axial forces. A bending moment acting on this model is resisted by equal but opposite forces in the two "r" members. The result is that no elongation occurs and there is no interaction of the flexural strength with axial load. The behaviour with this arrangement is very similar to the plastic hinge model used in the beams of most dynamic analyses, in which the positive and negative bending moment flexural strengths are specified at a node or column face in a beam.

**SEISMIC ANALYSIS OF A MULTI-STOREY FRAME**

**Frame Description**

To investigate the significance of elongation of the seismic performance of a structure a frame was sized. Two computer models were developed for this frame and both were used in time history analyses. In the first computer model the potential plastic hinge zones in the beams were represented by the sub-assembly of elements which allowed elongation effects to be incorporated, while the second model these zones were represented by the non-elongating sub-assembly. In the remainder of this paper these computer models are referred to as the elongating and non-elongating models.

The frame was designed to provide part of the gravity and lateral load resistance for the idealised six storey building with the idealised floor plan shown in Fig. 13. It was assumed that this structure is to be located in the most seismically active region in New Zealand (seismicity approximately equivalent to Zone 4 in UBC-91 code), and on soils of intermediate flexibility (approximately equivalent to S = 1.25 in UBC - 91). The lateral force resistance in the x direction is provided by the frames located on lines 1, 3 and 5. The frames on lines 2 and 4 were assumed to carry part of the gravity loading from the floors but be flexible with regard to the lateral forces. The frame on line 3 is the one which is analysed. As shown in the figure the precast flooring units are supported directly by the beams. The resultant gravity loading, of 53.7 kN/m acting on the beams at each level, is sufficient to ensure that unidirectional plastic hinge zones develop in the event of a major earthquake. The member sizes were proportioned to comply with the requirements of the New Zealand loadings code (3).

The structural walls on the lines A and D, in the idealised floor plan, resist the torsional actions and the seismic forces in the y direction. The
seismic mass associated with the frame on line 3 is 232 tonnes at each level. This is distributed in the ratio of 1 to 2 to the external and internal columns at each level respectively.

To determine the required beam strengths a gravity load analysis was made together with a modal response spectrum analysis for seismic actions. The seismic analysis was based on a structural ductility factor of 6 (approximately equivalent to an $R_w$ factor of 12.6 in the UBC code). The design flexural strengths of the beams at the potential negative moment plastic hinges were taken as the greater of, 1.4 times the dead load bending moments, or the sum of the dead and live load bending moments plus or minus the seismic bending moments. A very limited amount of moment redistribution was applied to equalise the required negative moment flexural strengths in the beams at each individual level.

Assuming that the actual initial negative bending moment yield strengths were equal to the design values, described in the previous paragraph, the locations and magnitudes of the corresponding positive moment plastic hinge zones were found. This procedure gave the minimum beam flexural (dependable) strengths which would satisfy the requirements of the NZ loadings code(3).

The initial flexural yield strengths of the column were found from the method recommended in the commentary to the NZ concrete code(12). This approach is described in detail in reference(14). The actual column yield strengths used in the analyses were on the conservative side of these values, as it was felt to be unrealistic to have too many changes in the height of the column.

The principal results of the gravity and modal analyses are summarised in Table 2 together with the initial flexural yield strengths of the potential plastic hinge zones. For this particular structure the base shear required by the NZ loadings code was close to 60 percent of the corresponding value from the UBC-91 code. The main reason for this difference lies in the restrictions that the UBC code places on the base shear when the fundamental period is determined by calculation rather than the empirical equation.

Strain Hardening Characteristics, Damping and the Earthquake Ground Motion.

An analysis of the results of a series of reinforced concrete beam tests (9) showed that the increase in strength, $\Delta M$, in reversing plastic hinges above the first yield bending moment, $M_y$, as a result of strain hardening, could be assessed from the expression -

$$\Delta M = 3.75 \theta M_y$$

(Eq. 3)
where $\theta$ is the angle expressed in radians sustained by the plastic hinge. This expression was used to determine the strain hardening characteristics of the potential plastic hinges in the columns and the negative moment plastic hinges in the beams. The coefficient of 3.75 was replaced by 1.0 to give the corresponding strain hardening characteristics in the positive moment plastic hinges.

In all the models the mass and stiffness damping coefficients were selected to give an equivalent of 5 percent viscous damping in the first and second modes.

The analyses were based on the El Centro 1940 N00E ground motion. This record was scaled so that the elastic response obtained from a single degree of freedom oscillator, with a period equal to the fundamental period of the structure, was equal to the design response spectrum value. This gave a scale factor of 1.2.

Results of Time History Analyses

The lateral deflection envelopes for the external columns in the frames with the elongating and non-elongating models are shown in Fig. 14. With the non-elongating model the beams act as stiff ties, which ensures that the deflection profiles of all columns are essentially the same. However, with the elongating model the growth in length of the beams causes the columns to be pushed outwards. From the two analyses it can be seen that the effect of elongation is to increase the interstorey deflection sustained in the lower stories at the external column lines. In addition to this, the maximum plastic hinge rotation at the base of these columns is approximately doubled and an additional plastic hinge is induced just below the first floor beams. Clearly the increase in rotation above that sustained by the non-elongating model would increase dramatically with the number of bays in the frame.

The bowed shape induced in the columns resulting from the elongation of the beams increases the bending moments which induce flexural tension stresses on the outside faces of the external columns. The maximum bending moments in these columns at the beam faces at each level are given in Table 3. The negative sign is assigned to the bending moment which induces flexural tension on the external face of the column. The gravity load action adds to the positive bending moments in the lower levels and the negative bending moments in the upper levels of the columns in each storey.

The elongations which develop at each level during the passage of the earthquake are shown in Fig. 15. At levels 3 and 4 values of close to 175 mm
are sustained, while in levels 1 and 6 the corresponding elongation is of the order of 95 mm. For level 6 the portal type action associated with the gravity loads induces appreciable axial compression in the beam. This increases the flexural strength and reduces the elongation. At level 1 the proximity of the ground and the stiffness of the columns act to partially restrain the elongation.

The maximum values of the axial forces in the beams, as predicted from the analyses with the two models, are reproduced in Fig. 16. As noted in the previous paragraph the gravity loads induce axial compression in the level 6 beams and the reaction to this generates some axial tension at level 5. The predicted magnitudes of these actions at these levels are similar for both models. In the other levels the difference in the predictions of the two analyses shows the effects of elongation. Axial compression is induced in the beams at level 1, and a consequence of this and the resultant bowed shape of the columns, is that significant axial tensile forces are induced in the beams at levels two, three and four. The corresponding axial forces in the non-elongating model are small.

The maximum negative and positive bending moments acting in the beams are compared with the first yield bending moments (neglecting axial load) in Fig. 17(a) and the negative moment plastic hinge rotations in Fig. 17(b). For the negative bending moments the strain hardening causes approximately a 20 percent increase in the maximum moment that is sustained. This value is consistent with the high inelastic rotation demand, which develops in unidirectional plastic hinges. In this case the plastic hinge rotation demands reached $3.2^\circ$; a value which is close to the limit that can be sustained by a beam detailed to satisfy the requirements contained in the UBC-91 or the NZ Concrete Code-82 (1,12). In Fig. 17(c) the maximum vertical deflection sustained by the beams mid points at each level are shown. It can be seen that these are of the order of 200 mm over the height of the frame.

The general order of structural actions predicted to arise in this frame as a result of elongation are in agreement with values obtained from previous analyses on a three and a different six storey frame using a number of different earthquake ground motions(15).

**DISCUSSION AND CONCLUSIONS.**

1. Elongation occurs when a plastic hinge forms in a beam. Experimental and analytical studies show that this action, which is neglected in current design practice, has important implications for the seismic performance of ductile reinforced concrete frame structures.
(2) Two forms of plastic hinge may develop in the beams of ductile frame structures in a severe earthquake, namely reversing plastic hinges, where both positive and negative inelastic rotations are successively imposed on the same zone, and unidirectional plastic hinges where the negative moment inelastic rotations accumulate close to the column faces and positive moment rotations at regions in the span of the beam. With the unidirectional plastic hinges each inelastic displacement of the structure causes the plastic hinge rotations to increase in magnitude. A consequence of this is that unidirectional plastic hinges are required to sustain substantially greater inelastic rotation demands than reversing plastic hinges.

(3) Measurements made on several series of reinforced concrete beams, which were detailed to satisfy the seismic requirements in the UBC and NZ concrete design codes (1,12) showed that plastic hinges elongate by 2 to 4 percent of the member depth before strength degradation occurs.

(4) In unidirectional plastic hinges, the moment rotation and elongation characteristics can be predicted from conventional flexural theory using the stress-strain characteristics of the concrete and reinforcement. However, with reversing plastic hinge zones the situation is more complex. In this case allowance has to be made for the Bauschinger effect in the reinforcement, the contact effect in the concrete associated with the closure of the cracks, the effect of shear and the change in the stress-strain characteristics of the concrete with cyclic loading.

(5) Measurements on test beams show that if inelastic rotation is imposed in one direction and this direction is reversed, the cracks in the compression zone do not close unless more than a critical level of axial load is applied to the beam. The magnitude of the critical axial load depends upon the shear force being sustained by the plastic hinge and the dislocation of the aggregate particles at the cracks (contact effect). The extension of the reinforcement in the compression zone, from one cycle to the next, provides the major contribution to the elongation which occurs in reversing plastic hinges.

(6) Changing the ratio of the top to bottom longitudinal reinforcement, or adding a composite slab to the beam, was found to have very little effect on the elongation which develops with cyclic loading.

(7) A model of a potential unidirectional plastic hinge zone, which is suitable for use in time history analyses and which allows elongation effects to be modeled, is described.
A six storey three bay frame, which was required to provide lateral resistance for earthquake actions and resist gravity loading, was designed to comply with the New Zealand loadings code (12). This frame was modeled in two ways. In the first, elongation in the beam plastic hinges was included and in the second it was neglected. Comparing the results from time history analyses of these two models enabled the structural effect of elongation to be assessed.

The analyses indicated that for an earthquake, which induces the design level of ductility demand, elongation has important implications. In particular, elongation caused the maximum interstorey deflection and the plastic hinge rotations of the column base in the first storey to be doubled. The elongation predicted in the different levels varied from 85 to 175 mm. Such movements have important implications for the detailing of the supports for precast floor components and the external cladding.

ACKNOWLEDGEMENTS.

Financial support received from the New Zealand Concrete Society and the Earthquake and War Damages Commission (of New Zealand) is gratefully acknowledged.

REFERENCES


4. Wight, J.K. (editor), "Earthquake effects on reinforced concrete structures", American Concrete Institute, Special Publication SP84, 1985, 428 p.


14. Paulay, T., "Deterministic design procedure for ductile frames in seismic areas", in Reinforced Concrete Structures Subjected to Wind and Earthquake Forces, American Concrete Institute, Special Publication SP-63, 1980, pp. 357-382.

Table 1 - Details of Beam Tests

<table>
<thead>
<tr>
<th>Ref</th>
<th>Beam</th>
<th>Shear span (f) (m)</th>
<th>(1)</th>
<th>f_y (MPa)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1) A_{top} (mm)</td>
<td>(2)</td>
<td>f_y (MPa)</td>
<td>(3)</td>
<td>G</td>
<td>M_s (kN)</td>
<td>M/t (kN)</td>
</tr>
<tr>
<td>10</td>
<td>F1</td>
<td>0.923</td>
<td>5-D20</td>
<td>290</td>
<td>5-D20</td>
<td>290</td>
<td>0</td>
<td>30.0</td>
</tr>
<tr>
<td>10</td>
<td>F2</td>
<td>1.329</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>F3</td>
<td>1.735</td>
<td>280</td>
<td>-</td>
<td>280</td>
<td>-</td>
<td>0</td>
<td>34.7</td>
</tr>
<tr>
<td>10</td>
<td>F4</td>
<td>2.142</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>F5</td>
<td>1.329</td>
<td>5-D20</td>
<td>-</td>
<td>5-D16</td>
<td>298</td>
<td>0</td>
<td>27.7</td>
</tr>
<tr>
<td>11</td>
<td>T1,1</td>
<td>1.500</td>
<td>5-D20</td>
<td>311</td>
<td>5-D20</td>
<td>311</td>
<td>0</td>
<td>42.1</td>
</tr>
<tr>
<td>11</td>
<td>T2,2</td>
<td>-</td>
<td>3-D20</td>
<td>307</td>
<td>3-D20</td>
<td>307</td>
<td>0</td>
<td>37.6</td>
</tr>
<tr>
<td>11</td>
<td>T3,2t (8)</td>
<td>-</td>
<td>5-D20+10-D10</td>
<td>312</td>
<td>5-D20</td>
<td>312</td>
<td>0</td>
<td>33.4</td>
</tr>
<tr>
<td>12</td>
<td>T1,1</td>
<td>1.500</td>
<td>5-D20</td>
<td>317</td>
<td>5-D20</td>
<td>317</td>
<td>0</td>
<td>33.4</td>
</tr>
<tr>
<td>12</td>
<td>T2,2D (6)</td>
<td>-</td>
<td>-</td>
<td>312</td>
<td>-</td>
<td>312</td>
<td>0.034</td>
<td>29.5</td>
</tr>
<tr>
<td>12</td>
<td>T3,2D (6)</td>
<td>-</td>
<td>-</td>
<td>312</td>
<td>-</td>
<td>312</td>
<td>0.068</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>S1</td>
<td>1.500</td>
<td>5-D20</td>
<td>321</td>
<td>5-D20</td>
<td>321</td>
<td>0</td>
<td>34.0</td>
</tr>
<tr>
<td>15</td>
<td>S2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.145</td>
<td>295</td>
</tr>
<tr>
<td>15</td>
<td>S3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.039</td>
<td>36.8</td>
</tr>
<tr>
<td>15</td>
<td>M1</td>
<td>1.300</td>
<td>2-D28</td>
<td>317</td>
<td>2-D28</td>
<td>317</td>
<td>0</td>
<td>43.0</td>
</tr>
<tr>
<td>15</td>
<td>M2U(7)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(1) Top and bottom longitudinal steel.
(2) P is the axial load applied during the test.
(3) f_y is the yield stress of longitudinal reinforcement.
(4) Theoretical flexural strength based on Whitney stress block.
(5) Shear strength provided by stirrups as per ACI 318 Code (V_s = A_s f_y d/s).
(6) Dynamic tests carried out at a rate comparable to major earthquake.
(7) Unidirectional plastic hinge test
(8) Tee beam, see Fig.5.
Table 2  Principal Results of Gravity Load and Modal Analysis of Frame.

<table>
<thead>
<tr>
<th>Structural periods</th>
<th>Proportion of mass participating in mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 = 1.65s$</td>
<td>$M_1 = 0.824$</td>
</tr>
<tr>
<td>$T_2 = 0.51s$</td>
<td>$M_2 = 0.104$</td>
</tr>
<tr>
<td>$T_3 = 0.27s$</td>
<td>$M_3 = 0.041$</td>
</tr>
</tbody>
</table>

Deflection of top level $= 44.3 \text{ mm}$,
Deflection at top level times structural ductility factor $= 266 \text{ mm}$,
Maximum interstorey deflection $= 10.2 \text{ mm}$,
Maximum interstorey deflection times structural ductility factor $= 61.3 \text{ mm}$

(0.018 interstorey ht.)

<table>
<thead>
<tr>
<th>Beam flexural strengths (kNm)</th>
<th>Column flexural strengths (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>+ve</td>
</tr>
<tr>
<td>-------</td>
<td>-----</td>
</tr>
<tr>
<td>6</td>
<td>-423</td>
</tr>
<tr>
<td>5</td>
<td>-468</td>
</tr>
<tr>
<td>4</td>
<td>-545</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
</tr>
<tr>
<td>2</td>
<td>-567</td>
</tr>
<tr>
<td>1</td>
<td>&quot;</td>
</tr>
<tr>
<td>G</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

Table 3  Maximum Bending Moments (kNm) in the External Columns.

<table>
<thead>
<tr>
<th>Storey</th>
<th>Level</th>
<th>Non-elongation Model</th>
<th>Ratio of Elongating Non-elongating values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-ve</td>
<td>+ve</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>-481</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-90</td>
<td>390</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-462</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-113</td>
<td>463</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>-532</td>
<td>203</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-109</td>
<td>446</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-573</td>
<td>232</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-122</td>
<td>473</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-567</td>
<td>243</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-245</td>
<td>564</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-438</td>
<td>116</td>
</tr>
<tr>
<td></td>
<td>G</td>
<td>-423</td>
<td>429</td>
</tr>
</tbody>
</table>
Fig. 1  Reversing plastic hinges

(a) Sway to right

(b) Sway to left

(c) Bending moments
End of 1st, 2nd etc. cycles?

(d) Deflected shape

Fig. 2  Unidirectional plastic hinges

(a) Sway to right

(b) Sway to left

(c) Bending moments
End 1st cycle

2nd cycle

(d) Deflected shape
Fig. 3 Test arrangement used by Zerbe and Durrani (5, 6, 7).

(a) Beam dimensions and reinforcement details

(b) Instrumentation on beam

Fig. 4 Details and instrumentation for beams M1 and M2U
All beams except T, 3T, M1 and M2U

Beam T, 3T:

Fig. 5 Reinforcement details for beams

At prescribed ductility

- 1st. down ½ cycle
- 2nd. cycle
- 1st. up
- 2nd cycle

Measurements in this zone

Top bars
Bottom bars

DUCTILITY OF 2

For $\mu > 4$ the $i$ and $ii$ values are virtually identical for the uni-directional plastic hinge

DUCTILITY OF 4

DUCTILITY OF 6, 8 & 10

DUCTILITY OF 12

(a) Uni-directional plastic hinge

DUCTILITY OF 8

(b) Reversing plastic hinge

Fig. 6 Longitudinal strains in unidirectional and reversing plastic hinges
Fig. 7  Shear actions in a beam

Fig. 8  Elongation in the reversing and unidirectional plastic hinges in beam M1 and M2U

Fig. 9  Effect of varying M/Vd ratio on elongation
Fig. 10  Effect of differing $A_s$ to $A_s'$ ratios on elongation

Fig. 11  Effect of differing axial load levels in elongation
(a) Plastic hinge model

(b) Stress-strain properties of truss elements.

Fig. 12 The sub-assembly used to represent a unidirectional plastic hinge
Fig. 13 Idealised plan on six storey building

Fig. 14 Deflected shape envelopes for external columns in the frame models
Fig. 15  Development of elongation with time at different levels

Fig. 16  Maximum axial forces sustained by beams in the two frame models
Fig. 17  Maximum bending moments, plastic hinge rotations and vertical deflections in the beams of the two frame models.