PRECAST CONCRETE FLOOR SUPPORT AND DIAPHRAGM ACTION

A thesis submitted in partial fulfilment of the requirements for the

Degree of Doctor of Philosophy

by

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Supervised by

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Dedicated to:

My late father
Michael James Herlihy

My wife Marinka and our children
Kate, Dacre and the Bump
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Abstract

Experimental research, engineering analysis and theoretical developments comprise a study in which various interactions between ductile moment resisting frames and precast prestressed hollow core flooring have been examined.

The most critical interaction tested involves support behaviour, and the ability of reinforcing details to provide control against loss of support and possible catastrophic flooring collapse under dilation effects. Plastic hinge dilation, also known as elongation or growth, is an inherent property of ductile concrete members when subjected to cyclic plastic deformations. Hence, the performance of floor support details is enveloped by the general design philosophy of seismic resisting structures. In the experimental phase, emphasis was placed on testing support construction joints from contemporary building practice, for direct comparison with special support tie details of known capabilities. The contemporary details were found to exhibit seriously flawed behaviour under monotonic and cyclic loading regimes.

Corroborative experiments were undertaken to establish direct shear capacities between typical composite bond surfaces. In particular, these tests addressed the discrepancy that has emerged between direct shear and shear flow strengths. Also, the continuity response of conventional and proposed support detail types was examined.

A composite section model was analysed to demonstrate the likely influence of prestressing steel on beam bending strength within a ductile frame environment. Likewise, the probable effects of prestressing steel on beam plastic hinge development were examined, but on a more theoretical basis.

Other elements of theory have been presented. These mainly concern the general topic of elastic-plastic response in reinforced concrete elements. The particular focus of this work has been to demonstrate a rational basis to stiffness transition and plastic buckling analysis. The important role of stiffness degradation in dynamic analysis has also been examined.
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Chapter 1

\( A_c \)  area of concrete cross-section
\( A_p \)  cross-section area of prestressing steel at bottom of the section
\( A_{ps} \)  area of prestressed reinforcement in flexural tension zone
\( A_s \)  area of non-prestressed tension reinforcement
\( A_{vf} \)  area of shear-friction reinforcement
\( b_w \)  web thickness
\( c \)  clear concrete cover
\( C \)  compression in concrete, in equilibrium with \( T \)
\( d \)  distance from extreme compression fibre to centroid of tension reinforcement
\( d' \)  distance from extreme compression fibre to centroid of compression reinforcement
\( d_b \)  nominal diameter of prestressing strand or bar
\( d_c \)  distance from extreme compression fibre to centroid of prestressing steel
\( D \)  the diagonal vector sum reacting against tension components \( \Sigma T_x \) and \( \Sigma T_y \)
\( e, e_o \)  eccentricity of prestressing force
\( f'_{c} \)  characteristic concrete cylinder compressive strength
\( f_{ci} \)  concrete compressive strength at transfer of prestress
\( f_{cj} \)  mean concrete cube compressive strength at time \( j \)
\( f_{ck} \)  characteristic concrete cylinder compressive strength
\( f_{k_cub} \)  characteristic concrete cube (150 mm) compressive strength
\( f_{cki} \)  concrete cylinder compressive strength at transfer of prestress
\( f_{cm} \)  characteristic cylinder strength of mortar in a joint
\( f_{ct} \)  mean tensile strength of cast-in-place concrete or grout
\( f_{ctd} \)  \( f_{ck} / 1.4 \)
\( f_{ctf} \)  mean concrete tensile flexural strength
\( f_{ctfk,50} \)  the flexural tensile strength below which 5\% of all tests may be expected to fall
\( f_{ck} \)  characteristic concrete tensile strength
\( f_{ckk,50} \)  the compressive strength below which 5\% of all tests may be expected to fall
\( f_{ctf} \)  characteristic concrete tensile flexural strength
\( f_{ckj} \)  the characteristic concrete tensile strength at time \( j \)
\( f_{ctm} \)  the mean concrete tensile strength
\( f_{ctmj} \)  the mean concrete tensile strength at time of release
$f_{pc}$ compressive stress in concrete (after allowance for all prestress losses) at centroid of cross-section resisting all applied loads

$f_{pu}$ ultimate tensile strength of prestressing steel

$f_{si}$ initial (jacking) stress in prestressing steel

$f_y$ lower characteristic yield strength of non-prestressed reinforcement

$F_{ps}$ the effective tensile force of prestressing steel

$F_{st}$ tensile capacity of tie configuration

$h$ height, inter-storey height

$h$ total member depth (thickness)

$I$ second moment of inertia

$I$ structural importance factor

$I_c$ second moment of inertia of the concrete section

$k = 2I_c / hA_c$

$k$ ratio of the distances between beam plastic hinge zones to column centrelines (<1.0)

$\ell_{a+}$ additional development length in embedded cores

$\ell_b$ development length of deformed bar

$\ell_{crit}$ a length equal to the transfer length of prestressing strand

$\ell_d$ total development length

$\ell_p$ plastic hinge length

$\ell_s$ support length

$\ell_t$ transfer (transmission) length

$\ln$ natural logarithm

$L$ length of a structure or an element of structure

$L_d$ limit state factor for the serviceability limit state (=1/6)

$L_u$ limit state factor for the ultimate limit state (=1.0)

$M^*$ design bending moment at section at the ultimate limit state

$M_o$ decompression moment in prestressed member

$M_x$ bending moment in cross-section at a distance $x$ from support

$n$ number of pretensioned strands contributing to flexural tension reinforcement

$N^*$ design axial load at the ultimate limit state normal to the cross section occurring simultaneously with $V^*$ to be taken positive for compression, negative for tension, and to include effects due to temperature and shrinkage

$p_w = (A_s + A_{ps}) / b_w d$

$P$ prestress force

$P_o$ prestressing force immediately after release

$R$ risk factor for a structure

$S$ static moment of inertia of the section

$T$ tension in reinforcing steel in equilibrium with $C$
$T_{x,y}$  tension component in direction x or direction y as given

$u$  perimeter of core or alternately 2h if embedded in joint

$\nu_b$  basic shear stress

$\nu_{cn}$  nominal shear stress provided by concrete when cracking results from combined shear and flexure

$\nu_{cw}$  nominal shear stress provided by concrete mechanisms when diagonal cracking results from excessive principal tensile stress.

$V_{pc}$  shear forces due to prestressing components

$V^*$  design shear force at section at the ultimate limit state

$V_{Rd11}$  design value of shear force resistance in region cracked in flexure

$V_{Rd12}$  design value of shear force resistance in the region uncracked in flexure (i.e., web shear)

$V_{tp}$  shear force component caused by transfer of pretension force

$V_x$  shear force in cross-section at a distance x from support

$V_{tr}$  horizontal shear strength developed along a cracked construction joint

$W_{px}$  the weight supported by a diaphragm and the elements tributary thereto at a level

$x$  distance along member from support or end of member, (x axis)

$Z$  zone factor

$\alpha$  reduction factor for prestressing force with regard to end transfer

$\delta$  displacement

$\Delta_{ct}$  proposed limiting measure of free end slip of prestressed strands for quality assurance of flexural bond performance

$\Delta_{l_b}$  additional length of bar to allow deformation capacity of $\delta_u = \varepsilon_{lim} \Delta_{l_b}$

$\varepsilon_c$  strain in concrete at extreme compression fibre

$\varepsilon_{lim}$  the limiting steel strain for a tie

$\varepsilon_s$  strain in steel

$\varepsilon_{sp}$  spalling strain of concrete

$\varepsilon_y$  steel strain at yield

$\phi$  strand diameter

$\phi$  strength reduction factor

$\gamma$  factor of safety

$\gamma_c$  partial safety factor for concrete ($= 1.4$ for good quality control)

$\phi_p$  curvature in a plastic hinge

$\psi_u$  curvature in plastic hinge at design level of structural ductility

$\phi_y$  curvature at yield

$\mu$  structural ductility factor

$\mu_f$  coefficient of friction
\( \mu_p \) structural ductility factor used for the design of a part
\( \theta \) an angle, slope or rotation
\( \theta_d \) plastic hinge rotation at design level of structural ductility
\( \theta_Y \) plastic hinge rotation at yield
\( \rho_p \) prestressed reinforcement ratio
\( \sigma \) stress
\( \sigma_c \) compressive stress in concrete or joint
\( \sigma_{cp} \) average prestress in concrete
\( \sigma_p \) effective prestress in steel
\( \sigma_{po} \) stress in tendons immediately after release
\( \sigma_{sp} \) maximum spalling stress
\( \tau \) shear stress
\( \tau_{max} \) maximum shear stress in a mortar joint just before initiation of first crack
\( \tau_r \) residual shear stress in a cracked mortar joint
\( \tau_{RD} \) design value of resistant shear stress
\( \xi \) depth factor, \( = 1.6 - d \) (metres), but not taken less than 1.0 m

Chapter 2

\( a \) acceleration
\( A \) area
\( A_c \) area of concrete cross-section
\( A_s \) area of reinforcing steel
\( A_{vf} \) area of shear-friction reinforcement
\( b \) width of concrete compression block
\( B \) width of a structure or an element of structure (breadth)
\( c \) distance from extreme compression fibre to the section neutral axis
\( d \) distance from extreme compression fibre to centroid of tension reinforcement
\( d' \) distance from extreme compression fibre to centroid of compression reinforcement
\( e \) exponential function (\( \approx 2.7183 \))
\( E \) modulus of elasticity
\( EI^* \) equivalent flexural rigidity
\( E_c \) elastic modulus of concrete
\( E_s \) elastic modulus of steel
\( f'c \) characteristic concrete cylinder compressive strength
\( f'_{ci} \) concrete compressive strength at transfer of prestress
\( f_c \) concrete compression stress
$f_{cr}$: modulus of rupture (stress at first cracking)
$f_s$: steel stress
$f_y$: lower characteristic yield strength of non-prestressed reinforcement
$F_{ps}$: the effective tensile force of prestressing steel
$g$: gravitational acceleration ($\approx 10\text{m/s}^2$)
$h$: height, inter-storey height
$H$: height function, total building height (storeys $0\rightarrow n$)
$I_{cr}$: second moment of inertia based on cracked elastic section analysis
$I_e$: effective second moment of inertia
$jd$: lever arm of moment resisting section
$k$: ratio of the distances between beam plastic hinge zones to column centrelines ($<1.0$)
$K$: structural stiffness, stiffness matrix
$K_N$: interstorey stiffness after $N$ damaging load cycles have elapsed
$K_0$: initial interstorey stiffness
$l$: distance between column centres
$l'$: effective interstorey distance
$l'_{\text{min},\pm}$: minimum effective length of beam between opposing plastic hinges
$L$: length of a structure or an element of structure
$m$: mass per unit length
$M$: bending moment
$M$: mass, mass matrix
$M_{oxy}$: vector sum of overstrength moments
$M_p$: bending moment in a plastic hinge zone (corresponding to $\varphi_p$)
$M_y$: bending moment at first yield (corresponding to $\varphi_y$)
$n$: number of strands contributing to flange tension under negative bending moment
$n$: modular ratio
$N$: number of load cycles (damage cycles) causing loss of horizontal stiffness at storey level
$N_{oxy}$: couple force corresponding to vector sum of overstrength moments $M_{oxy}$
$P$: force
$q_{\text{tr}}$: force developed in shear friction per unit length of beam
$s$: spacing of starter reinforcement
$s_{x,y}$: required spacing of starter reinforcement in respective $x$ or $y$ direction
t  
T  

a period of vibration

u  

strain energy density (strain energy per unit volume of material

U_b  

strain energy due to bending

V  

shear force

V'  

nominal shear force at a plastic hinge

V_{rr}  

horizontal shear strength developed along a cracked construction joint

x  

distance in direction of x axis

y  

distance in direction of y axis

y  

distance to neutral axis in cracked elastic section analysis

z  

distance from critical section to the point of contraflexure

Z  

section modulus

α  

structural sway constant (relationship between mass and stiffness)

β  

bisexion angle of beams intersecting at a column (for beams at right angles, $\beta = 45^\circ$)

δ  

displacement

δ_b  

maximum amplitude of displacement at base of structure

δ_o  

displacement response with nil resonance

Δ  

inter-storey deflection

ε_c  

strain in concrete at extreme compression fibre

ε_o  

concrete strain at peak compression stress

ε_s  

strain in steel

ε_sp  

concrete spalling strain

ε_y  

steel strain at yield

φ  

strength reduction factor

η  

shape factor (cumulative distribution of building mass with height)

φ  

curvature

φ_p  

difference between plastic and yield curvatures in plastic hinge zone ( $\varphi_p - \varphi_y$)

φ_p  

curvature in a plastic hinge

φ_u  

curvature in plastic hinge at design level of structural ductility

φ_y  

curvature at yield

κ^2  

separation constant for solution to PDE

μ_f  

coefficient of friction

ν  

angle made between force $N_{oxy}$ and the corner bisexion angle $\beta$

θ  

an angle, slope or rotation (building angle of drift)

θ'  

angle of beam plastic hinge rotation in side-sway mechanism

θ_u  

plastic hinge rotation at design level of structural ductility

θ_y  

plastic hinge rotation at yield
\( \sigma \) stress
\( \omega \) circular frequency of vibration
\( \Psi_\Delta \) strain energy per unit length due to inelastic portion \( (= M_p \Phi_\Delta) \)
\( \Psi^* \) sum of strain energy per unit length of contributing portions of a section
\( \Psi_c \) strain energy per unit length in concrete portion
\( \Psi_{se} \) elastic strain energy per unit length in steel portion
\( \Psi_{sp} \) plastic strain energy per unit length of steel portion
\( \zeta \) stiffness reduction coefficient (damping)

Chapter 3

\( a \) transformed area of concrete section
\( A_c \) area of topping bond interface
\( A_{pc} \) gross area of precast section
\( A_s \) area of non-prestressed tension reinforcement
\( c \) depth from neutral axis to extreme compression fibre
\( d_b \) nominal diameter of prestressing strand or bar
\( e \) eccentricity of pretension force
\( E_c \) elastic modulus of concrete
\( f_t \) principal tensile stress
\( f_c \) concrete stress
\( f_y \) yield stress of reinforcement
\( f_t \) reinforcing steel stress
\( f_c \) compressive strength of concrete
\( f_{tx} \) axial tensile stress in hollow core section at fracture
\( F_p \) force in prestressing steel
\( F_s \) force in non-prestressed steel
\( G \) modulus of rigidity \( (= 0.4E) \)
\( h_l \) topping thickness
\( H \) overall height (thickness) of member
\( L \) length
\( L_g \) gauge length
\( M \) bending moment
\( P \) axial tension force applied by test rig
\( P_{ef} \) effective (final) pretension force after losses
\( P_i \) initial pretension force (before transfer)
$P_{pc}$  force in precast section
$t$  effective thickness (depth) of shear volume
$u$  bar bond stress
$u'$  strain energy density (strain energy per unit volume)
$\bar{u}$  average bar bond stress
$U_e$  external work done
$U_i$  internal strain energy
$U_{pc}$  strain energy stored in precast section
$V_n$  shear force transferred by shear friction across a construction joint
$V_{se}$  effective embedded volume of reinforcement
$\hat{y}_s$  distance from section bottom fibre to centroid of strands
$\hat{y}_x$  distance from section bottom fibre to neutral axis
$Z$  section modulus

$\Delta L$  change in length
$\varepsilon_u$  ultimate strain
$\varphi$  curvature
$\theta_p$  plane of principal stress, relative to plane of axial tensile stress
$\tau$  shear stress
$\tau_b$  topping bond shear stress

Chapter 5

$A$  area
$A_b$  steel bar area
$A_s$  area of reinforcement
$b$  width of slab
$c$  depth from neutral axis to extreme compression fibre
$C$  end condition constant
$d_b$  nominal diameter of prestressing strand or bar
$E_c$  elastic modulus of concrete
$E_s$  elastic modulus of steel
$f_c$  concrete stress
$f_c'$  compressive strength of concrete
$f_r$  modulus of rupture
$f_s$  reinforcing steel stress
$f_y$  yield stress of reinforcement
h  topping thickness
H  overall height (thickness) of member
I  second moment of area
L  Length
L_{cr}  critical buckling length
L_{Ecr}  critical buckling length calculated by Euler theory
M_{cr}  flexural cracking moment
M_e  bending moment corresponding with elastic buckling
M_p  plastic bending moment
M_{\Theta(i)}  Moment capacity of the i^{th} plastic hinge
n_f  number of fixed ends
n_\theta  number of plastic hinge rotations in buckling mechanism
N^s  design axial force
r  radius of gyration
S  plastic section modulus
u_c  strain energy density (strain energy per unit volume) of concrete
u_s  strain energy density (strain energy per unit volume) of steel
U_b  strain energy due to bending
U_c  strain energy due to axial compression
V  volume (concrete or steel reinforcement)
y  distance to centroid
Z  elastic section modulus

\delta  displacement
\delta_{cr}  critical displacement
\delta_{ie}  critical displacement of intermediate elastic members
\delta_p  critical displacement of plastic members
\delta_{se}  critical displacement of slender elastic members
\delta^*  design critical displacement
\varepsilon_c  concrete strain
\varepsilon_o  strain corresponding with peak stress
\varepsilon_s  steel strain
\varepsilon_u  ultimate strain
\varepsilon_y  yield strain of steel
\phi  curvature
\lambda  length division factor: 1 for sway members, 2 for braced members
\sigma  average stress
Chapter 6

A bond surface area
$\Gamma_c'$. concrete crushing strength
$V_h$ horizontal shear force

Chapter 7

$A_s$ area of reinforcing steel
$b$ width of concrete compression block
d distance from extreme compression fibre to centroid of tension reinforcement
$E$ elastic modulus
$\Gamma_c'$ measured concrete cylinder compressive strength
$f_y$ measured yield strength of reinforcement
$I$ second moment of area
$K_0$ rotational stiffness (from test data)
l_p plastic hinge length
$L$ Length
$M$ bending moment
$M^*$ design bending moment in the Ultimate Limit State
$M_p$ plastic bending moment
$M_y$ first yield bending moment
$w$ weight per unit length
$z$ distance from critical section to the point of applied force (shear span)

$\delta$ displacement
$\epsilon_c$ strain in concrete at extreme compression fibre
$\epsilon_{pu}$ concrete strain at peak compression stress
$\epsilon_s$ steel strain
$\epsilon_y$ steel strain at yield
$\phi_p$ curvature in a plastic hinge
$\phi_y$ curvature at first yield
$\phi_\Delta$ difference between plastic curvature and yield curvature in plastic hinge zone
$\theta$ rotation at support
$\theta_p$ rotation at plastic bending moment
$\rho$ rotation at simple induced by applied loads
$\Psi_p$ sum of strain energy per unit length at bending moment $M_p$
$\Psi_\Delta$ strain energy per unit length due to inelastic portion ($= M_p \cdot \phi_\Delta$)
Chapter 8

\( A_b \) area of bar
\( d_b \) bar diameter
\( E \) elastic modulus
\( f_{ck} \) cylinder crushing strength
\( f_s \) steel stress
\( f_y \) steel yield stress
\( I \) second moment of area
\( l_d \) development length
\( L \) total embedment length of bar
\( M \) bending moment
\( M^* \) design bending moment in the Ultimate Limit State
\( n \) shape coefficient for bond capacity function
\( N \) normal force due to curvature
\( q \) bond capacity
\( q_u \) ultimate bond capacity
\( s \) slip
\( T \) tension force
\( T_o \) initial tension force
\( T_{5d_b} \) reduced tension force at five bar diameters (\( 5d_b \)) embedment average bond stress
\( V \) shear force
\( V_o \) initial shear force
\( w \) weight per unit length
\( x \) length along bar

\( \alpha \) shape coefficient for bond-slip function
\( \mu \) friction coefficient
\( \rho \) rotation at simple induced by applied loads
\( \tau \) bond stress
\( \tau_{max} \) maximum (peak) bond stress
\( \Sigma_0 \) sum of surface area of bar per unit length
1

Introduction

1.1 THE USE OF PRETENSIONED FLOORS IN BUILDINGS

1.1.1 BACKGROUND

The first instance of pretensioned floor slab manufacture appears to be connected with the experimental work of the German engineer B. Hoyer at Braunschweig Technical University in the mid 1930s. The technique of long-line prestressing that has since evolved is synonymous with the pioneering work of Hoyer, and is often referred to as the Hoyer system. Typically, this method involves the pretensioning of prestressing steel through a form work that is placed between bulkheads. Concrete is cast into the form around the prestressing steel and is cured until a degree of compressive strength is attained. At this point, the force sustained by the prestressing steel is released and transferred to the formed concrete section. The innovative significance of Hoyers system, in relation to contemporary techniques employed in prestressed concrete, was the method of force transfer. The Hoyer system relies on the establishment of transfer bond stresses at the interface of the prestressing steel and the surrounding concrete matrix without the specific use of anchorages. Provided that sufficient transfer bond between the prestressing steel and concrete elements is attainable, this system becomes ideally suited to the mass production of precast units. With respect to buildings, the Hoyer, or long-line system, is especially suited to the manufacture of modular flooring units of uniform sections and load carrying requirements.

The experimental slabs constructed by Hoyer were 50 mm thick and 1.2 m wide, and a multiple number of 2.0 mm diameter cold-drawn high strength piano wires were used for prestressing steel. Such small diameter wires were necessary because of the high bond stresses demanded by wire with a typical ultimate strength of more than 2400 MPa. The experiments concluded that pretensioned slabs are strong, flexible and durable. Unfortunately, an excessive number of piano wires were required to construct a reasonable span, making this system an interesting concept but rather impractical for economical deployment in construction. However, it is evident that pretensioning was used in Germany during the second world war period in the construction of submarine bases along the Atlantic and North Sea coasts.

1.1.2 THE COMMERCIAL DEVELOPMENT OF PRETENSIONED CONCRETE

It was not until about 1950 that pretensioning began to expand into the commercially successful industry as we know it today. A number of North American engineers had long recognised the potential of this form of construction, and had followed pre-war developments in European...
prestressing technology, the work of Freyssinet, Magnel, Hoyer and Abeles. However, it was a United States invention that provided the genuine means of economical and dependable pretensioning, and that was the development of seven wire stress relieved prestressing strand by the John A. Roebling & Sons Corporation.

Concurrently, the building industry across the United States was in a boom situation. The demand for longer floor spans in the absence of a reliable structural steel supply became an influential factor in the creation of a pretensioned concrete industry. As a result of the demonstrated viability and cost effectiveness of pretensioned concrete products, investors were guided towards enhancing the existing concrete industry operations by promoting and undertaking this form of construction. The commercial market remained buoyant and specifiers became increasingly enthusiastic about the use of pretensioned concrete. Thus, engineers were encouraged to diversify and refine applications, and to explore options such as the development and standardisation of sections. Consequently, many pretensioned flooring products were devised that went on to become industry standards such as double tees, single tees, flat slabs and joist-infill systems.

The first extruded hollow core flooring produced in the United States was probably in 1954 and this was achieved using modified German extrusion plant. Early precast concrete manufacturers had also attempted to produce non-extruded hollow core units using either paper voids or systems involving core forming dies that were drawn through the concrete by external machinery. In the late 1950s, successful extruded hollow core plant was in active service but it was mostly in the 1960s that the extruded hollow core flooring products were developed to their potential.

General references on the advent of pretensioning in North America are contained in the book “Reflections on the Beginnings of Prestressed Concrete in America” [PCI, 1981].

Although New Zealand engineers generally recognise that most of the fundamental advances in prestressed concrete technology are European in origin, the development of pretensioned concrete in New Zealand has been more strongly influenced by the North American experience. Moreover, it can be said that the backgrounds of commercial development of pretensioned concrete in North America and New Zealand have a similar economic theme. As neither the North American continent or New Zealand were physically attacked during the second world war, post-war construction was driven almost entirely by boom economy investment in contrast with the immediate civil reconstruction needs of post-war European countries.

Like their counterparts in America, the early entrepreneurs in New Zealand recognised the potential of prestressed concrete as a diverse and competitive construction material. However, pretensioned flooring manufacture in New Zealand did not develop as dramatically as other products such as pretensioned piles, power poles and bridge beams. This was partly due to the small population of New Zealand coupled with a strong tradition of timber frame housing on freehold land titles, which tended to limit the use of precast flooring to projects other than domestic construction. Another very influential factor was that much of the readily accessible money for construction came through local and central government expenditure and not from

Chapter 1: Introduction: The Use of Pretensioned Floors in Buildings
the financial backers of private development. This generally resulted in a demand for products that are synonymous with an expanding infrastructure, such as power poles and bridge beams.

By the mid 1960s, pretensioned flooring had gained more momentum in the construction market and during that decade utilisation expanded to include most of the standardised sections that had originated in North America. Extruded hollow core flooring systems, however, did not appear on the New Zealand construction scene until the latter half of the 1970s.

The 1980s saw a major building boom in New Zealand that resulted in a considerable attrition of construction methods, with one of the critical requirements becoming speed of erection on a short lead time. This resulted in a tendency towards multi-storey buildings constructed from precast concrete frame elements and occasionally from precast wall elements; a trend inspired by some creative structural designers that proved to be a very efficient form of construction [O'Grady, 1988; Wood, 1988]. Moreover, the mass production capabilities, cost effectiveness and structural efficiency of hollow core flooring made this product the ideal collaboration in suspended flooring.

The increasing success of pretensioned one-way systems over cast-in-place floors may also be attributed to market forces that have existed in New Zealand for some time. Because of the general non-availability of quick reusable forms and the continual high cost of ordinary formwork and labour, cast-in-place floors appear to have become part of the attrition process. In the authors own experience, the last cast-in-place floor tendered against was in 1988 when a precast alternative of 250 mm double tees was accepted on a medium-rise frame structure that was detailed as a cast-in-place slab with secondary beams.

In the present decade, pretensioned flooring has maintained its strong position in New Zealand construction, and this should continue as such. However, the concrete industry in the 1990s is generally showing more diversity than earlier decades and there also appears to be better integration of ideas than was previously demonstrated. The use of cast-in-place systems will always be an option to designers, and there has been a resurgence of post-tensioned floors in certain parts of the country as well as some proposals of lift-slab construction. The construction industry infrastructure has, though, become very much geared toward the erection of precast flooring systems. In the authors opinion, it would take an impressive swing of the pendulum to change this construction culture in New Zealand.

1.1.3 APPROPRIATE DESIGN STANDARDS

An important ingredient that prompted New Zealand engineers into quickly adopting the American state-of-the-art in pretensioning (besides its suitability and extensiveness) was obviously language. The proceedings from ACI-ASCE committees that produced substantiated code requirements were immediately comprehensible to New Zealand engineers, and the use of English units of measurement was agreeable as well. The ACI 318 codes, in particular ACI 318-63, were to become standard documents that covered concrete design in New Zealand during the 1960s. By 1968, the Standards Association of New Zealand had produced a Standard Recommendation for prestressed concrete NZSR 32:1968 [SANZ, 1968]. Although this
document made extensive use of existing American, British and Australian guidelines, it most importantly established prestressed concrete in context with contemporary New Zealand standards relating to construction materials and key chapters (i.e., chapters 8, 9 and 11) of the then current Model Building Bylaw, NZS 1900.

NZSR 32: 1968 was not superseded until the publication of the comprehensive New Zealand concrete structures standard “Code of Practice for The Design Of Concrete Structures” NZS 3101: 1982, parts 1 and 2 [SANZ, 1982]. This standard devoted a chapter to prestressed concrete and included information in its references from both broader and more specific sources than the previous document. This is indicated by referrals to European (CEB-FIP) material and recommendations from New Zealand research on seismicity in post-tensioned frames that is associated with Professor R Park. The issue of seismicity in the prior standard NZSR 32: 1968 formed perhaps the shortest section in the entire document (section 13) and is quoted thus: “Firm recommendations to cover seismic design of prestressed concrete structures have not yet been established”. The use of pretensioned flooring in seismic resisting buildings is implicit in this statement.

The new order of construction that emerged during the 1980s often saw precast frame and wall elements replace cast-in-place construction, and because of cost effectiveness an almost complete preference developed for pretensioned flooring systems. A number of the provisions in the 1982 concrete standard were challenged by precast elements because this standard was, of course, developed on the basis of cast-in-place concrete construction. The cast-in-place design philosophy also embraced many of the assumptions relating to the generalised performance of structural diaphragms. In particular, the requirements for diaphragms were briefly considered in the section of the code that covered structural (shear) wall construction. Although this standard provided reasonable guidance for the consideration of actions that may arise in a diaphragm, the provisions for detailing were not of a comprehensive nature. This has been noted in a compilation text [Booth (Ed.), 1994] that compares United States, New Zealand and Japanese practices, which quotes on diaphragms thus: “The detailing requirements given in the concrete code (i.e., NZS 3101: 1982) are minimal compared with those for other elements”.

In recognition that there was a need for some congruity in the use of precast concrete elements as functional parts of seismic resistant structures, a joint study group was appointed by the New Zealand Concrete Society (NZCS) and the New Zealand National Society of Earthquake Engineering (NZNSEE) in the latter 1980s to address this situation. The outcome of this joint study group was the publication of a useful text containing applicable guidelines (though not code requirements) for general precast concrete construction [NZCS-NZNSEE, 1991]. In this publication, separate chapters have been included for both the use of pretensioned flooring and the distinctive role of floor diaphragms in seismic resistant buildings.

In 1995, the revised New Zealand concrete design standard was published [Standards New Zealand, 1995], and this comprehensive document has incorporated standard requirements that are relevant to precast concrete construction. A separate chapter has also been provided for the specific requirements of diaphragms, which includes recommendations for methods of analysis and general considerations of design actions.

Chapter 1: Introduction: The Use of Pretensioned Floors in Buildings
Although there is an abundance of design information that is available from standards and other notable references, much of the onus in prestressed concrete design will remain with the designers. It would be extremely difficult, if not impossible, to provide a standardised set of design rules that can encompass all aspects of prestressed concrete design and construction. This is largely due to the nature of prestressed concrete itself. Prestressed components may appear in a multitude of forms with each requiring its own levels of design and manufacturing input, considerations of construction materials, construction procedures on site and considerations of elastic behaviour and ultimate limit state performance. Perhaps due to the reward that comes from extra effort, prestressed concrete has traditionally been associated with those engineers who are willing to work from first principles as the requirements dictate.

1.1.4 PRETENSIONED FLOOR SYSTEMS IN COMMON USE

1.1.4.1 General

The concrete industry in New Zealand has developed and popularised a diverse array of building applications and products in recent years. As a result, a greater proportion of domestic dwellings are constructed from concrete elements such as tilt panels, masonry blocks and expanded polystyrene (EPS) systems with infilled concrete cores.

Table 1.1  Typical simply supported span-to-depth ratios of commonly used pretensioned concrete members [after Collins and Mitchell, 1987]

<table>
<thead>
<tr>
<th>Type of element</th>
<th>Live load (kPa)</th>
<th>Span/depth ratio (l/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>less than dead load</td>
<td>40</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>2.4</td>
<td>40-50</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>4.8</td>
<td>32-42</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>2.4</td>
<td>20-30</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>4.8</td>
<td>18-28</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>2.4</td>
<td>23-32</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>4.8</td>
<td>19-24</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>less than dead load</td>
<td>30</td>
</tr>
</tbody>
</table>
These forms of wall construction immediately facilitate the use of pretensioned flooring units and in many situations (especially with tilt panels) a suspended concrete floor fully complements the design of the supporting wall structure by providing a rigid diaphragm element. At this lighter end of the construction market the floor requirements are often influenced by the relative geometric complexities of house construction. In this role, pretensioned flat slab units and joist-infill systems have proved most effective.

Flat slabs and joist-infill systems have also been widely used in commercial construction, however, there is a limitation with these systems when significant spans are desired without the additional expense of having to provide temporary shoring. In this role extruded hollow core systems are ideal, allowing longer spans to be achieved at the optimum of economy. Where long spans are required to support more intense loads (especially from shear or impact) either double tee or single tee sections may become the more appropriate option to the designer. Pretensioned tee sections also allow the use of partial prestress techniques which can improve both the flexural performance and economics of the section.

1.1.4.2 Flat Slabs

The typical pretensioned flat slab that is consistently manufactured in New Zealand is commonly known as Unispan and has a section that is 1.2 m wide and 75 mm deep (Figs. 1.1 and 1.2). Various thicknesses of composite topping can be placed to facilitate particular load and span requirements and also the addition of continuity reinforcement over the supports. This system almost always requires either one or two rows (depending on span) of temporary shoring that is positioned before the cast-in-place topping is added (Fig. 1.3). This shoring acts to pre-set a construction camber into the floor for deflection control and alleviation of bottom fibre tension stresses under service loads.

The thickness of flat slabs may be increased to form either stiffer composite floors or pre-finished flooring units that do not require temporary shoring. Where a composite topping is not employed, special detailing must be considered in order to achieve dependable diaphragm performance under seismically induced forces [Standards New Zealand, 1995].

Pretensioned flat slabs are generally favoured in short-span domestic and light commercial construction due to their efficiency in terms of delivered cost, relatively light weight and simplicity to erect. Because they are shored at construction it is not difficult to achieve a uniform soffit that may be either lined, textured or painted, thus permitting a minimum of compartment head-room without compromising fire resistance ratings and sound insulation properties. One of the features of flat slabs is that they permit inexpensive chases and rebates to be formed for floor penetrations. Raked and curved end seatings can also be accommodated without too much difficulty, which adds to flat slab popularity as suspended flooring for architecturally designed houses.

Provided that sensible load-span ratios are chosen and that on-site construction is properly carried out, such as the temporary shoring and cast-in-place topping, pretensioned flat slabs are an ideal flooring element. The upper surface of flat slabs can be broomed to achieve a good
texture for composite bond with the cast-in-place topping, and tie reinforcement (typically R6 spirals) can be incorporated to enhance the interface bond capacity. The generally low magnitude of axial prestress in flat slabs coupled with small internal lever arms results in a composite section that exhibits stable behaviour under long-term creep and shrinkage actions.

Fig. 1.1  Typical composite section of pretensioned flat slab (Unispam) flooring

Fig. 1.2  Manufacture of pretensioned flat slabs
1.1.4.3 Joist-Infill Systems

Although joist systems involving precast concrete infill blocks have been used in New Zealand construction, progressive refinement has led to the general use of pretensioned concrete joists with sawn timber infills in combination with a cast-in-place concrete topping. The sections of pretensioned joists that are commonly manufactured in New Zealand (e.g., Fig. 1.4) only differ in the basic dimensions of the joist units themselves and the nominal configurations of composite ties. These sections are designed on the assumption of a one-way joist flooring system and certain alleviations are permitted for these joist floors with respect to shear reinforcement, provided that specified dimensional and spacing limitations are adhered to [Standards New Zealand, 1995].

![Diagram of joist-infill section](image)

**Fig 1.3** Temporary shoring of a suspended floor constructed from flat slabs

**Fig 1.4** Typical joist-infill section used in New Zealand construction

*Chapter 1: Introduction: The Use of Pretensioned Floors in Buildings*
The joist units may be manufactured to any suitable depth which usually ranges between 75 mm and 250 mm deep. At construction, temporary shoring is necessary to pre-set a construction camber and residual compression stresses into the bottom fibre of the precast unit similarly to pretensioned flat slab units. Although joist-infill systems are more efficient than flat slabs in terms of floor self weight and span capabilities, the structural advantage of joist-infill systems is often offset by additional materials and labour costs associated with manufacture and on-site construction.

Joist-infill systems are well suited to floors where difficulties arise from seating layouts, point loads and penetrations. Because only the discrete length of the joist unit needs to be considered, geometrically difficult layouts are often more manageable than with slab systems where a reasonably accurate end seating profile must be formed. Where significant point loads or additional superimposed patch loads are supported by the floor, joists may be conveniently grouped together under the load to provide for the increased demand in floor capacity. Another distinct advantage with joist-infill systems is for floor layouts that require many small and seemingly random penetrations for services, such as with hospital buildings. These types of penetrations can be better managed on-site by simply locating and forming these duct holes through the timber infills before the topping concrete is placed.

An important design consideration with joist-infill systems is the proportion that the composite topping makes to the finished composite section. The ratio of the composite section second moment of inertia to that of the bare prestressed rib may be in the order of twelve or more. The relatively large lever arm that may occur between the centroid of the topping slab and the composite section axis can result in excessive long term deflections due to shrinkage strains in the topping element. The possible effects of these separate section portions must always be considered in the design of joist flooring.
1.1.4.4 **Double Tee and Single Tee Units**

Before the advent of hollow core flooring, tee sections were the most popular pretensioned flooring unit for constructing long spans and for situations involving larger superimposed floor loads without the need for temporary shoring. In New Zealand, the double tee unit has been most commonly used, although single tee units have appeared on occasion and have proven to be an effective unit for longer span applications.

![Diagram of a double tee section with composite topping](image)

**Fig. 1.7** Typical metric double tee section with composite topping

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A useful feature of tee sections is that partial prestress methods may be used and this can result in a more efficient design in certain situations. Partial prestressing may also compensate for using less pretension force in the section, thus producing a unit which has less tendency to creep than a fully pretensioned section. This is especially beneficial for longer spans where the impact of volume changes upon the support structure must always be considered. Because of the relative size of tee sections, creep and shrinkage movements in particular have been known to cause problems with relatively flexible support elements such as walls.

With tee floors it is possible to include cast-in connection details, which makes possible the use of these units in untopped floor diaphragms. Additional tie reinforcement may also be included to enhance the composite bond between the precast unit and topping concretes. Because of the end seating configurations available it is not difficult to detail tee units for additional shear force or support restraint actions that may arise.

Many of the major difficulties with double tee units occur at the manufacturing stage. In floor layouts that involve irregular floor penetrations it can be a difficult detailing exercise getting the units to line up with the penetrations without interfering with the tee legs. Placing rebates in these units can likewise cause difficulties at the production stage. The compounding cost of moulding time and the generally higher labour and raw materials content required in tee manufacture can quickly make this type of unit less attractive than hollow core flooring systems. With single tee units, storage and transportation must always involve support frames to keep the units upright. This requirement can cause logistical problems where single tee units are to be stored in the manufacturers yard, and can be especially difficult when units need to be set down on the construction site.

Fig. 1.8 Single tees used in the construction of the new Civil and Mechanical Engineering buildings at the University of Canterbury School of Engineering (1996).
1.1.4.5 Extruded Hollow Core Flooring

Extruded hollow core is probably the most economical of the longer span pretensioned flooring sections available. An automated production process coupled with the flexural efficiency of prestressed hollow core sections results in a flooring element that is ideal for the repetitive production runs demanded by many of the larger building projects. Because of the so called "zero slump" concrete that is required for the extrusion process, early release strengths are obtainable, which adds to the mass production capabilities of hollow core flooring.

There are, however, some aspects of hollow core units that are potentially not so ideal as their raw production advantage. The most obvious is that the extrusion process does not allow for the inclusion of shear reinforcement in the webs of hollow core units, and another important point is that the surface of extruded concrete is difficult to significantly roughen by traditional brooming methods. These characteristics are unique to extruded products and may be either emphasised or discarded (in terms of structural importance) depending on the particular usage that the flooring unit is put to. However, these facets surrounding the construction of hollow core units will always remain as important design considerations and form the basis of much of this thesis. Extruded hollow core flooring is the subject of further discussion in the following section.
1.2 PRECAST PRESTRESSED HOLLOW CORE FLOORING

1.2.1 GENERAL

1.2.1.1 Cross Sectional Sizes

A variety of extrusion machines have been used in New Zealand since hollow core manufacture began in about 1976. The earlier plant tended to be either “Dycore” or “Stresscore” extruders but these brand names have been superseded in recent times by the “International Hollow Core Engineering (IHE)” and “Partek” machines. Differing forms of cross-section may be associated with the individual brands of extrusion plant, however, the fundamentals of concrete extrusion apply in a general way to all of the typical hollow core machines used in New Zealand.

A number of cross-section depths have been made available (Fig 1.9), with the most commonly used sizes being 200 mm and 300 mm deep. However, units as shallow as 150 mm are also manufactured, and a 400 mm deep unit was introduced into the Auckland market by Firth Industries during April of 1996.

Fig. 1.9 Typical hollow core sections produced by Firth Industries
Fig. 1.9 (continued) Typical hollow core sections produced by Firth Industries

Table 1.2 Section properties corresponding to cross-sections shown in Fig. 1.9

<table>
<thead>
<tr>
<th>Unit</th>
<th>Area (m²)</th>
<th>Y_b (mm)</th>
<th>I_x (m⁴)</th>
<th>Self Wt. (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 Dycore/Partek</td>
<td>0.1192</td>
<td>100</td>
<td>0.00065</td>
<td>2.4</td>
</tr>
<tr>
<td>200 Stresscore</td>
<td>0.1264</td>
<td>99</td>
<td>0.000629</td>
<td>2.6</td>
</tr>
<tr>
<td>250 Stresscore</td>
<td>0.1336</td>
<td>125</td>
<td>0.001123</td>
<td>2.8</td>
</tr>
<tr>
<td>300 Dycore/Partek</td>
<td>0.1606</td>
<td>153</td>
<td>0.00204</td>
<td>3.2</td>
</tr>
<tr>
<td>300 Stresscore</td>
<td>0.1718</td>
<td>145</td>
<td>0.00199</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring
As with all flooring systems used in New Zealand, hollow core units are required to satisfy a number of serviceability and ultimate limit state performance criteria. The serviceability requirements associated with pretensioned floors will almost always include such obvious items as the control of flexural stresses and short-term and long-term deflections, although acoustic, thermal and vibration properties may also need to be considered in the serviceability checks [Standards New Zealand, 1992].

1.2.1.2 Design for Comfort and Fire Safety

As part of a building space that is designed for human occupancy, hollow core units need to comply with certain requirements relating to comfort and safety that are distinct from purely structural behaviour considerations. The four main categories involve three serviceability requirements relating to comfort which are Vibration Behaviour, Acoustic Properties and Thermal Properties (ambient), along with the ultimate limit state category of Fire Resistance.

(a) Vibration Behaviour

For typical applications and span lengths of pretensioned flooring systems there appears to be no need to overly consider the problems of vibration due to vertical accelerations caused by foot traffic and seismic actions. This is mainly due to the relative stiffness of the floor component, which is usually achieved in design work by a sensible choice of span-to-depth ratio. However, with pretensioned floors there is a real temptation for the designer to push the span out. Hence, where long design spans employ continuity reinforcement to allow redistribution of design bending moments, care should be exercised to ensure that the floor proportions are not decided on the basis of strength alone. This may be especially true when there is a potential for vibration stimuli within the structure, or there is little by way of effective damping in the form of partition walls, etc.

With longer spans, the natural frequency of hollow core units with composite topping may coincide with the broader range of characteristic frequencies that are generated by human activities (typically 2-8 Hz). In New Zealand it is specified [Standards New Zealand, 1992] that floors with fundamental frequencies of less than 8 Hz will require an investigation based on dynamic analysis if the proposed occupancy involves rhythmic activities, such as with dance venues and gymnasiums. Of course, any situations involving the support of frequently reciprocating loads and vibrating machinery should be treated in the same manner.

Long span hollow core flooring units may also be susceptible to vibration caused by the vertical accelerations associated with earthquakes. A feature of the recent Northridge (1994) earthquake was the propensity of significant vertical accelerations measured in both structures and the free-field. For example, a vertical acceleration of 0.52g was measured in a six storey parking structure and up to 1.83g was recorded on an interchange bridge with a corresponding maximum horizontal acceleration of 1.0g [NZNSEE, 1994]. Or, several occasions, and with a wide dispersion of epicentral distances, the vertical acceleration components resulting from Northridge were sufficiently manifest to warrant extra consideration when reviewing the performance of structures. The design vertical acceleration due to earthquakes is normally taken

*Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring*
as being two thirds of the horizontal component [Standards New Zealand, 1992]. The Northridge event is justification that measures to avoid unfavourable effects caused by significant vertical accelerations should be embodied in the overall design philosophy.

Simplified approaches to what otherwise can be a complex problem are available for guidance [ACI, 1979; CEB, 1991], which includes useful information on damping ratios and the comparative behaviour of calculated response versus the measured response of floor systems. It is interesting to note that with realistic estimates of flexural stiffness, damping and vertical acceleration, it is feasible that long span hollow core floors could exhibit a deflection amplitude that would be described as “very disturbing” as a subjective measure of human response to the serviceability level earthquake.

Bearing vertical accelerations in mind, it is essential that designers approach longer spans from the point of view of providing enough flexural stiffness over the basic strength and deflection requirements under static gravity loads. This is because the vertical response amplitude alone, before the application of a suitable dynamic amplification factor, may already be unacceptable in the serviceability limit state environment. Both the measured and calculated values of damping for floors supporting non-structural partitions point to characteristic values that range between 5% and 10% of critical damping, therefore, the provision of an adequate flexural stiffness may be the most efficient method of amplitude control.

Although vibration problems have traditionally been counteracted by prescribing floor deflection limits under superimposed loads, such as span/360 (1/30 in. per ft. of span), these types of empirical formulae are directed towards controlling vibrations induced by foot traffic and not seismic actions. As shown at Northridge and at other places, there are always elements of uncertainty and magnitude when detailing for seismic response. In the case of long span hollow core floors it is envisaged that open plan floors in stiff low-rise structures may be the most susceptible to the effects of vertical seismic accelerations.

(b) Acoustic Properties

The requirements for noise control are usually most acute when floors act as a partition between different occupancies in a residential building. In New Zealand, a standard for sound transmission is set out in section G6 of the New Zealand Building Code [Building Industry Authority, 1991]. In this document the minimum requirements for a sound transmission classification (STC) between party walls and floors and an impact insulation class (IIC) for floors is given as 55. This figure is supported by recommended practice from other parts of the world that have sufficient experience of noise control in multi-storey apartments. However, emphasis is still placed on the need to plan carefully so that louder sound sources such as laundries are not positioned in near proximity to designated quiet areas, otherwise an STC of up to 70 is recommended [CPCI, 1987].

Since a 200 mm deep hollow core unit with 65 mm topping has an STC of about 54 and an IIC of 30, it is apparent that additional sound proofing of the floor will be required in most structures. In this situation the shortfall of IIC could be overcome by specifying dense carpet.
with a soft underlay on the floor unit, giving an additional IIC of 25 to meet the building code requirements.

For qualitative information on the acoustic properties of individual types of hollow core sections the licence holder (precast manufacturer) will have access to specific test data that is the property of the suppliers of the particular brand of hollow core extruder.

(c) Thermal Properties

Similarly to acoustics, the Building Code [Building Industry Authority, 1991] contains section G3 on the requirements for thermal insulation in buildings. The minimum thermal resistance (R) value will usually vary for different compartments within a structure, ranging from a minimum R value of 0.6 for concrete walls to an R value of 1.5 for roofs. When designing for thermal requirements, the prescribed R value is achieved, in its simplest form, by the addition of the R values of individual components (including air gaps) that make up the thermal barrier. It is evident that hollow core units with composite topping alone are not adequate to achieve the required external R value. For 300 mm hollow core with 65 mm topping, this value is only about 0.5 [CPCI, 1987].

There are methods for improving the thermal resistance of bare hollow core units from within, such as applying insulation materials to the hollow core voids. Again, the precast manufacturer will have access to proprietary information on the thermal properties of particular extruded hollow core units and methods for improving their thermal properties. With one of the larger manufacturers of hollow core machinery being a Finnish company, this is hardly surprising.

(d) Fire Resistance

Many tests have been performed on the fire resistance properties of pretensioned flooring units including hollow core flooring [CPCI, 1987]. The standard for practice in New Zealand relating to the fire resistance rating (FRR) of hollow core flooring [Standards New Zealand, 1995] has been developed around the findings of European (CEB-FIP) and North American test laboratories. Based on an equivalent thickness, it is apparent that a simply supported 200 mm deep hollow core unit without topping and with normal height of pretension strand will achieve the prescribed integrity, insulation and stability end point criteria when exposed to a 90 minute fire. Because the vast majority of new buildings in New Zealand have a required FRR of 60 minutes or less [Building Industry Authority, 1991] there should be no foreseeable problems with the fire safety aspect of composite topped and partially continuous hollow core floors.

For applications where abnormally high fire loads are present or other concerns may exist within the fire compartment with regards to unusual geometry and combinations of building materials, consultation should be made with a Fire Engineer [Thomas, 1996].
1.2.1.3 The Flexural Behaviour of Hollow Core Flooring

(a) Serviceability Limit State

The combination of a dry-mix concrete and the high frequency vibration that is required for extruded concrete manufacture results in a concrete constituent that is dense with good binding between the aggregate and cement paste. This is beneficial to the flexural cracking modulus of these elements and it is anticipated that the allowable tensile stress values for uncracked pretensioned members as given in standards [Standards New Zealand, 1995] will be conservative for hollow core units (although this may be affected by aggregate source). Well compacted and dense concrete coupled with a characteristically high release strength will also benefit the pretension strand transfer bond performance when control is achieved through radial tension inducing mechanisms such as Hoyers effect [Lin and Burns, 1982].

Referring to Fig. 1.10, it can be seen that hollow core slabs are flexurally efficient as an unpropped flooring system. This efficiency is partly due to the comparatively large pretension eccentricity that can be utilised in cored slab systems without exceeding the allowable combined stress parameters (compressive or tensile) at the prestress transfer stage. Because most of the deeper hollow core units shown in Figs. 1.9 are pretensioned at eccentricities that are well outside the middle third kern of the section, tensile stresses will result in the top flanges at the ends of these members. Under a non-continuous support configuration (i.e., at a true simple support) and after term losses of prestress force, the top fibres of these units may continue to act in tension throughout the service life of the structure.

Due to flexural proficiency, the provision of continuity reinforcement is not a strict requirement for the satisfactory strength performance of hollow core flooring under either service or ultimate limit state loadings. For longer spans, however, the allowable deflection criteria will become an increasingly relevant consideration and the full use of continuity bars should be specified for these applications.

Full scale gravity load tests conducted on both a simple span [Scott, 1973] and continuous spans [Rosenthal, 1978] indicate that much of the flexural behaviour of hollow core slabs can be predicted by established principles of analysis. In his paper, Scott concluded that the methods prescribed by the ACI building codes [ACI 318-71, 1971] will, with sufficient accuracy, predict the flexural behaviour of a hollow core slab at all the critical phases in a test-to-failure. These observations are supported by other references in which the more precise moment-curvature analysis techniques are also presented [Lin and Burns, 1982. Collins and Mitchell, 1987].
Figs 1.10  Load-span relationships of hollow core flooring with 65 mm thick composite topping and differing levels of prestress force. The numbers of 12.7 mm diameter strand are indicated in the legend.
Fig. 1.11 Typical load-deflection curve for a simply supported hollow core slab with composite topping [after Scott, 1973]

The European recommended [FIP, 1988] approach to flexure in hollow core flooring is perhaps the most discriminating (ultimately due to shear implications) in that it tends to regard extruded hollow core units as a non-generic form of pretensioned floor construction. In the FIP publication on hollow core, emphasis is placed on eliminating flexural cracks that may otherwise occur in zones that involve the development of prestress force. This is due to the likelihood that cracks generated in these regions, without the benefit of reinforcing ties, will penetrate the full depth of the section resulting in reduced aggregate interlock and a reliance on prestressing steel dowel action to sustain shear forces.

The FIP recommends that flexural tension allowed in the extreme fibre of hollow core sections over the prestress transfer zone should be limited to an average of about 85% of the flexural tensile stress that is allowed in zones of full prestress force, as derived from Equation 1.1 for common depths of hollow core section. The FIP recommendations for flexural stress in extruded hollow core systems are somewhat more restrictive than those adopted by the New Zealand standard because the tensile stress used in calculations is limited to the lower five percentile value, which is about 71% of the mean value of concrete tensile strength taken from tests (refer columns 3 and 4 in Table 1.5). In accordance with Table 1.4, this leads to a maximum allowable flexural tension stress of approximately 70% of the ACI building code values in zones of full prestress force at a given strength of concrete. In the aforementioned zones of prestress development, allowable flexural tension stress will reduce to about 60% of that allowed by ACI building codes (see Table 1.3).

The FIP publication [FIP, 1988] provides another example where flexural tensile strength and axial tensile strength have been made distinct. This is in the limits that are placed on flexural tensile stresses that may occur due to bending in the transverse direction. The lesser axial tensile stress value has been employed in this situation because the presence of cores in the section does not support assumptions regarding flexural tension.

Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring
Table 1.3 shows the maximum allowable concrete stress for uncracked sections at the serviceability limit state used in New Zealand. The corresponding FIP values are shown in Table 1.4.

**Table 1.3**  Maximum allowable concrete stresses for uncracked sections at the serviceability limit state [after Table 16.1, Standards New Zealand, 1995]

<table>
<thead>
<tr>
<th>Stress case</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress case</td>
<td>Immediately after transfer before time dependant losses</td>
<td>Permanent loads plus variable loads of long duration or frequent repetition</td>
<td>Specified service loads for buildings where load category II does not apply</td>
<td>Permanent loads plus infrequent combinations of transient loads</td>
</tr>
<tr>
<td>Compression</td>
<td>$0.6f'_d$</td>
<td>$0.4f'_c$</td>
<td>$0.45f'_c$</td>
<td>$0.55f'_c$</td>
</tr>
<tr>
<td>Tension</td>
<td>$0.5\sqrt{f'_d}$</td>
<td>zero</td>
<td>$0.5\sqrt{f'_c}$</td>
<td>$0.5\sqrt{f'_c}$</td>
</tr>
</tbody>
</table>

For calculation of the effective flexural tensile stress, the axial tensile stress is modified as a function of the section depth. For the application of stresses given in Table 1.4, the concrete flexural tensile stress $f_{ctf}$ is related to the axial tensile strength $f_d$ by:

$$f_{ctf} = f_d \left(0.6 + \frac{0.4}{d^{0.25}}\right)$$  \hspace{1cm} (1.1)

where the effective section depth $d$ is in metres, and is not taken greater than 1.0 m.

**Table 1.4**  Maximum allowable concrete stresses for uncracked hollow core sections at the serviceability limit state [after FIP, 1988 and CEB-FIP, 1978]

<table>
<thead>
<tr>
<th>Stress case</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress case</td>
<td>Stress in concrete at transfer</td>
<td>Stress at top or bottom fibre in transfer zones for any load combination</td>
<td>As for load category II in zones of full prestress development</td>
</tr>
<tr>
<td>Compression</td>
<td>$0.6f_{ck}$</td>
<td>$0.6f_{ck}$</td>
<td>$0.6f_{ck}$</td>
</tr>
<tr>
<td>Tension</td>
<td>$f_{ctm}/1.3$</td>
<td>$f_{ctk,0.05}/1.3$</td>
<td>$f_{ctk,0.05}/1.3$</td>
</tr>
</tbody>
</table>

*Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring*
From the FIP recommendations, the tensile stress values referred to in Table 1.4 may either be obtained directly or derived from the values given in Table 1.5 below:

Table 1.5  Relationships between ①cube strength, ②cylinder strength, ③lower limit tensile strength and ④mean tensile strength of concrete, used in Table 1.4 [FIP, 1988]

<table>
<thead>
<tr>
<th>① f_{ck, 150 mm}</th>
<th>② f_{ck}</th>
<th>③ f_{ct, 0.05}</th>
<th>④ f_{cm}</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>25</td>
<td>1.8</td>
<td>2.5</td>
</tr>
<tr>
<td>35</td>
<td>30</td>
<td>2.0</td>
<td>2.8</td>
</tr>
<tr>
<td>40</td>
<td>35</td>
<td>2.2</td>
<td>3.1</td>
</tr>
<tr>
<td>45</td>
<td>40</td>
<td>2.4</td>
<td>3.4</td>
</tr>
<tr>
<td>50</td>
<td>45</td>
<td>2.6</td>
<td>3.7</td>
</tr>
<tr>
<td>55</td>
<td>50</td>
<td>2.8</td>
<td>4.0</td>
</tr>
<tr>
<td>60</td>
<td>55</td>
<td>3.0</td>
<td>4.3</td>
</tr>
</tbody>
</table>

(b) Ultimate Limit State

For the ultimate limit state the FIP approach to composite hollow core flooring is less specific and follows general provisions [CEB-FIP, 1978] that are applied on the basis that adequate composite bond is maintained between the precast and cast-in-place components of the section [FIP, 1982].

For calculation of section capacity the New Zealand Standard and the ACI building codes treat hollow core units identically to other prestressed construction. By simplified methods, the stress in the strand at the flexural strength of the unit is obtained by making a suitable reduction to the ultimate tensile capacity of the strand to allow for the characteristic stress-strain curve of the prestressing steel. Alternatively, moment-curvature analysis may be used with strain compatibility assumptions being applied over the actual stress-strain characteristics of the constituent materials in the section. For hollow core floors the simplified method of analysis has proved adequate for design purposes [Scott, 1973].

Another consideration that has been of interest in the flexural strength design of hollow core flooring is the establishment of flexural bond in the prestressing strand. This pertains mainly to highly loaded short spans where flexural bond demands may extend into the transfer bond regions at the ends of a unit. Tests have shown that sudden collapse of these types of members is unlikely. However, important conclusions were drawn on quality control aspects at manufacture [Anderson and Anderson, 1976].
1.2.1.4 The Shear Behaviour of Hollow Core Flooring

(a) Flexure-Shear

Hollow Core slabs have demonstrated adequate shear capacity to resist the customary levels of uniformly distributed load that are associated with the floors of buildings [Scott, 1973]. In situations where concentrated loads occur near the ends of units, the support details may require extra attention if the allowable shear and bond stresses are not to be exceeded. If required, the shear capacity can be greatly enhanced by specifying concrete infilled voids and embedded reinforcing bar details for end seatings that are subjected to increased shears [Mejia-McMaster and Park, 1994].

It is widely considered that extruded hollow core slabs should not be used in structural applications that involve large shear forces, especially large shear forces in combination with impact such as is caused by heavy motor vehicles.

In keeping with section 1.2.1.3, a comparison drawn between New Zealand practice [Standards New Zealand, 1995] and the recommendations of the FIP [FIP, 1988] indicates that a more conservative stance has been taken by the Europeans with respect to shear in hollow core flooring. However, the New Zealand standard approach to shear in prestressed concrete does concur with CEB-FIP methods, and some important qualifications on shear formulae given in the New Zealand standard are outlined in the commentary (part 2) of the standard.

Although the FIP recommendations could appear to be conservative with respect to flexure-shear in hollow core systems, this document has only existed as a recommendation against a large background of European national codes. Written along the lines of a guide to good practice, it is understandable that the authors would advocate design criteria that can guarantee absolute performance for expressly “precast prestressed hollow core flooring”.

Flexure-shear due to predominant point loads would appear to be an acute consideration for extruded hollow core units because of the likelihood that flexural cracks will coincide with the development of shear mechanisms as the ultimate limit state of loading is approached. Because there is an absence of shear reinforcement, longitudinal splitting cracks may be expected to propagate along the hollow core webs, possibly resulting in a catastrophic shear failure (see Fig. 1.12). This peeling effect is caused by dowel action on vertically unrestrained longitudinal reinforcement as part of the shear resistance mechanism and has been widely observed in many flexure-shear tests involving beam sections without web reinforcement [Park and Paulay, 1975].

To avoid flexure-shear problems the FIP recommendations place extra limitations on the shear capacity of extruded hollow core members that are cracked in flexure. It should also be noted that the allowable flexural tension stresses have already been restricted for the serviceability limit state, as discussed in the previous section on hollow core flexure. The CEB-FIP model code method is followed [CEB-FIP, 1978] for calculation of flexure-shear capacity. However, the design values of resistant shear stress (as a function of design concrete compressive
strength) have been scaled to an average of 73% of those calculated by the model code formula, and are listed in Table 1.6.

Fig. 1.12 Longitudinal peeling in flexure-shear zones caused by the dowel action of principal reinforcement in the absence of stirrups [BS 8110: 1995 Handbook, 1995]

Table 1.6 FIP recommended values of resistant shear stress for use in flexure-shear capacity calculations (Eq. 1.2) involving extruded hollow core flooring [after FIP, 1988]

<table>
<thead>
<tr>
<th>f_{ck} (MPa)</th>
<th>\tau_{Rd} (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.27</td>
</tr>
<tr>
<td>30</td>
<td>0.29</td>
</tr>
<tr>
<td>35</td>
<td>0.31</td>
</tr>
<tr>
<td>40</td>
<td>0.33</td>
</tr>
<tr>
<td>45</td>
<td>0.34</td>
</tr>
<tr>
<td>50</td>
<td>0.36</td>
</tr>
<tr>
<td>55</td>
<td>0.36</td>
</tr>
<tr>
<td>60</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Thus, the flexure-shear capacity in the ultimate limit state is given by:

\[ V_{rd} = \tau_{Rd} b_w d \xi \left( 1 + 50 \rho_p \right) + \frac{0.9 M_y V_x}{M_x} \quad (1.2) \]

Actually, both terms of this equation differ from the treatment of flexure shear in the CEB-FIP model code because an additional factor of 0.9 placed on the proportion of shear resistance that can be utilised in relation to the section decompression moment. In the CEB-FIP model code, the resistant shear stress is given by:

\[ \tau_{Rd} = 0.068 \sqrt{f_{ck} + 5/y_e} \quad (1.3) \]
where the partial safety factor γ_c is taken as 1.4 for well placed concrete. If Eq. 1.3 is substituted in Eq. 1.2 and the factor of 0.9 is ignored in the second term, a very similar value of flexure shear capacity will result to that calculated by the corresponding equation in the New Zealand standard [Standards New Zealand, 1995]. Equation 9.9 in this document gives the nominal shear stress provided by concrete in regions subject to flexure-shear cracking for non-composite uniformly loaded beams as:

$$v_{en} = v_b + \frac{V' M_o}{b_w d_e M^*}$$

(1.4)

The basic shear stress $v_b$ is applied identically to all members incorporating tension reinforcement or prestress steel in a flexural tension zone, and is given by:

$$v_b = (0.07 + 10 p_w) \sqrt{f'_{c}}$$

(1.5)

For situations where the section is of composite construction and does not support a uniform load (which usually infers a live load), a fundamental approach to flexure and shear interaction must be adopted. This allows for a more exact analysis to be performed with regards to predicting the shear force that corresponds with the designated critical moment in the section. In Eq. 1.4, the critical moment is taken as the decompression moment for the prestressed section and is that which will cause an extreme fibre stress of zero after consideration of all losses. Consequently, this equation is conservative for some sections because the mechanics of flexure-shear are such that first cracking due to flexural bending stresses is the point from which a shear failure is instigated. The ratio of first cracking moment to decompression moment will vary with differing configurations, and is most significant for members with light, concentric prestress and reduces with increased prestress force and eccentricity.

The development of flexure-shear equations for specific load configurations is discussed in various publications [ACI, 1968] where the dead and live load components are isolated and treated accordingly. Analysis of flexure-shear capacity for composite topped and shored flooring systems involves extra steps with the separation of components and this has been discussed in relation to the shear design of pretensioned joists [Herlihy, 1995].

(b) Web Shear

For web shear capacity, the formula given in the FIP recommendations [FIP, 1988] is described as the shear capacity for a region that is uncracked in flexure. Because the critical section for web shear is at the interior face of the end support, the FIP method directly modifies the beneficial effect of axial prestress force over the transfer length of the pretensioning steel. The shear capacity formula is given as:

$$V_{Rd12} = \frac{1b_w}{S} \sqrt{f'_{cd}^2 + 0.9 \alpha \sigma_{sp} f_{cd}}$$

(1.6)

where the average prestress in the concrete is:
\[ \sigma_{cp} = \frac{A_p \sigma_p}{A_c} \quad (1.7) \]

and the reduction factor \( \alpha \) is based on a parabolic prestress force transfer distribution, given as:

\[ \alpha = 1 - \left( \frac{t_s}{t_i} \right)^2 \quad (1.8) \]

It will be recognised that the FIP equation is a manipulation of the classic formula for shear flow and, therefore, the section is indeed assumed to be uncracked.

Further information on the shear and transverse loading performance of hollow core flooring is presented in a comprehensive FIP Technical Report [FIP, 1982].

Equation 9-11 in the New Zealand standard [Standards New Zealand, 1995] works along the same lines as the FIP equation in that allowances are made for the effects of axial prestress and the contribution of concrete principal tensile stress, however, these are then incorporated into a shear index (V/\(bd\)) format:

\[ v_{ew} = 0.3 \left( \sqrt{f_c} + f_{pe} \right) \quad (1.9) \]

The New Zealand standard also permits calculation on the basis of limiting the principal tensile stress at the centroid of the composite section to 0.33\( \sqrt{f_c} \). In this regard, the classic elastic shear formula may also be used with due considerations being made for axial prestress effects and other such stresses that may bear an influence under Mohrs circle failure criterion.

Attention to secondary transverse stresses is also important because the phenomena of bursting and splitting stresses are notable side-effects that also exist in the transfer zone. These and other aspects relating to production technique are discussed in the following subsection on hollow core manufacture.

1.2.2 THE MANUFACTURE OF HOLLOW CORE FLOORING

1.2.2.1 Machinery and Concrete

There are two recognised modern processes for automated hollow core flooring production, these are the extrusion and slip-forming methods. There are some notable differences between these processes, and there has also been some opinion as to the performance of each.

The slip-form method utilises a higher slump concrete than the extrusion process but also requires a higher cement content in the mix. A direct benefit with slip-forming is that less formidable vibration is required to form deep sections, which until recently has been a limiting factor with the extrusion process. Many slip-forming machines also allow the manufacture of a diverse range of industry products by means of an interchangeable profile forming insert.
Almost all the hollow core flooring produced in New Zealand have been manufactured using the extrusion process. The combination of a “zero slump” dry-mix concrete with high frequency vibration has produced perhaps the most cost effective and reliable hollow core sections available. Because extruded mixes require a comparatively small amount of cement and use a low water-cement ratio, yet achieve a well compacted high modulus concrete, there is good control of creep and shrinkage effects. This has proven to be a very important factor with hollow core flooring because the pretensioned hollow section is otherwise poorly conditioned to resist actions that cause volume changes.

The “shear compaction” machinery that has recently appeared in New Zealand symbolises the state-of-the-art in hollow core manufacture. This improved method makes the extrusion of deep hollow core sections possible without the problems of suitable high frequency vibration, and has greater tolerance of concrete slump variations.

When producing dry-mix concrete the proportion of water can be critical, with the mix being sensitive to within 10 litres of the ideal quantity of added water per cubic metre batched. Anything in excess of this limit can result in a mix that is too dry to be properly extruded or one that will tend to collapse. The batching part of the operation is therefore of utmost importance with extruded products and to achieve a workable dry-mix concrete requires carefully graded aggregates, selective use of plasticisers and an industrial strength pan mixer.

In New Zealand, the mix design will be strongly influenced by the regional availability of mineral aggregate. The main centres in New Zealand where hollow core flooring has been produced are the Auckland region, Hamilton (discontinued), Wellington region and Christchurch. In the Auckland and Wellington regions the predominant aggregates are crushed basalt and greywacke and crushed greywacke respectively. Although of differing geological derivation, these are very commendable aggregates for hollow core production, being hard, inert and angular stone [C&CA, 1990].

In the Hamilton area, the aggregate source has been either crushed basalt or andesite. Andesite is a least preferred aggregate for use in concrete, generally having inferior crushing and abrasion resistance than basalt and greywacke. Nevertheless, the porous texture of andesite can promote bond between the aggregate and cement paste components of a mix, thus benefiting concrete tensile properties.

Canterbury province in the South Island has benefited from good quality river run greywacke aggregate. However, it is common for these mixes to contain comparatively small percentages of crushed coarse aggregate. Although this does not necessarily affect the compressive strength of concrete, the assumed flexural tensile strength (when taken as a function of compressive strength) may not be so consistently high in practice. Production experience suggests that mixes containing a large proportion of smooth rounded stones (also known in the vernacular as “ball-bearing mixes”) can show limitations with regards to tensile strength properties. This observation is somewhat under-researched but logically it would suggest that unlike the andesite aggregates mentioned earlier, it is difficult to establish good bond between a hard, polished stone surface and cement paste (see Fig. 1.13).
The incidence of reduced bond strength that is attributed to rounded aggregate does not go without mention in the literature, and the reasoning given is similar: “There is little influence of the type of the aggregate on the direct and splitting tensile strengths, but the flexural strength of concrete is greater when angular crushed aggregate is used than with rounded natural gravel. The explanation is that the improved bond of crushed aggregate holds the material together but is ineffective in direct or indirect tension” [Neville and Brooks, 1987].

**Fig. 1.13** Differing failure modes of the aggregate component in concrete cylinders subjected to the standard splitting test. Fractured light-weight expanded clay shale aggregate (at left) and debonded greywacke smooth pebble aggregate.

In the context of hollow core performance, units comprised of mainly smooth round aggregates as those described could have a reduced shear capacity. In particular, flexure-shear capacity is determined by first flexural cracking followed by aggregate interlock across an inclined flexure-shear crack that is subjected to further opening. These actions invoke the exact elements of performance that have been found lacking in rounded stone mixes through research and production experience.

No special guidance is given by the manufacturers of hollow core machinery on the derivation or geometry of aggregates, which is understandably a topic that is beyond their professional obligation.

A mix design for extruded concrete may typically have a ratio of about 2:1 by weight of aggregate to sand, and a ratio of 5:1 by weight of aggregate plus sand to cement (see Table 1.7).
Table 1.7  Typical proportions of solid components in an extruded hollow core mix per cubic metre batched

<table>
<thead>
<tr>
<th>Constituent</th>
<th>kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 mm agg.</td>
<td>590</td>
</tr>
<tr>
<td>7 mm agg.</td>
<td>390</td>
</tr>
<tr>
<td>Sand</td>
<td>460</td>
</tr>
<tr>
<td>Cement</td>
<td>280</td>
</tr>
</tbody>
</table>

An incorrect concrete mix or faulty machinery can result in a product that may typically exhibit collapsed flanges over the hollow core voids or tearing of the concrete along the outer webs of the unit. In accordance with what has been discussed earlier on flexure-shear, tearing cracks in the outside webs may be detrimental to the performance of hollow core units.

Insufficient concrete compaction at manufacture has also been revealed as a prime cause of strand pull-in. Under flexural testing, the strands that exhibited a large pull-in at transfer were observed to undergo a premature flexural bond failure. An important conclusion from these tests was that: “good workmanship and proper maintenance of production equipment are vital for quality assurance” [Anderson and Anderson, 1976]. In their research paper, an empirical quality assurance method was proposed for flexural bond on the basis of measured strand pull-in, $\Delta_{ct}$:

$$\Delta_{ct} \leq \frac{f_{u}d_{b}}{6500} \tag{1.16}$$

which for the most commonly used supergrade (1860 MPa) strand is equal to $0.2d_{b}$.

1.2.2.2 Production Process and Quality Control

(a) Surface Roughening

From a composite construction perspective, one criticism that has often been directed at extruded flooring is the problem of achieving a significantly roughened bond surface. In ordinary precasting, obtaining good surface roughening is a simple exercise that is usually accomplished with a clean yard broom [SANZ, 1987]. The hard, compacted surface of extruded zero-slump concrete is not conducive to brooming and excessive pressure applied to the surface to achieve this will cause a cave-in over the hollow core voids (see Fig. 1.14).

The degree of surface roughening specified for extruded flooring in the New Zealand standard is a minimum of 2 mm peak to trough. Reliance is placed on the shear keys between units to assist the composite bond and the specified minimum key width is 20 mm. It has been proven by tests that smooth bond surfaces without the keys are adequate for the development of composite bond under gravity loads [Scott, 1973. CTA, 1976]. Although automated roughening
of the top surface of extruded systems is feasible, this may seem an excessive measure in view of results from the composite bond tests reported.

![Brooming of an extruded hollow core surface](image)

**Fig. 1.14 Brooming of an extruded hollow core surface**

The information reported by Concrete Technology Associates [CTA, 1976] is quite thorough. A number of deleterious practices were imposed on the bond surface to cause variations in performance and reduce the composite bond capacity. Of these, it was found that a heavy coating of oil or unvibrated low slump concrete produced the lowest bond strengths. Putting these observations in context with the construction environment, it is insufficient vibration of topping concrete that would be of greatest concern.

**b) Saw Cutting and Transfer of Prestress**

Saw cutting is the necessary method of separating extruded units from the bed and transferring the prestress force into the section. With deeper units, pre-cutting is often done through the upper half of the section with a cross-cut blade to prevent the mechanical saw blade from jamming when cutting the strands. Care must be exercised when pre-cutting plastic concrete not to disturb the bond between strands and concrete, hence, a maximum depth of pre-cut should be specified.

The effects of shock release that are associated with saw cutting of strands have been suitably well researched. It is concluded that sudden release by saw cutting has no obvious effects on the flexural bond capacity of strands provided that the pull-in limits prescribed by Equation 1.10 are not exceeded [Anderson and Anderson, 1976]. Where the prescribed limit is exceeded, some downgrading of the unit capacity should be considered in accordance with the observed pull-in of individual strands.
The most fundamental parameters used for defining prestress transfer length are the diameter of strand, the initial prestress and the crushing strength of the surrounding concrete at the time of release [Zia and Mostafa, 1977]. More elaborate models have also been developed in which fuller representation is made of variations in the prestress force, strand slippage and bond stress distributions over the transfer length [Balázs, 1992]. However, it remains difficult to determine what the actual transfer bond length will be for a given situation. Analysis of research indicates that there is considerable scatter in experimental results for both sudden and gradual release. Therefore, a simple approximation for transfer length is satisfactory for design purposes.

Fig. 1.15 Transfer length versus $\frac{f_{sl}d_b}{f'_{ci}}$ for sudden release [Zia and Mostafa, 1977]

What appears to be consistent is that the maximum expected transfer length for typical configurations of strand and concrete strengths of hollow core units is about 80 times the strand diameter. The transfer length for sudden release has been proposed as a linear relationship based on regression analysis of a body of experimental results (Fig. 1.15) [Zia and Mostafa, 1977]. Substituting typical values into equation 1.11 (the metric equivalent of line A in Fig. 1.15) the ratio of maximum transfer length to the average transfer length under sudden release will be in the order of 4/3.

$$\ell_t = 1.5 \frac{f_{sl}d_b}{f'_{ci}} - 120$$

(1.11)
The design figure for transfer length under sudden release is often taken as a factor of 1.2 to 1.3 times that for gradual release [Libby, 1977], which results in an estimated range of 60 to 65 strand diameters.

For design purposes, the distribution of force over the transfer length is either taken as being linear over 50 strand diameters [ACI, 1991; Standards New Zealand, 1995] or parabolic over usually about 60 strand diameters (Equation 1.8) [FIP, 1988]. As a result, the calculated strand force at a fraction of the transfer length is not particularly sensitive to the chosen method. For in-depth analyses a power curve relationship has proved to correlate well with test results, as well as facilitating a solution to the governing second order differential equation [Balázs, 1992; Bruggeling and Huyghe, 1991].

Part of the significance of transfer length and the force distribution over this length is the resulting component of shear force generated in the support zone. Since the bending moment induced in a section by prestress force is given as the product of force \( P(x) \) times the eccentricity \( e(x) \) (which is constant in \( x \) for hollow core), the induced shear force may be calculated from the slope of the non-uniform bending moment:

\[
V_{Po(x)} = \frac{d}{dx} \left( P \cdot e \right)_{(x)} = P \frac{d}{dx} e + e \frac{d}{dx} P \tag{1.12}
\]

The vertical component of the force \( P \) may be taken into account in the shear capacity design of prestressed members with sloping tendons, and is termed the transverse component of effective longitudinal prestress [Standards New Zealand, 1995]. As mentioned, the eccentricity is constant over the length of a hollow core unit and this component (i.e., the first term in Equation 1.12) disappears. The second term which relates to the shear component induced through transfer bond is not recognised in the standards, although its contribution has been successfully employed in the exact formulation of hollow core web shear capacity for comparison with the results of FEM analysis [Lin Yang, 1994].

For an estimate of the shear capacity development due to transfer bond, some knowledge of the bond envelope is required. A reasonable approximation of strand force distribution over the transfer length is given by the parabolic FIP equation:

\[
P(x) = P \left[ 1 - \left( \frac{t_t - t_x}{t_t} \right)^2 \right] \tag{1.13}
\]

Hence, the shear force induced by transfer of prestress force via equation 1.12 is:

\[
V_{tpo} = 2P e \frac{t_t - x}{t_t^2} \tag{1.14}
\]

The magnitude of this force depends on the prestress force, the eccentricity of prestress and the length terms as expressed in the equation. The direction in which this force acts is entirely
dependent on the position of the eccentricity above or below the neutral axis of the section. For a positive eccentricity (i.e., below the neutral axis) the bottom fibre will be subjected to increasing compression with distance from the end of the member. This corresponds with an upward and decreasing component of shear force in the section over the transfer length.

(c) Spalling and Splitting Stresses

![Diagram of spalling and splitting stresses](image)

**Fig. 1.16** Spalling, splitting and bursting stresses in hollow core sections [FIP, 1988]

Cracks caused by spalling and splitting stresses (and to a lesser extent bursting stress) (see Fig. 1.17) need to be considered at the manufacture stage. Typically, the type of crack propagated by spalling stresses will occur horizontally along the hollow core web and results in a direct reduction to the shear capacity of the member. For standardised hollow core sections that conform to dimensional limitations imposed to counter splitting problems in the transfer zone [FIP 1988], spalling cracks will usually be the result of insufficient tensile strength of concrete at release.

It can be seen from Fig. 1.15 that comparatively short transfer lengths have also been measured for 3/8 inch (9.5 mm) and 1/2 inch (12.7 mm) diameter strands. For hollow core units the abscissa on this graph will fall at about 22, which corresponds with a minimum recorded transfer length of 12 inches (305 mm). Lower bound values such as this need to be taken into account for evaluation of potential spalling stresses in the webs of hollow core sections at release.

The maximum spalling stress is a function of web thickness, the eccentricity and magnitude of prestress force and transfer length. Spalling stress should be checked in the web for which the most effect is generated, and can be done either by a calculation method or by use of Fig. 1.17 which have been partly derived by finite element analyses [FIP, 1988]. In this, the transfer length is taken as a lower bound value which is about 40 strand diameters for hollow core units, or 80% of the mean transfer length nominated in codes or standards.

The spalling stress \(\sigma_{sp}\) as derived from the FIP recommended method should be limited to less than \(f_{ct}t_j\); the characteristic concrete tensile strength at time \(j\).
Control of splitting and bursting stresses is provided by sufficient concrete cover to the strands. In respect of this, the FIP recommendations state that clear cover should be provided of:

\[ c \geq \frac{0.05 \varphi \sqrt{\sigma_{\text{up}} f_{\text{ecf}}}}{f_{\text{ck,j}}} \]  

(1.15)

![Graph](image)

**Fig. 1.17**  Spalling stress as a function of eccentricity and transfer length [FIP, 1988]

(d)  **Cleaning, Stacking and Handling**

Adequate cleaning of units is perhaps one of the most important operations in the quality control process at manufacture. As discussed in quality assurance literature [e.g., FIP, 1992], unwashed hollow core units may have acute difficulties with composite bond performance, and the problem is not one that is likely to be rectified at the construction site.

The need for thorough washing stems from the fact that saw cutting generates a fine slurry that settles into the slumping depressions over the hollow core voids (Fig. 1.18). If this slurry is not washed away it dries to form an obstructive caked layer that inhibits composite bond.

![Diagram](image)

**Fig. 1.18**  Slurry from saw cutting settles into slump depressions and must be washed off
As with all slender pretensioned products, careful stacking of hollow core units is important, and especially if they are to be stored for some time or uniform deflections are required. A limit must be placed on the height of stacks and because of the eccentricity of prestress the length of overhang must be restricted and placement of stacking spacers must be uniform. Consideration must also be made for the position of slab cut-outs both when handling the unit and at storage.

![Diagram of hollow core unit stacking](image)

**Fig. 1.19**  Transportation of hollow core slabs with openings [FIP, 1992]

![Diagram of slabs and spacers](image)

**Fig. 1.20**  Storage of hollow core slabs [FIP, 1992]

Most issues relating to production are largely a matter of care, common sense, training and experience. Useful and readable guidelines on production process are given by authorities on prestressed concrete [FIP, 1988; FIP, 1992; PCI, 1985].
1.2.3 SUPPORT DETAILS FOR HOLLOW CORE FLOORING

1.2.3.1 General

In this section, current support details that are associated with hollow core flooring are presented. The specific role of connection details in diaphragm action is deferred until the following section.

The nature of support details adopted for hollow core units should be influenced by the governing conditions of structural design. These conditions can be categorised on the basis of a reasonable expectation of the overall performance of the structure. For example, either a bomb blast that results in loss of support or strong seismic forces may cause serious and irreparable damage to structures. Nevertheless, a properly designed earthquake resistant structure should not lose entire support members, nor will a blasted building be subjected to prolonged and intense cyclic base shears. Hence, although cross-crediting of respective diaphragm and support details that arise from these separate design philosophies is acceptable, some rationalisation must be involved to effect a good design.

In a build-up to the analysis of existing and prospective support details, consideration must be given to contemporary construction culture and the underlying requirements of design. From this point onward, optimisation or avoidance of known causes and effects (depending on the consequences of their influence) should result in an appropriate solution. Much of this can only be accomplished with particular knowledge that is gained from experimental research.

1.2.3.2 European Details

In primarily aseismic European building design there is strong emphasis placed on avoiding structural failure through progressive collapse caused by impact, explosion, fire or other effects including unlikely seismic events [FIP, 1986]. As a result, wording in the FIP Recommendations for hollow core flooring is such: “The hollow core units should be directly or indirectly tied to the support structures at both ends. The structural connections, especially the connections at the supports, should be designed and detailed aiming at structural integrity and ductility during collapse” [FIP, 1988].

In effect, these statements set a benchmark for the typical configuration of support details that are recommended by the FIP. Because earthquake risk is quite low over most of Europe there is a general tendency to avoid structural toppings. However, toppings are advocated in Europe for situations where difficulties arise in detailing for the transfer of diaphragm forces in untopped floors, which is usually at the corner connections of strut and tie fields [Bruggeling and Huyghe, 1991].

The principle of tie connections for the purposes of resisting progressive collapse is that they have sufficient strength and ductility to allow full redistribution of the gravity load support mechanism. There are obvious preferences in the type of detail employed to accomplish this because the performance of ties is influenced by a number of geometric and material
parameters. The requirement is for: "high deformability, high force transferring capacity and high energy absorption" (Fig. 1.21) [FIP, 1982]. It is also desirable that an even force-displacement relationship is obtained, as this helps with predicting the net results for systems that involve the interaction of a group of support details under varying rotations (Fig 1.22).

![Diagram](image)

**Fig. 1.21** Properties of a force-displacement curve for support connection details that are of interest: ultimate deformation, maximum force transferred, internal strain energy and the initial stiffness [FIP, 1982]

![Diagram](image)

**Fig. 1.22** The force-displacement relationship should be as even as possible. Connection detail (2) is preferable despite it having a lower peak strength [FIP, 1982]

One of the important properties of tie connections is their ability to absorb a quantity of internal strain energy. This becomes a critical design parameter when sudden loss of support causes a downward movement, with the potential energy of the total vertical shear force being converted into kinetic energy. The internal potential energy capacity provided by the support tie details must then equal or exceed the maximum kinetic energy that results from this action. This is referred to as the condition of energy equilibrium for the system, and is an important criterion in connection design under sudden loss of support.

*Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring*
A comprehensive body of research has been undertaken on the performance of ductile tie connections for hollow core flooring systems, in particular [Engström, 1992]. In this research, verifications were made of the typical material and construction configurations that appear in the FIP recommendations on hollow core flooring [FIP, 1988]. Further, general information is provided on the performance expectations of reinforcing bar tie connections.

Details from European Literature:

Fig. 1.23 The wall structure is damaged by accidental loading. In order to prevent an extension of the damage and total collapse, the damaged structure must remain stable and an alternative load-bearing system must bridge over the damaged area. [Engström, 1992]

Fig. 1.24 Tie-to-compression arch detail for an untapped hollow core floor slab involving grouted cores [Bruggeling and Huyghe, 1991]

Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring
Fig. 1.25 Grouted key tie bar details (coupling bars) [Bruggeling and Huyghe, 1991]

Fig. 1.26 Hollow core floor construction with composite reinforced structural topping [Bruggeling and Huyghe, 1991]
The anchorage length and allowable forces of tie (coupling) bar details shown in Figs 1.24 to 1.26 is also referenced in the comprehensive and all encompassing FIP Recommendations handbook [FIP, 1988].

(a) **Bars grouted into shear keys**

For straight deformed bar ties, the total development length is the basic development length of the bar $l_0$ plus a margin $\Delta l_b$ to allow for full plastic deformation over a utilised length. It should be noted that these quantities are additional to the minimal requirement of the prestress transfer length $l_{crit}$, and hence, the total embedment will be:

$$l_d = l_{crit} + l_b + \Delta l_b$$  \hspace{1cm} (1.16)

Of the straight ribbed bar details tested [Engström, 1992] it was found that 12 mm diameter bars could develop their full tensile strength over an embedment length of 50 bar diameters (i.e., 600 mm) at a corresponding ultimate force of about 70 kN. The displacements at fracture were measured between 7.1 and 20.3 mm, however, an average of about 18.5 mm at fracture is a more typical value. The reinforcing steel in question had a measured yield strength of 525 MPa, an ultimate strength of 725 MPa and 13.9% elongation at fracture. It was not possible to develop the tensile strength of 16 mm diameter bars in this way over a 50 bar diameter (i.e., 800 mm) embedment length.

The provision of a specific transverse clamping action is essential for the reliable deployment of key embedment details. In the above tests, pull-out and shear interface failure of the 16 mm bar tests and one pull-out failure involving a 12 mm bar was synonymous with rupture of the transverse tie-beam joint. Premature rupture of the transverse tie joint was caused by pre-cracking of the longitudinal (tie embedment) joint and was a common denominator for all the tie details that failed below the expected load.

It was observed that the quality of grout fill had an important effect on the performance of these details. Deformed bars of 150 to 200 bar diameter anchorage lengths were observed to pull out if the grouting did not fully encase this reinforcement. It was thus concluded that even deformed bars should be provided with hook ends when used in this form of detail [Engström, 1992]. Some straightening of hook ended bars was observed at the plastic stage and it is considered that this was the result of insufficient hook cover. Because of the accumulation of tie force in the hook region, it is particularly important that sufficient end cover is provided for smooth bar details.

(b) **Bars grouted into cores**

For bars that are grouted into cores, the embedment is taken as the prestress transfer length $l_{crit}$ plus an additional length that is determined by the type of anchorage employed. For hook returns, this additional length can be determined by:

$$l_{add} = \frac{F_{st}}{f_{ct} u}$$  \hspace{1cm} (1.17)

*Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring*
To avoid fracture of the section (see Fig. 1.27) the allowable magnitude of tie force for embedment in individual cores has been provisionally recommended at 80 kN, with a maximum of 160 kN per end for 265 mm deep hollow core units [FIP, 1988]. Subsequent testing has shown that considerably larger forces may be developed by these details, but considerations must also be made for concurrent stresses acting in the ends of units that may contribute to section fracture [Engström, 1992]. It is indicated that forces of up to 100 kN may be safely resisted by a single core with a maximum of 200 kN per end of the member. There was no indication of shear failure between the hollow core and the infill grout interface.

In the role of embedded ties, smooth bars with hook ends have demonstrated greater endurance than deformed bars and there is less concern with secondary effects that may cause failure of the detail. Due to the efficient nature of deformed bar bond, the ensuing shorter embedded gauge length results in a combination of higher splitting stresses and lower deformability than with properly anchored smooth bars (Fig. 1.28). Neither of these characteristics are desirable for the environment in which ductile tie connections for hollow core units must act. Conversely, smooth bar details have demonstrated the potential of strength and durability required to perform the ultimate limit state task (Fig. 1.29) [Engström, 1992; Mejia-McMaster and Park, 1994].

Fig. 1.27   Splitting of hollow core webs due to high strength embedded deformed bars under tension [after Engström, 1992]

![Fibbed tie bars, Kg 40](image)

Fig. 1.28   Force-displacement characteristics of deformed bars embedded in hollow core units [Engström, 1992]
Fig. 1.29 Force-displacement characteristics of smooth bars embedded in hollow core units [Engström, 1992]

1.2.3.3 North American Details

Both composite topped and untopped hollow core floors have been advocated for use in the seismic regions of North America and a variety of connection details have been considered [ATC-8, 1981]. Unlike European practice, more emphasis has been placed on the development of strength and stiffness for floor diaphragm purposes (strong connections) than the potential loss of support to flooring units. As a result, North American details do not appear to exhibit the general requirement of ductility that is prescribed by FIP literature [FIP, 1982. FIP, 1988]. For example; of nine typical North American support connection details reported to the ATC-8 workshop, all were expressly designed on the basis of strength and stiffness. Although attention was drawn to the fact that some "strong" connections may exhibit ductility, no intimation of this quality was attached to any of the details presented [Becker and Sheppard, 1981].

Because a degree of strength development is rudimentary to all connection details, the issue of ductile performance is usually only a matter of specific detailing practice (i.e., the way that strength is developed and maintained under foreseeable actions). In review of typical North American details there is no evidence of the ductile embedded bar-in-core type details that have been tested and endorsed by the Europeans.

However, it would be incorrect to suggest that potential loss of support actions in precast flooring has been overlooked by American engineers. In a paper to the PCI, the precast construction pioneer Alfred Yee has discussed the detailing necessary to resist collapse of precast floor slabs caused by support damage [Yee, 1991] (see Fig. 1.30). Moreover, a number of design aspects that are covered in his paper relate to the basis of the present study in a more direct way than European design criteria. The emphasis is on the effects of seismicity with
regards to connections in particular. Also, the type of precast construction that is referred to can be identified as similar to New Zealand practice.

The importance of compatibility of connections for precast components in seismic resisting diaphragms (and hence, ductility requirements) has also been stated in American literature. Based on the findings of ATC-8, it was reported that to preserve stability: “precast diaphragms should be designed to maintain their in-plane stiffness, even after the lateral force resisting system has entered the inelastic range” and furthermore, that: “Sufficient ductility should be provided in inter-element connections to prevent brittle failure of the diaphragm due to out-of-plane deformations imposed in the vertical subsystems. Similarly, connections between diaphragms and vertical subsystems must be ductile and tough enough to sustain inelastic, cyclic rotations of a magnitude consistent with the lateral yielding of vertical subsystems” [Clough, 1982].

With regards to ductile connections between diaphragms and vertical subsystems, there are no actual recommendations given by either ATC-8 or in the PCI journal of how details that meet the stated requirements might look. Some information is given on ductile inter-element connections (i.e., connections between flooring units), however, it must be remembered that the details presented in ATC-8 were given as a representation of contemporary connection practice and of proposed connections only. Of the fifteen diaphragm connection details reported [Becker and Sheppard, 1981] only four had supporting test data, none of which was for expressly ductile connections. It would appear (and is a reasonable expectation) that the onus should lie with practising engineers and research institutions to verify that existing and proposed connection details can perform in a manner conducive to the recommendations from ATC-8. After all (as stated in the preface of the document) the purpose of ATC-8 was to: “summarise the current state-of-practice and research and to prioritise research needs...”. The wholesale condonation of an arbitrary group of mostly untested connection details would not be the objective of an astute American research council. Examples of the various diaphragm connection details that were reported by ATC-8 are shown in Figs 1.31 to 1.32.

**Details from North American Literature:**

![Diagram of Slab seating connections designed to prevent collapse](Image)

*Fig. 1.30  Slab seating connections designed to prevent collapse [after Yee, 1991]*

*Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring*
Fig. 1.31  Connection of floor slab to perimeter shear (structural) wall [ATC-8, 1981]

Fig. 1.32  Untopped hollow core floor diaphragm connection detail utilising chord-tie reinforcement and grouted tie bars [after Moustafa, 1981]
1.2.3.4 New Zealand Details

Composite topping of floor diaphragms has been a basic New Zealand code requirement for seismic resisting buildings, and floor diaphragm actions are assumed to be carried by this structural topping alone. The typical connection details employed between precast hollow core and support elements are very similar to those used for composite topped slabs in North America (see Figs 1.35 and 1.36). A special support detail that involves tie reinforcement has been used on occasions where precast units have been supplied undersized (Fig. 1.34). This detail has only been employed for situations that involve short seatings and gravity loadings, it has rarely been used to mitigate the potential of floor collapse caused by imposed actions of support structures.

Unlike North America, the overall seismic performance of New Zealand structures is governed by the principles of capacity design. Capacity design is described as the procedure whereby:
“elements of the primary lateral force resisting system are chosen and suitably designed and detailed for energy dissipation under severe deformations. All other structural elements are then provided with sufficient strength so that the chosen means of energy dissipation can be maintained” [Standards New Zealand, 1995]. Until recently, the need to maintain deformation compatibility between the interconnected components of floor diaphragms and members of the seismic resistance system has not been stated in so many words.

The end support zones of simply supported precast flooring units frequently coincide with regions of supporting members that have been detailed to partake in energy dissipation through inelastic deformation. A proportion of the composite floor slab is considered to contribute to the strength and stiffness of the post-elastic section of the support element; a width that is derived from the total thickness of the floor element (see Fig. 1.33). This places more emphasis on the reliability aspect of connection details; especially with regards to the toughness requirements, in the likelihood that deformation compatibility is expected in the vicinity of flooring members supports.

\[
\begin{array}{c|c|c}
\text{For} & \text{Flexural strength} & \text{Stiffness} \\
\hline
\leq b_w + 15h_b & \leq b_w + 5h_b & \leq b_w + \frac{\zeta_m}{2} \\
\leq b_w + \zeta_mh_b & \leq b_w + \zeta_m/2 & \leq b_w + \zeta_m/4 \\
\leq \frac{\zeta_m}{8} & \leq \frac{\zeta_m}{8} & \leq \frac{\zeta_m}{26}
\end{array}
\]

\(\zeta_m = \text{span length of beam}\)
\(\zeta_m = \text{clear distance to the next web}\)
\(\text{*} = \text{flange in compression}\)

Fig 1.33 Assumed effective widths of beam flanges used in seismic design [Paulay and Priestley, 1992]

Chapter 1: Introduction: Precast Prestressed Hollow Core Flooring
In order to achieve the strength capacity that is provided by starter bars embedded into composite topping, a positive composite bond must exist between the precast flooring element and the topping slab. This is fundamental to the performance of the typical New Zealand support detail and, as a result, composite ties between the precast unit and topping concrete have occasionally been specified for regions of diaphragms where stress concentrations may occur. However, many designers have been reluctant to include devices that will improve or ensure composite bond unless the allowable value of topping slab shear given in the concrete design standard is likely to be exceeded. This is probably an adequate approach for many situations provided that non-exceedance of the given limiting shear value of $0.33 \sqrt{f'_c}$ can actually be verified. In reality, this may not be so straightforward as generalised topping slab shear stresses calculated on the breadth of flooring might suggest. At floor supports of ductile structures, the likely sources of critical shear and bond demand will come from localised interactions between the diaphragm element and ductile members of the primary seismic resistance system.

Many of the requirements for precast floor support connections are either directly referenced in (or are inferred by) various sections of the current New Zealand concrete design standard [Standards New Zealand, 1995] and the loadings code [Standards New Zealand, 1992]. The current concrete standard recognises specific interactions between floor elements and the primary resistance members, the importance of maintaining load paths and the restraint that may be applied to vertical systems via the floor element. A number of prescriptive measures of support control have also been introduced for precast flooring, such as minimum seating lengths.

**Details from New Zealand Literature:**

The following details are for hollow core units as used in New Zealand Construction. The support detail for insufficient seating (Fig. 1.34) is a widely accepted method for overcoming disparities between supplied lengths of precast members and support dimensions:

![Composite Topping with Mesh](image)

**Composite topping with mesh**

**Hollow core flange removed for grouting**

**6.10 central in voids**

**Prop at construction**

**Gap**

**Fig. 1.34** Embedded bar detail for situations where sufficient seating length is unobtainable.
Fig. 1.35  Typical interior support detail

Fig. 1.36  Typical external support detail

Fig. 1.37  Typical hollow core flooring layout
1.3 DESIGN ASSUMPTIONS ASSOCIATED WITH FLOOR DIAPHRAGMS IN SEISMIC RESISTING STRUCTURES

1.3.1 GENERAL

As structural entities, diaphragms have generally demanded somewhat less detailed analysis, design and construction effort than other massive critical path elements of a building. In general deployment, the flooring members that make up diaphragms are required to perform a variety of simply defined serviceability and ultimate limit state functions. As described earlier (Section 1.2.1) these functions relate to deflection control, minimum durability, fire safety requirements and ultimate limit state gravity load carrying capacity.

In addition to transferring gravity loads, diaphragms are laterally stiff and dominant components within the structure that may generally (and conveniently) permit simplified assumptions regarding in-plane force transfer. Yet, in broad terms of seismic resistance, diaphragms are usually considered as passively interacting horizontal plate elements that subscribe to the actions of primary seismic resistance members.

Hence, the most crucial function of a diaphragm will be the performance spectrum during a severe earthquake. It is generally recognised that this function will amount to numerous combinations of simultaneously occurring complex actions. Simply described, it will involve the distribution of horizontal in-plane self-weight (inertia) and transfer forces into the primary seismic resisting system via perimeter connections (i.e., the support interface) and the receipt of forces (often large planar reactions in the case of transfer forces) and deformations imposed by the primary resisting system through the same connections. Historically, the observed failure of connections between these elements has proved to be a major concern with so-called seismic resisting diaphragms [Gupta (Ed.), 1981].

From a developer’s perspective, it should be noted that diaphragms (pertaining to the precast elements) are expected to consistently maintain an economic margin in building construction. In the author’s experience, the floor elements of a structure can be the only large portions where the most (apparently) favourable cost and convenience may wholly governs the consultants selection process.

1.3.2 THE SOURCES OF IN-PLANE DIAPHRAGM FORCES

1.3.2.1 General

The basic sources of horizontal in-plane forces that are associated with fundamental diaphragm actions in seismic resistant buildings are separated into two classes; namely, simple and transfer diaphragm actions. The relative magnitudes and incidence of these actions within a structure will mainly depend on the overall configuration of the primary seismic resisting system. In general, seismic resistant structures are categorised as either moment resisting frames, structural (shear) wall systems or dual systems (hybrid structures) comprised of both structural walls and
frame elements. The deformation response of each of these structure types to earthquake induced ground motions will be somewhat different (Fig. 1.38). Because diaphragms subscribe to actions imposed by primary seismic resisting members, the characteristic in-plane force components of diaphragms in each of these systems will also be different.

![Diagram showing deformation patterns due to lateral forces of a frame, a wall element, and a dual system](image)

Fig. 1.38 Deformation patterns due to lateral forces of a frame, a wall element, and a dual system [Paulay and Priestley, 1992]

### 1.3.2.2 Simple Diaphragm Actions

Simple diaphragm actions will occur in all diaphragms and are due to the inertia forces that result from earthquake induced ground motions. The magnitude of these actions is determined by mass-acceleration relationships at the given levels that diaphragms occupy within a structure. Since the self-weight and payload of floor diaphragms may typically be around 70% of a building’s total weight, simple diaphragm action is the instigator of most of the horizontal shear forces that are caused by ground shaking.

The forces developed in simple diaphragms are distributed into members of the vertical primary seismic resisting system in accordance with relative stiffness, relative proximities and the availability of suitable load paths. Hence, strut and tie models are widely accepted as being the most pragmatic method for representing the flow of forces in these diaphragms. Analysis of typical strut and tie models will immediately indicate that force concentrations occur in the peripheral regions between the floor plate and vertical members.

Although simple diaphragms subjected to in-plane forces can usually be designed to behave as predominantly elastic elements, it is possible under severe actions that redistributions in force transfer capabilities will occur due to support member elongation and yielding of connections at the floor supports. For this reason it has been recommended that alternative load paths are considered for the inclusion of a viable tension resisting mechanism (Figs 1.39 and 1.40) [Paulay, 1999].
For the purposes of determining design forces, simple diaphragms are usually modelled as deep beams that react against uniformly distributed inertia forces derived from seismic response analysis. The interstorey shears resisted by vertical primary resisting elements (as well as secondary gravity systems) should have already figured in the derivation of horizontal inertia forces at given levels, and need not be considered further. However, the distribution of horizontal in-plane forces within the diaphragm itself must still be resolved. Because diaphragms account for such a large share of the overall seismic weight, it is important that their inertia forces can be distributed into vertical resisting members in near proportion to the stiffnesses attributed to these members in the response analysis. If suitable load paths cannot be secured, the contributions (actions and reactions) of potentially stiff primary resisting members at floor level may need to be reviewed. If load paths are not robust then damage (in the form of inelastic deformation) could be expected to occur within nominally elastically behaving diaphragms and result in the redistribution of load paths. A situation that warrants attention in this regard is where a sizeable floor opening lies adjacent to a vertical member.

Fig. 1.39  Resistance of a diaphragm to alternate directions of inertia forces using diagonal compression fields [Paulay, 1999]

Fig. 1.40  Resistance of a diaphragm to alternate directions of inertia forces using diagonal tension fields [Paulay, 1999]
1.3.2.3 Transfer Diaphragm Actions

Acute variations in the stiffness of primary seismic resisting systems over the height of a structure introduce large in-plane forces. These forces will need to be transferred to the locations of other primary seismic resisting members via the floor diaphragm plate at the level affected by the stiffness change. The typical such case is where a structure is comprised of a monolithically connected tower and podium (Fig. 1.41). The accumulated seismic shears from levels above the podium height will be distributed into the perimeter members of the podium through the transfer diaphragm at podium level.

Obviously some very large forces can be generated, and it is not uncommon for special detailing measures to be incorporated to assist (and direct) force transfer in these floors. These generally appear in the form of well reinforced infill strips that interconnect the vertical elements involved in the force transition (Fig. 1.42). Depending on the magnitudes of forces, additional reinforcement has often been included in the topping slabs as drag bars to achieve tensile force transfer capacity within transfer diaphragms. For the inclusion of any special reinforcing steel to be fully effective, some inelastic deformation within the diaphragm element must occur. Hence, the assumptions of ductile design may need to be applied to some portions of the diaphragm, in particular if concurrent rotations in the horizontal floor plane are likely to occur. For example, yielding of reinforcement may be expected where a floor plate containing drag bars connects with a ductile structural wall.

A similar situation that requires significant force transfer capabilities is where diaphragms interconnect between the structural walls and frame members that make up dual systems (Fig. 1.38). As indicated, the diaphragms of these systems act as in-plane restraints between two dissimilar structural systems so that deformation compatibility is maintained. The result is that net in-plane tension and compression forces must be developed through the floor slab over the height of the wall-frame interface (Fig. 1.43).

![Diagram of a typical transfer diaphragm at podium level](image)

*Fig. 1.41 Typical case requiring a transfer diaphragm at podium level*
Infill strip with drag bars that connect between rigid elements of the transfer diaphragm

Flooring units Confinement as required

Fig. 1.42 Infill strips and drag bars in a typical transfer diaphragm

Fig. 1.43 Restraint provided by the diaphragms of dual systems structures [Paulay and Priestley, 1992]

1.3.3 ANALYSIS AND DESIGN PROCEDURES FOR SEISMIC DIAPHRAGMS

1.3.3.1 General

In order to obtain design forces for either simple or transfer diaphragms, some reasonable estimates must first be made of the horizontal shears that may be attributed to the inertia forces of simple diaphragms within a structure. These shears have been described as estimates because it is generally agreed that even the most elaborate procedures for the seismic response analysis of primary seismic resisting systems are only approximations [Paulay and Priestley, 1992]. Hence, diaphragm response, which is derived as a next step from the same global response approximations, can only be taken as an estimate.

The following is a generalised overview of code provisions and methods of analysis used for the determination of diaphragm actions and detailing purposes:

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Chapter 1: Introduction: Floor Diaphragms in Seismic Resisting Structures
1.3.3.2 The Uniform Building Code (UBC:94) Approach to Diaphragms

Unlike earlier editions of this document, floor diaphragms are treated as being different to shear walls. The requirements of UBC:94 are mainly directed at local inertia forces and structural irregularities as given by section 1631.2.9, and special detailing in accordance with section 1921.6:

Local inertia forces are distributed over the height of a structure by a special formula which gives a lower limit of $0.35ZIW_{px}$ and need not exceed $0.75ZIW_{px}$. Because the lower limit is independent of both structural period and ductility, it will govern for tall ductile structures. Inertia forces calculated in this way are then directly added to forces that may arise from transfer diaphragm actions.

Special requirements are given for structures of irregular plan shape, which may experience differing responses within the same storey level and thus produce out of phase actions. For these diaphragms, drag bars and chords of reinforcing will need to be introduced unless a three dimensional analysis model can demonstrate that the configuration is otherwise sufficient. Limits are placed on the allowable stresses of connector elements in diaphragms that are subject to torsional response, or which contain re-entrant corners or abrupt discontinuities in geometry or stiffness.

Most of the detailing requirements of section 1921.6 are identical to the ACI code provisions for diaphragms that are subject to seismic forces. The method for calculating boundary forces in diaphragms is also as specified by the ACI code.

1.3.3.3 The ACI-318.95 Code of Practice

Section 21.5 of the ACI code [ACI-318, 1995] gives some special requirements for stiff structural systems, which specifically includes diaphragms, struts, ties, chords and collector members used to transmit forces induced by earthquake.

With regards to diaphragms, emphasis is placed on limiting extreme fibre stresses which are calculated using a linear elastic model and gross floor section properties. Unless the entire diaphragm already contains substantial transverse reinforcement (section 21.4.4), boundary members must be provided at diaphragm edges and at the perimeters of openings where extreme fibre stresses under factored loads exceed $0.2f_c'$. Boundary members required as such must also contain the transverse reinforcement specified by section 21.4.4 of the code. Where the calculated compressive stress is less than $0.15f_c'$, these boundary members may be discontinued.

Boundary members are proportioned to resist the sum of factored axial forces acting in the plane of the diaphragm plus the couple force that corresponds with the factored moment acting at the diaphragm section. The lever arm of the couple force is taken as the perpendicular distance between boundary members.

Chapter 1: Introduction: Floor Diaphragms in Seismic Resisting Structures
In review of the above requirements, boundary members containing a minimum of transverse reinforcement and fully anchored longitudinal reinforcement are prescribed for the control of seismically induced diaphragm forces. The code provisions appear to be straightforward in their implementation and should achieve a good level of protection for diaphragm elements. Because boundary members at the diaphragm perimeter may also form part of a primary seismic resisting system, the requirements for both transverse and longitudinal reinforcement given in section 21.4.4 of the code should be checked against the minimum requirements for members in potential plastic hinge zones.

1.3.3.4 The New Zealand Standards Approach to Diaphragms

The two documents associated with seismic diaphragm design for concrete structures are the Loadings Standard [Standards New Zealand, 1992] and the Concrete Structures Standard [Standards New Zealand, 1995]. In the Loadings Standard, diaphragm response is considered (as it has been traditionally) along with the “Requirements for parts” of a structure. The Concrete Structures Standard has introduced a section that specifically treats diaphragms as elements with separate detailing requirements.

The Loadings Standard advocates the use of the equivalent static, modal response spectrum and numerical integration time history methods of seismic response analysis. For structures that do not meet horizontal regularity requirements, three-dimensional analysis methods are necessary to determine seismic response behaviour.

With regard to diaphragms and structural regularity, the Loadings Standard gives relevant precautions on avoiding geometric and stiffness irregularities within diaphragms that may result in unfavourable response behaviour. Likewise, it is stated that diaphragms of a potentially flexible nature should be investigated to establish whether significant force redistributions may occur. In this way, the Loadings Standard emphasises the need for designers to recognise the areas of concern with diaphragms, and to devise solutions which are consistent with seismic analyses. Ultimately, the design process should incorporate the use of detailing methods such as those provided for diaphragms in the Concrete Structures Standard [Standards New Zealand, 1995]. With the general availability of powerful analysis software, this type of approach may be more in keeping with contemporary design practice than the prescriptive requirements of code documents.

Floor coefficients can be calculated from the “Requirements for parts” section of the Loadings Standard for corresponding equivalent static and response spectrum methods of analysis. For the ultimate limit state, the floor coefficient taken at or below the base of the structure is $0.25RZL_u$, which incidentally may exceed the floor coefficient at the adjacent first floor level. Thus, values taken from the ductile seismic response of the primary structure are scaled by appropriate factors to give a design envelope of floor coefficients that will generally correspond with fully elastic ($\mu = 1$) global response.

The application of elastic diaphragm response, as determined by a floor coefficient, should warrant some consideration. It is essential to fully consider diaphragm response if any inelastic behaviour is expected to occur, as permitted for parts by table C4.12.1 of the Loadings

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*Chapter 1: Introduction: Floor Diaphragms in Seismic Resisting Structures*
Standard. The reason is that unlike the true “parts” of a structure, yielding diaphragms can cause a redistribution in the transfer of forces which could in turn influence the response behaviour of the structure. Furthermore as already mentioned, greater ductility demands than the allowable specified ($\mu_p = 3$) for diaphragms under the “Requirements for parts” may occur in certain regions of diaphragms. This ductility demand will not necessarily be due to the proposed inertia forces, but rather through the accompaniment of inelastic deformations that are characteristic of ductile support members. The Concrete Structures Standard [Standards New Zealand, 1995] states that a special study or theoretical analysis is required for diaphragms that may exhibit inelastic deformations. However, this is only a requirement for inelastic deformations that are due to in-plane forces.

Based on the above, it is the author’s opinion that the Loadings Standard [Standards New Zealand, 1992] may not go far enough to distinguish diaphragms from their traditional classification as a “part” of a structure. This assertion is supported by the fact that the definition of a “part” of a structure does not encompass the broader function of diaphragms.

A “part” is described in the Loadings Standard as: “An element which is not intended to participate in the overall resistance of the structure to lateral displacement under earthquake conditions in the direction being considered”. This definition is contradictory of diaphragms when we consider, for example, the essential coupling role of diaphragms in dual system structures or the major stiffness modifications effected on beam elements due to their rigid connection to diaphragms. These influences are neither unintentional or unquantifiable at the initial design stage. Hence, the influence of diaphragms may be well integrated into both the response analysis and the impending assumptions on post-elastic behaviour of the primary structure. Because the disposition of true parts (e.g., cable train hangers, stand alone partition walls and coffee machines) is far more simply defined than as for diaphragms, it seems logical that diaphragms should be identified by the Loadings Standard as distinct elements with their own detailed requirements.

The new diaphragms section of the Concrete Structures Standard [Standards New Zealand, 1995] summarises the ordinary structural requirements for diaphragms such as minimum topping concrete thickness and maintaining composite tie action. These requirements are essentially identical to those of the preceding concrete design standard [SANZ, 1982]. However, the new section places direct emphasis on providing details that can perform to a level characterised by particular structural actions. This is indicated by the content of section 13.3.7.4 where it is stated that: “Connections by means of reinforcement and shear transfer mechanisms from precast concrete diaphragms to components of the vertical primary lateral force resisting systems shall be adequate to resist the relevant design forces”. This requirement becomes fundamental to the present course of study when we consider the full ramifications of the term “relevant”. The commonly employed tie connection detail (see Fig. 1.36) for floor construction has had no supporting data from tests involving actions that are characteristic of seismic resisting diaphragms.

In summary, the Concrete Structures Standard has introduced a section which gives useful guidelines for achieving structural control through diaphragms. The commentary (part 2) should
be read in conjunction with the standard to gain an understanding of some of the given requirements. Yet, it is evident that on the subject of seismic resistant diaphragms there is a lack of the gritty underlying experimental data that has been a hallmark of the New Zealand Standard. The diaphragms section of this document is more descriptive than prescriptive, which, for the greater part, may actually be the most pragmatic approach to the design of diaphragms. Nevertheless, the remaining issue is to resolve the capabilities of diaphragm connections that have been based on strength design parameters, but which must perform in context with a capacity design philosophy.

1.3.3.5 Rigid Diaphragms in Response Spectrum Analysis

Modal response spectrum analysis has been by far the most common form of computer analysis used in New Zealand. Until recently, most modal analysis software packages, such as ETABS, have modelled diaphragms as infinitely rigid plates. Thus, only three degrees of freedom feature in the floor analysis, which amounts to two translations and one rotation of the entire floor plate in the horizontal plane at a given level. This modelling infers that the floor portions of diaphragms do not accompany support member rotations through the horizontal plane, which is not even partially correct (otherwise floor serviceability deflections would not require checking under mere gravity loads). However, because diaphragms are such a dominant element under in-plane actions, the assumption of a rigid diaphragm has generally been considered adequate for analysis.

Assuming a rigid diaphragm may result in a lower estimated value of structural period, for use with basic seismic hazard spectra, than would result if floor deformations were included. For the majority of cases, the predominant first (fundamental) mode of vibration will therefore correspond with a higher estimate of spectral acceleration for the determination of seismic shears, which produces a conservative design. This is especially the case for many of the medium-rise structures that are common in this country. In the New Zealand Loadings code, the basic seismic hazard acceleration coefficient for subsoil category (a) (i.e., rock or very stiff soils) increases by more than a factor of two for fully elastic response as natural periods move between 1.0 seconds and 0.45 seconds [Standards New Zealand, 1992]. An overestimate of seismic shears should (albeit indirectly) result in the use of larger magnitude actions for the design of diaphragms.

Where higher mode effects such as significant torsion response of tall or irregular structures may be of concern, a diaphragm that is detailed to maintain structural compatibility and survive the concurrent overstrength actions of support members (particularly in the lower storeys of frames) under first mode response is likely to be adequate to ensure higher mode actions. At any rate, extra care with the analysis and detailing for interactions between the diaphragms and support members of torsionally sensitive structures is an implicit requirement of contemporary design practice [Standards New Zealand, 1995]. In many cases, this will involve such measures as introducing seismic gaps in diaphragms of irregular shape to produce a series of well conditioned sub-elements (Fig. 1.44).
The assumption of an infinitely rigid diaphragm can cause a potentially dangerous oversight, as it tends to produce an underestimate of horizontal storey deflections. This is mainly of concern with taller frame structures detailed for ductility in that P-delta effects will start to influence behaviour if limits on inter-storey drift are exceeded [Standards New Zealand, 1992]. In the analysis, the effects of diaphragm flexibility can sometimes be allowed for by reducing the stiffness of primary seismic resisting members that receive forces at the floor level connections [NZCS-NZNSEE, 1991].

![Plan configuration simplified by introducing a seismic break](image)

**Fig. 1.44** Plan configuration simplified by introducing a seismic break

### 1.3.3.6 Flexible Diaphragms in Response Spectrum Analysis

More recently, Response Spectrum Analysis programs that allow flexible diaphragm modelling have been made commercially available (e.g., ETABS version 6.0). The term “modelling” hasequipped because only rigid elements can have mass assigned to them for purposes of generating diaphragm inertia forces. Therefore, if the effects of diaphragm response are to be incorporated, the designer must construct a grid (checker board) model of rigid elements that are interspersed with flexible elements [Robertson, 1996]. Obviously in doing so, some care must be exercised with how the model is formed, especially with regards to such matters as maintaining compatibility at the boundaries of floor plate elements.

With flexible diaphragm analysis, the results should be subjected to a careful reckoning procedure because a large body of data is extracted giving only the absolute magnitudes of induced actions. The actual task of collating output data to the point of distinguishing concurrent principal actions can be time consuming and may also expose the analyses to interpretation errors (making post-processing an advisable option). Furthermore, the output of response spectrum analysis gives the envelopes of peak response as derived from the global response of the structure. Consequently, it is not feasible to check the equilibrium of diaphragm forces against the structural action envelopes generated for other (primary resistance) structural members.

Response spectrum analysis incorporating flexible diaphragms is generally considered to be a useful design tool for examining the elastic response behaviour of certain types of structures,

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especially for deflections criteria. Nevertheless, the detailing of reinforcement for diaphragms will usually need to follow more straightforward methods that involve basic equilibrium equations, such as with the strut and tie methods. This is partly because modal response spectrum analysis does not present the simultaneous coalition of forces and moments which act through a particular plane of reference, making it difficult to keep track of equilibrium in structural interactions. More importantly, response spectrum analysis cannot incorporate the influence of post-elastic behaviour nor the effects of overstrength actions in primary seismic resisting members. To remain consistent with the situation that exists in real members of the primary resistance structure during a severe earthquake, force transfer in diaphragms may best be accomplished by using equilibrium models and the capacity design approach to detailing [Hare, 1996].

1.3.3.7 Numerical Integration Time History Methods of Analysis

Both the linear and non-linear time history methods may be utilised to determine the particular response of primary seismic resisting elements and diaphragms. However, because of the computational effort involved, these methods are generally limited to checking the response of either unique or important structures. The Loadings Standard [Standards New Zealand, 1992] requires that at least three appropriately selected and scaled earthquake accelerograms are used, and that at least 15 seconds of strong motion shaking is featured in the analysis.

Like response spectrum analysis, the linear elastic time history method is strictly limited to linear elastic structural behaviour. Hence, an informed choice of initial stiffness values for the various structural elements is important. It is apparent that some care and foresight must be exercised when selecting nominal member stiffness for use in what are essentially elastic analyses. It must also be appreciated that alternating combinations of direct-axial forces and sway actions will continuously alter the stiffness of concrete members during an earthquake event.

The linear time history analysis method has a major advantage over response spectrum analysis in that it provides definitive information on structural performance, such as the likely magnitude and frequency of damage cycles caused to key structural elements. Because of the equilibrium equations which must be satisfied as part of the analysis, it is also possible to directly extract a particular regime of forces and moments acting in a member at a given interval.

Non-linear time history analysis involves much computational effort and is probably not appropriate for the routine analysis of diaphragms. Furthermore, the analysis procedure relies on elasto-plastic stiffness models which in turn rely on a predetermined configuration of reinforcing steel, both longitudinal and transverse in effect. Hence, a sensible choice of reinforcement configuration is normally required before the analysis can proceed without the risk of doing costly re-runs. As such, non-linear time history analysis is often considered as a useful devise for checking the performance of existing designs, and ultimately as a tool for researchers [Paulay and Priestley, 1992]. The reinforcing details in diaphragms of structures so examined will probably have been based on simple equilibrium models and the principles of capacity design.

Chapter 1: Introduction: Floor Diaphragms in Seismic Resisting Structures
1.3.3.8 Strut and Tie Methods (Simple Equilibrium Models)

Design is always an iterative process that involves progressive refinement, and as inferred by the preceding paragraph, even the most elaborate analysis methods may require fairly accurate input. Thus, a designer armed with this ability should at least have the skill to envisage the format of an acceptable engineering solution. In this regard, strut and tie methods form an extremely useful approach to resolving force transfer within diaphragm elements. Strut and tie methods (Fig. 1.45) are strongly advocated for the practical detailing of diaphragms.

A fundamental requirement of correct detailing practice is that equilibrium is achieved through viable load paths. Moreover, the capacity of reinforcing steel provided for this must be adequate to sustain the overstrength actions associated with the inelastic behaviour of supporting primary seismic resisting systems. Good detailing practice requires that continuous chords are developed by the tension reinforcement in an otherwise brittle medium. Hence, it is often more expedient for the designer to exercise this option in the first instance by using strut and tie methods.

It is widely recognised that a potential danger of designing the density of reinforcing steel placement to exactly defined zones of peak stress as derived from elastic analysis, is that consideration is not given to tension shift effects. Considering both response spectrum analysis and linear time history analysis: being purely elastic methods that are subject to the selective nature of input data, the peak actions derived at nominally selected node points within a flexible diaphragm are only discrete results (in the statistical sense of a result). Regardless of how much value the designer may place on the derived planar forces, it is fundamentally important that variations in the magnitude and location of tensile forces can be accommodated within the diaphragm element. The only safe and practical way that this requirement can be accomplished in ordinary reinforced concrete is by implementing a continuous distribution (chord) of fully anchored reinforcing steel. This naturally returns the designer to using simplified detailing practice that is based on strut and tie models.

Fig. 1.45 Strut and Tie models [Standards New Zealand, 1995]
1.4 PRECAST FLOORING AND SUPPORT MEMBER INTERACTION

1.4.1 GENERAL

As briefly discussed in section 1.2.3.4, much of the concern with the seating of precast flooring units can be attributed to their interaction with ductile members of the support structure. As a consequence, many of the assumptions used in the analysis of diaphragms may be of secondary importance to the present study, other than to help provide an overall basis for designs that will not exacerbate the problem. Actually, for the diaphragms of most buildings it is considered that variations in analysis input parameters (e.g., diaphragm flexural and shear stiffness) should not significantly affect the output [NZCS-NZNSEE, 1991]. With flooring unit support, the critical factor is global ductility demand and the need to maintain deformation compatibility with ductile members of the primary seismic resisting system.

The ductility demand of supporting members may set up patterns of inelastic deformation that induce certain reactions in flooring units. The deformations of members of the support structure are a by-product of the means by which seismic energy is dissipated through the formation of plastic hinges. As such, the nature of these actions is somewhat absolute and the flooring system is obliged to maintain deformation compatibility with the support structure.

1.4.2 THE INFLUENCE OF STRUCTURAL SYSTEMS

1.4.2.1 General

The manner in which plastic hinge formations may affect floor connections is a direct function of the type of structure involved. There are three main categories of ductile seismic resisting structural systems to consider: ductile moment resistant frames, structural wall systems and dual systems which utilise both frames and structural walls.

1.4.2.2 Ductile Moment Resisting Frames

Medium sized and tall moment resistant frames are designed in accordance with capacity design principles and the provision of beam sidesway (strong column-weak beam) mechanisms [Park and Paulay, 1975]. Accordingly, the structure is analysed and designed on the basis that plastic hinges will only form in beam members (usually where they connect to columns) and at the column bases as necessary (Fig. 1.46). The column sections above the base of the structure are designed to remain elastic or (ultimately) not to allow the formation of plastic hinges under severe seismic actions. For a given structure geometry, the degree of plastic hinge rotation generated in the beams is a function of the global displacement ductility demand and plastic hinge lengths. Analyses of frames has indicated that the displacement ductility factor of a frame is of the same order as a single degree of freedom system having the same force displacement characteristics and designed for the same fraction of elastic response loading [Park and Paulay, 1975]. Subsequently, the curvature ductility factor at beam plastic hinges may be estimated from Equation 1.18:
\[ \frac{\varphi_u}{\varphi_y} = 1 + \frac{\ell}{6r_p} \left( \frac{\theta_u}{\theta_y} - 1 \right) \] (1.18)

Fig. 1.46  Beam sway mechanism of a building frame

For a symmetrical beam side-sway mechanism, the rotational ductility factor at the beam ends \( \theta_u/\theta_y \) is usually found to be about twice the displacement ductility ratio \( \mu \). The ratio of beam length to plastic hinge length \( l/l_p \) may typically be about nine. Based on these parameters, the curvature ductility ratios \( \varphi_u/\varphi_y \) that must be attained in plastic hinge zones for given displacement ductility ratios \( \mu \) may be estimated for ductile moment resisting frames (Table 1.8):

Table 1.8  Typical estimates of curvature ductility ratios corresponding with displacement ductility factor ratios of frames, as calculated by Equation 1.18

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varphi_u/\varphi_y )</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>17</td>
</tr>
</tbody>
</table>

The relationship between curvature ductility factor ratios and strength development in plastic hinge zones of beams may not only depend on the quantity and effective depth of beam reinforcement but also on the layout of precast prestressed flooring elements. Figure 1.47 shows the formation of plastic hinges adjacent to the corner columns of a ductile frame structure.

Because of the complexity of earthquake ground motion, structural elements as such (Fig. 1.47) are designed to resist seismic forces acting independently along each axis of the structure.

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[Standards New Zealand, 1992]. Under combinations of oblique earthquake motion and induced torsion, it is anticipated that a severe earthquake will cause the simultaneous formation of plastic hinges in each frame axis [Park and Paulay, 1975].

Fig. 1.47 Plastic hinge formation at the junction of perimeter beams, indicating the likely floor plane fracture pattern of a composite hollow core flooring unit.
A detailed study of the mechanism of cast-in-place floor slabs acting as beam flanges has been conducted at the University of Canterbury [Cheung, 1991]. However, because of the very different nature of flooring systems involved there is a limit to the applicability of results from this research to the present study. The work of Cheung is based on traditional cast-in-place (two-way slab) floor systems, which for flexural reasons [Park and Gamble, 1980] contain a considerable amount of (often continuous) slab reinforcement that is well anchored into the support elements, including beam plastic hinge zones. Furthermore, a monolithic cast-in-place slab approaches a state that is both homogeneous and isotropic in its original unloaded condition, which is certainly not the case for modular pretensioned flooring systems.

By contrast, one-way prestressed flooring (especially hollow core) systems are generally designed as simply supported members, and tend to contain the minimal amount of "starter" reinforcement needed to form construction joints between the composite topping and beam members. This reinforcement is typically lapped with hard-drawn wire mesh in the composite topping slab and thus curtailed within 600 mm from the support (see Fig. 1.36). Moreover, the likely response of prestressed flooring units to the formation of beam plastic hinges will depend on the obvious differences in section for each direction of floor span (Fig. 1.47). Subsequent behaviour will be conditioned by the type of prestressed flooring sections involved, the strength and stiffness of shear connections between the beam and flooring elements and the actual position along beams where plastic hinges form.

Although the influences described in the foregoing paragraph may appear to present complications, they actually allow for simplifications in modelling the interactions between prestressed flooring systems and ductile beam members. Based on what are considered to be admissible simplifications, comparisons are made below with the findings of research into the contribution to beam tension of cast-in-place slabs [Cheung, 1991].

(a) Orthotropic (Orthogonally Anisotropic) Pretensioned Flooring

The orthotropic nature of modular pretensioned flooring is almost certain to limit the effective floor section contributing to beam tension in plastic hinge zones to the width of one unit module. This also results from inelastic curvature \( \varphi_p \) of the end support beam, which may produce a fracture along the weak joint between unit modules (Fig. 1.47). The result is a structural discontinuity that will interrupt the shear flow between prestressed units \((i \rightarrow j)\) and also the spread of flexural cracks into the sections of consecutive \((i \rightarrow j \rightarrow k \ldots)\) prestressed units (Fig. 1.48). It is a reasonable expectation that cracks will develop in the prestressed section (i.e., unit i) adjacent to a beam plastic hinge, in direct coalition with beam cracking. However, the ensuing degree of crack propagation and participation of flooring units in developing flange tension will be strongly influenced by the configuration of the prestressed floor section in relation to the adjacent beam (Fig. 1.48).

The section of composite topping over the adjoining edges of prestressed units is most often only lightly reinforced with hard drawn wire mesh. Tensile strains developed in the topping slab due to plastic hinging of the end support beam will quickly engage the mesh reinforcement and draw it to strains that correspond with tensile fracture. The elongation capacity of New Zealand

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manufactured hard drawn wire mesh has been shown by tests as generally less than 5% [Mejia-McMaster and Park, 1994; Herlihy, 1994]. Obviously, significant crack openings will occur and spread along the joints between prestressed units, causing a marked decrease in shear transfer across this region.

Fig. 1.48 Likely effective widths of tension flange for prestressed flooring systems with nominal side seatings onto ductile perimeter beams

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The flange tension mechanism associated with cast-in-place slab systems (Fig. 1.49) relies on developing orthogonal tension components to react against concrete strut segments. To achieve this, similar capacities of reinforcing steel $\Sigma T_x$ and $\Sigma T_y$ must act in each direction to satisfy equilibrium with the vector sum $D$ of the diagonal components (Fig. 1.50). The relatively small proportion of slab starter reinforcement involved with composite toppings coupled with short embedment lengths would not support the formation of this type of mechanism in pretensioned flooring systems. Furthermore, the axial component of prestress force would oppose the development of radial patterns of strut segments in pretensioned slab systems with adequately bonded toppings, as would the discontinuity caused by prestressed stems in tee and joist construction. Hence, the development of cracks in an orthotropic floor system will be most strongly influenced by the initial controlling factors of concrete section properties and relative stiffness in the orthogonal directions.

![Diagram](image)

**Fig. 1.49** Effective widths of tensile flanges for cast-in-place floor systems [Cheung, 1991]

![Diagram](image)

**Fig. 1.50** Equilibrium criteria for the mechanism of tension flanges in cast-in-place slabs [Cheung, 1991]
(b) Shear Flow and Construction Joints

The shear transfer across construction joints in diaphragms is normally based on the principles of shear friction [Kolston and Buchanan, 1980], in which the required area of shear friction reinforcement should be calculated by Equation 1.19 [Standards New Zealand, 1995]:

$$\mathbf{A}_{vf} = \left( \frac{V^*}{V_{fr}} - N^* \right) \frac{1}{f_y}$$

(1.19)

In the case where no external force is acting, Equation 1.19 assumes that a sufficient dislocation has occurred along the opposing faces of the construction joint to develop the yield force of shear reinforcement $A_{vf} f_y$. An amount of shear resistance can also be attributed to the dowel mechanisms of shear friction reinforcement, which become increasingly important at larger shear displacements (Fig. 1.51). However, the shear stiffness across typical floor construction joints is likely to be somewhat reduced at the stage when contributions from dowel action become significant [Paulay, Park and Phillips, 1974].

Fig. 1.51 The mechanisms of dowel action across a shear face [Park and Paulay, 1975]

As indicated by tests, the strength and stiffness of shear planes will not reduce until cracking has occurred and degradation of the joint concrete ensues. When crack widths are relatively small and maintained constant along the shear plane, it has been shown that appreciable shear stresses can be developed through aggregate interlock [Paulay and Loeber, 1974]. Under cyclic actions, it is recognised that the rate of stiffness degradation is more sensitive to the magnitude of prior loading cycles than the frequency of loading. This is because the mechanism of aggregate interlock, an important component of shear resistance, tends to become ineffective due to aggregate dislodging under intense cyclic forces (also see section 1.2.2.1).

For the edge shear keys of hollow core members (Fig. 1.52), equations are recommended for estimating the shear capacity of joints in both the uncracked and cracked states (Equations 1.20 and 1.21) [Brugelging and Huyghe, 1991]. Similar to other such methods, the maximum and residual shear stresses are taken as a function of mortar strength at the interface $f_{im}$ and the presence of a normal clamping stress $\sigma_C$. A safety factor of $\gamma = 2.5$ has been applied in these equations. Hence, the respective maximum $\tau_{max}$ and residual $\tau_r$ shear strengths may be taken as:

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Before cracking:  \[ \tau_{\text{max}} = 0.9 \tau_c^{0.7} (\ln \tau_{\text{cum}})^{0.18} + 0.05 f_{\text{cm}} \]  (1.20)

After cracking:  \[ \tau_r = 0.95 \sigma_c (\ln f_{\text{cm}})^{0.25} \]  (1.21)

---

**Fig. 1.52** Joint geometry and normal clamping stress [Bruggeling and Huyghe, 1991]

Due to volume changes, a topping crack may develop at the interface between the perimeter beam and the edge of first flooring unit (i) (Fig. 1.48). However, in one-way flooring construction this should remain as a hairline crack and significant concrete tensile capacity will exist across the edge joint under service loads. Hence, if the normal stress \( \sigma_c \) is taken as the tensile capacity of topping concrete (the key infill mortar in composite construction) downgraded to 0.33 \( \sqrt{f'_{\text{c}}} \), then Equation 1.20 will give a \( \tau_{\text{max}} \) of at least 2.5 MPa.

For most situations, a resisting shear stress of 2.5 MPa would be adequate to develop shear flow forces between the perimeter beam and floor unit. The value of \( \tau_{\text{max}} \) calculated as such should be considered conservative because a more effective shear strength will be developed across the monolithic portion of the composite topping, and a generous safety factor has been applied.

As the ultimate limit state is approached during a severe earthquake, extensive cracking is expected to occur in and around the perimeter region of the structure and Equation 1.21 will become more appropriate. An important consideration is that the effective clamping stress \( \sigma_c \) will be continually changing throughout the seismic attack, and this should reflect on the participation of flooring units in beam actions. Clamping stresses that occur in a construction joint during the compression run of a load cycle will overshadow the tensile contributions from starter reinforcement in developing shear capacity. Thus, assuming that the composite topping remains adequately bonded to the precast units, starter reinforcing will provide the base level of clamping stress to the adjacent construction joint, similarly as provided by \( A_v f_y \) in Equation 1.19.

Combinations of structural actions will produce tension \( T \) and compression \( C \) in the top fibres (the floor plane) of beam plastic hinges in regular ductile frames (Fig. 1.53). The combinations shown are all likely events during an earthquake, and are as induced by components of translation only as caused by ground motion that is oblique to the axes of the structure, and translation with torsion. Therefore, these combinations are characteristic of seismic response in all such structures, regardless of (but strongly influenced by) eccentricity of mass and building height etc.
At each of the corner columns (1, 2, 3 and 4 in Fig. 1.53) there are four combinations of actions to consider due to the formation of plastic hinges in beams under two-way frame actions. Because of the orthotropic nature of pretensioned flooring systems, each combination of plastic hinging in a specific column region will produce a different effect on the horizontal shear capacity of floor construction joints. For an isolated corner column region, which is taken at column 3 in Figure 1.53, the effects of combined actions under the given modes of frame displacement are presented in Table 1.9:

Fig. 1.53  Modes of frame displacement causing tension T and compression C in the floor plane adjacent to plastic hinges
Table 1.9 Combinations of two-way frame actions at column (Fig. 1.53) producing tension T or compression C in the top fibre of floor slabs constructed from modular pretensioned units with composite topping

<table>
<thead>
<tr>
<th>Frame displacement (translations only &amp; translation+rotation)</th>
<th>At beam plastic hinges column</th>
<th>Effect produced in flooring units adjacent to respective beam plastic hinges</th>
<th>Combined effect on floor system</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T + R</td>
<td>grid a</td>
<td>along grid line 1</td>
</tr>
<tr>
<td>(a-1)</td>
<td>(a-1)</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(a-2)</td>
<td>(a-2)</td>
<td>C</td>
<td>T</td>
</tr>
<tr>
<td>(b-2)</td>
<td>(b-1)</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>(b-1)</td>
<td>(b-2)</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although real seismic response may involve somewhat more complexity than graphic models of translation and rotation, certain fundamental elements of behaviour can be postulated from these. Referring to Figure 1.53 and Table 1.9, it can be concluded that tension should exist in the floor plane of a corner column region during three out of every four seismic response displacements and that response cycles will change the sign (sense) of each force. Thus, regions of in-plane compression C-C or tension T-T will tend to directly alternate, as will regions where the floor plate is subjected to vertical shears, C-T and T-C. This has important implications on providing sufficient capacity in the floor connections and construction joints to withstand recurring alternations of forces, be they in-plane axial forces or shears.
(c) **Strength Development in Pretensioned Flooring Tension Flanges**

As discussed, the magnitude of flange tension will be influenced by various aspects which relate to the type of pretensioned section involved and the affected shear capacities of construction joints under seismic actions. Another very important factor is the actual location where plastic hinges are formed along the frame members.

Where plastic hinges occur close to column faces, the tension developed by prestressing steel in an adjacent floor section will be limited by strand bond capacity with the surrounding concrete. From a design standards perspective, the length $L_d$ that is required from the free end of a member to develop the full tensile strength of prestressing strand is defined by envelopes of distance versus steel stress (Fig. 1.54).

![Diagram of steel stress vs distance]

**Fig. 1.54** Variation in steel stress with distance from the free end of a pretensioned unit [Standards New Zealand, 1995]

Accordingly, the length in the transfer zone $f_{se}d_{y}/21$ corresponds with a bond strength of 5.2 MPa and the interior portion $(f_{ps} - f_{se})d_{y}/7$ with a bond strength of 1.7 MPa [Collins and Mitchell, 1987]. The greater bond strength in the transfer zone is attributed to the Hoyer effect, which is not effective in the interior portion. However, it is important to note that significantly higher bond stresses, in the range of 3 to 5 MPa, have been obtained in tests where the Hoyer effect has been absent [Stocker and Sozen, 1970]. Other tests specifically performed on saw cut pretensioned hollow core units showed that an average of 85% of the design tensile strength of ½ inch 270 ksi (12.7 mm 1860 MPa) strand could be developed over a bond length of 48 inches (1.22 m) [Anderson and Anderson, 1976]. This length is about 60% of $L_d$ as derived from Figure 1.54 for the full strength development of strand, and indicates an average bond strength of 3.2 MPa over the bond length.

When flexural cracking occurs at a distance along the member which is less than required to develop the tensile strength of strand through bond, it is expected that strand slippage will result. This slippage will produce a mode of plastic mechanism which is characterised by
increasing slip displacement at peak bond stress (Fig. 1.55), and will therefore depict the upper limit of strand contribution to the development of flange tension.

![Graph showing force-slip relationship of prestressing strand](image)

**Fig. 1.55** Force-slip relationship of prestressing strand [Collins and Mitchell, 1987]

As the location of plastic hinges move away from columns, as is commonly achieved by bar curtailment and beam haunches (Fig. 1.56), the strength development of strands in the adjacent floor slab will tend towards the tensile capacity of prestressing steel. Furthermore, increased length into the floor section will facilitate shear flow bond between the pretensioned unit and the frame element. Thus, the full tensile strength of prestressing steel may contribute to the strength development in plastic hinges located away from column faces.

![Diagram showing plastic hinges](image)

**Fig. 1.56** Plastic hinges located away from the columns [Standards New Zealand, 1995]

For hollow core flooring, the contribution of individual strand numbers to tensile flange strength may be estimated from a simple model that is based on the diameter and grade of strand most commonly used and the results from equations and tests as referenced. Assuming that the tensile capacity of 12.7 mm supergrade (1860 MPa) strands is resisted by a uniform horizontal shear stress $\tau_r$ (see Equation 1.21) acting along the construction joint between the hollow core flooring unit and edge beam, then equilibrium of forces is achieved by:

$$V_{tr(t)} = F_{pr(t)}$$

(1.22)
Accordingly, the number \( n \) of strands considered effective in developing the design ultimate tensile strength of prestressing steel (186 kN) at distances 600 mm, 1.2 m and 1.8 m from the unit end may be calculated:

over transfer length at 600 mm: \[ n_{0.6} = \frac{\tau_r h}{200} \] (1.23)

average bond failure at 0.85\( f_{pu} \): \[ n_{1.2} = \frac{\tau_r h}{130} \] (1.24)

where strand bond will not fail: \[ n_{1.8} = \frac{\tau_r h}{100} \] (1.25)

Thus, for example, if \( \tau_r \) is taken as 1.5 MPa (0.33\( \sqrt{f'c} \) for 20 MPa topping concrete) and nominal side seating is provided giving a construction joint depth \( h \) of 265 mm (200 unit + 65 topping). The number of strands contributing to flange tension are estimated (Equations 1.23, 1.24 and 1.25) as two at 600 mm, three at 1.2 m and four at 1.8 m from the unit end.

The contribution of prestressing steel to the strength developed at plastic hinges will obviously be significant since a 12.7 mm supergrade strand has about the equivalent tensile strength of a 24mm Grade 430 bar at yield. For comparison, the negative bending moment capacity has been calculated for a typical configuration of perimeter beam plus bonded floor section (Fig 1.57) at first yield and at displacement ductility factors of \( \mu = 3 \) and \( \mu = 6 \) (see Table 1.8 and Fig. 1.58). The analysis has allowed for \( n = 0, 1, 2, 3 \) and 4 strands to participate in flexure.

![Diagram of beam junction](image)

**Fig. 1.57** Plan of beam junction in a ductile moment resisting frame incorporating a pretensioned hollow core floor unit, indicating progressive development of strand tension forces with distance from unit end

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Fig. 1.58  Negative bending moment development corresponding to the number of prestressing strands participating in flange tension at given structural displacement ductility factors (see Table 1.8 and Fig. 1.57).

Fig. 1.59  Moment-curvature relationships corresponding to the number of prestressing strands participating in flange tension at given structural displacement ductility factors (see Figures 1.57 and 1.58).

For the chosen section geometry and configuration of reinforcing bars and strand, the increase in bending moment capacity is 40 kNm per strand at first yield and around 57 kNm per strand at $\mu = 3$ and $\mu = 6$.

In the derivation of bending moment capacity the concrete spalling strain $\varepsilon_{sp}$ was limited to 0.004. Hence, the bending moment capacity decreases slightly due to the physical loss of section under curvature ductility factor demands corresponding with $\mu = 6$ (Fig. 1.59). The maximum concrete strain obtained from this analysis was 0.0146 ($3.65\varepsilon_{sp}$) for $n = 4$ strands at $\mu = 6$. The maximum concrete strain obtained at $\mu = 6$ for this section with $n = 0$ strands (i.e.,

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no tensile contribution from strands) was 0.00595 (1.49\varepsilon_{sp}). This difference in maximum concrete strain is quite pronounced, making it essential that effective confining reinforcement is provided in the plastic hinge zones of these structures. It should be noted that there has been no allowance in the analysis for the overstrength effects of reinforcement. The overstrength contributions of reinforced cast-in-place slabs have been summarised with similar conclusions by Cheung [Cheung, 1991].

(d) Frame Dilation

One of the recognised by-products of cyclic plastic hinge deformations of beams is the phenomenon of plastic hinge dilation, which involves the incremental elongation of beams (growth) due to the non-closure of cracks. Although documented in many research reports over the years, the actual effects of elongation on the performance of frame structures has only been postulated in fairly recent times. Many of the assumptions regarding the probable effects of plastic hinge dilation on frame structures (i.e., frame dilation) would appear to be upheld by the findings of several bodies of research work [Fenwick, 1990; Fenwick, Ingham and Wu, 1996].

The magnitude of beam plastic hinge dilation may be estimated as a function of beam depth and the degree of hinge rotation, and is given as [NZCS-NZNSEE, 1991]:

\[ \text{extension} = \frac{\theta (d - d')}{2} \]  \hspace{1cm} (1.26)

The combined elongations of more than one plastic hinge within a frame may depend on several factors, especially the level of axial restraint. For reversing plastic hinges (see Fig. 1.60), the rotation \( \theta \) applied at the hinge zone can be related to the interstorey deflection of the building by the expression:

\[ \theta = \frac{\Delta}{h \, k} \]  \hspace{1cm} (1.27)

where \( k \) is the ratio of the distances between the hinge zones in a beam to a column centreline and \( h \) is the storey height.

Because cracks in the compression zone of reversing plastic hinges essentially remain open unless appreciable axial restraint is applied, the total storey level extension based on two plastic hinges forming in each beam is:

\[ \text{extension} = \frac{2 \, n \, \Delta}{h \, k} \left( d - d' \right) \]  \hspace{1cm} (1.28)

where \( n \) is the number of bays in the frame.

For the case of unidirectional plastic hinges forming in beams (see Fig. 1.61), the total of extension is given by:
extension = \frac{n \sum \theta}{2k} (d - d') \tag{1.29}

where \( \sum \theta \) is the sum of plastic hinge rotations occurring in each bay of the structure. Although \( \sum \theta \) cannot be directly related to inter-storey drift in the straightforward manner of reversing plastic hinges, it has been tentatively suggested that \( \sum \theta \) should be taken to equal 5\( \Delta /h \).

Although it is considered that uni-directional plastic hinges will cause the greatest beam elongations, it is reversing plastic hinges that are likely to be synonymous with the type of frame elongation that can cause loss of support to pretensioned flooring units. This is because uni-directional hinges form in beams that are subjected to considerable positive bending moments under gravity loads, and include effects from end moment actions generated by frame sway during an earthquake. Where such a beam lies adjacent and parallel to the span of pretensioned flooring units, the pretensioned units will already have been designed to carry their entire self weight plus all superimposed loads assigned to them, and at a limited midspan deflection. Hence, there will generally be little gravity load distributed into beams that span parallel to pretensioned flooring units, and the development of uni-directional plastic hinges seems very unlikely.

Conversely, the beams that support the ends of pretensioned flooring units will be subjected to a considerable line load and unidirectional plastic hinges are much more a possibility. The consequence of plastic hinge dilation occurring along the axis of these beams is that openings may appear in the construction joints between flooring units, but there should not be a direct loss of support to flooring units through dilation effects. Again, because pretensioned flooring is designed on a modular basis, the degradation of construction joints between units will not normally jeopardise gravity load carrying capacity. However, designers should bear this in mind where heavy point loads have been distributed over more than one modular flooring unit. These units should be effectively tied together by ductile transverse reinforcement in the topping slab that extends at least a development length beyond the outermost contributing floor units.

Tests have shown that the total longitudinal elongation of a plastic hinge is in the order of 2% to 8% of beam depth, and depends on the degree of axial restraint provided. For the general case of beams in a frame configuration, extensions in the order of 2% to 4% are probably realistic considering the variability of axial loads along beam lines as larger ductility ratios are approached during a severe earthquake. Where this causes concern is with the seating of precast flooring when long spanning pretensioned units cover more than one span of ductile beams. The accumulated elongation of each beam span needs to be considered. Although it is evident that an amount of the accumulated elongation will be absorbed at crack openings occurring in the flooring unit adjacent to plastic hinges, it is likely that these actions will also weaken the shear capacity over the construction joint between the flooring unit and beam. Thus, it is feasible that much of the flooring unit length will be mobilised and net elongation will occur at one end of the unit. This, when combined with seating loss through the spalling of support beam cover concrete, gives sufficient reason for the review of construction practice.
Fig. 1.60  Reversing plastic hinges in a frame bay [NZCS-NZNSEE, 1991]

Fig. 1.61  Uni-directional plastic hinges in a frame bay [NZCS-NZNSEE, 1991]
1.4.2.3 Structural Wall Systems

The main consideration with ductile structural wall systems is that a severe earthquake will cause dilation effects in plastic hinge zones at the bases of large vertical wall elements. Although there has not been much specific research done on structural interactions resulting from wall elongation, there are accounts of wall dilation causing coupling effects in a full scale building test [Wight, 1985]. In the referenced test, coupling resulting from wall elongation was observed to cause an overall increase in structural stiffness and exerted an appreciable influence of the response behaviour of the building. By default, the flooring system is incorporated in these actions which may involve a number cyclic rotations about the horizontal plane of the floor plate (Fig. 1.62).

![Diagram of structural wall with vertical member coupling inducing bending at floor supports and plastic hinge elongation causing upward displacement of structural wall.]

Fig. 1.62 Coupling of the floor elements due to elongation of shear walls

The immediate implication of coupling rotations about the floor slab is that bending moment and shear demands are placed on the support details of flooring units (Fig. 1.62). A cantilever test that was conducted on a typical hollow core specimen with composite topping (see section 7.3.1) showed that the yield strength of deformed starter reinforcement was developed at a slab rotation of about 0.004 radians. If it is feasible that the elongation of a wall which resists both seismic forces and gravity loads will be around 1% of the wall length, then for a 6 m long wall the expected elongation is 60 mm. Typical floor spans are less than 16 m between supports (see Figs. 1.10), which strongly suggests that wall elongations of the aforementioned magnitude may cause starter bars strains which are well in excess of yield strain.

Hence, it is clear (Fig. 1.62) that for pretensioned floor spans which couple between ductile wall elements, there is a basic requirement that the wall support details are able to sustain cyclic
inelastic deformations. Furthermore, for the opposing supports, some consideration should be given to the potential spalling off of concrete in the seating regions.

1.4.2.4 Dual Systems

In the case of dual systems, there is an obvious hybridisation between moment resisting frame and structural wall systems. For ductile seismic resisting structures, both of these systems will integrally resist seismic shears, with deformation compatibility being maintained by coupling forces which act through the planes of the floor diaphragms [Goodsir, 1985](see Fig. 1.43).

With regards to dilation effects in primary seismic resisting members, consideration needs to be given to the characteristic elongation effects of both ductile frames and walls, as discussed in the previous sections. The added consideration is that the various tensile and compressive transfer (coupling) forces acting through the floor slab are essential to achieve dual system response. Consequently, the floor slabs in these structures need to be considered as an integral part of the primary seismic resisting system, and not as elements destined for little more than passive interactions. Hence, depending to a large degree on building configuration, sizeable axial forces will need to be developed concurrently with dilation effects which may arise either from frame members or structural walls, or both.

1.4.3 METHODS OF CONTROL FOR FLOORING-SUPPORT INTERACTIONS

1.4.3.1 General

The most important single attribute of support details which may be subjected to dilation type loadings is the ability to withstand axial deformation without appreciable loss of strength or viability of load carrying capacity. In brief, this is a prerequisite which may be classically termed a ductile support tie detail. Also, in developing such details, due consideration must be given to the mechanics by which ductility can be achieved. From a capacity design point of view, ductile tie details bear a fairly direct resemblance with the “weak link in the chain” analogy which is often used to explicate capacity design philosophy (Fig. 1.63). In using this type of approach, the various ductile and brittle components will need to be identified. The first caution is that a simple extrapolation from the common engineering perception which considers all steel reinforcement as ductile, is not sufficient for implementing fully effective tie details in modular flooring units.

Fig. 1.63 Chain analogy for illustrating capacity design [Paulay and Priestley, 1992]
Because tie details for hollow core units will involve embedment into a flexurally efficient section that contains no transverse reinforcement, simple but important rules for reinforcement type, reinforcement detailing and embedment length need to be carefully observed:

1.4.3.2 The Brittle Concrete Element

(a) Splitting Stresses

Due to the thin unreinforced webs of hollow core flooring units, the potential for splitting due to reinforcing bond stresses needs to be addressed. Experiments [Engström, 1992] have shown that the tensile extension of high strength deformed bars embedded in the cast-in-place concrete in the cores of hollow core units can cause web splitting. Since this is a highly unfavourable effect, the potential for web splitting must be eliminated from details that are required to perform in a ductile manner.

(b) Direct Tensile Fracture

The only necessary concrete fracture to form a ductile tie mechanism is that which occurs between the floor element and the support. The fracture line will be naturally instigated by the transition that occurs in materials and stiffness of the floor section where the precast unit meets the support. It is obvious that the propagation of further tensile fracture in the precast units should be avoided.

1.4.3.3 The Ductile Reinforcing Element

(a) Extendibility

Because of the potentially large ductility demand required in an extreme event, reinforcing steel must be of a ductile type with known qualities. Furthermore, tie reinforcement should be detailed so that large extensions can be obtained by yielding over a length where bond has broken down, and to undergo cyclic loadings without causing additional damage to the surrounding concrete. To achieve these requirements, straight, plain (i.e., non-deformed) reinforcing bar details are a necessity.

(b) Anchorage

Tension forces developed by tie reinforcement in hollow core units will need to be fully anchored in concrete zones where a sufficient compressive reaction exists. Plain bars will require a full hook at the end to achieve anchorage, with due care given to providing sufficient cover concrete to the end hook to avoid bursting and pullout at the hook.
1.5 OBSERVATIONS OF FAILURE

1.5.1 GENERAL

Although diaphragms are generally regarded as presenting less risk of serious failure than other components within seismic resisting structures, there have been occasions where considerable damage has been observed, sometimes resulting in complete or partial collapse.

There are records of multiple structural collapse involving untopped hollow core flooring, such as the Armenian earthquake of 1988. The Armenian disaster (as with earthquakes in similar regions) may have been somewhat predisposed, considering the low (often less than 0.05g) design base shears and general lack of ductile design attributed to the primary seismic resisting members. However, a number of important (if a touch primordial) observations were made regarding connections between floor diaphragm elements and support structures in Armenia. The following paragraphs are reproduced from a section titled “Many very old lessons relearned” taken from a paper which discusses the Armenian earthquake [Wyllie and Filson, 1989].

“Exterior walls must be tied to diaphragms. The complete lack of ties in the masonry buildings...has been seen in many earthquakes. Apparently they (i.e., the designers) relied on friction between precast plank (i.e., hollow core unit) and masonry at the bearings or they ignored the calculation for wall anchorage”. Furthermore “A structure must be tied together. The lack of building ties with the untopped precast hollow core floor planks without connections clearly illustrated this old lesson.”

Another observation of failure occurred in the Northridge earthquake of 1994, in which a section of the hollow core floor of a single storey car park building collapsed (Fig 1.64).

![Failure of a single storey car park floor at Northridge](image)

Chapter 1: Introduction: Observations of Failure
In the case of the Northridge structure, a hollow core floor with structural topping fell from steel beams which were supported on pin ended steel columns. Although exact details of this failure do not appear to have been reported, reconnaissance team photographs indicate that a chord tie failure may have played a part in the eventual collapse. The continuous reinforcing across the supports was comprised of wire mesh which appeared to fracture in tension and offer little resistance to the eventual collapse.

With respect to seismic design, the Northridge structure may have been of substandard construction when compared with other structures in California. The overall building layout consisted of an open fronted perimeter masonry wall with pin-ended steel columns supporting spans of steel beams onto which the hollow core was seated (Fig 1.65). The pin-ended columns would have offered little seismic resistance, and photographs indicate that the structure swayed outwards toward the open fronted side. There was no apparent damage to the masonry perimeter wall.

![Diagram of building layout](image)

**Fig. 1.65** Illustrative layout plan of the Northridge car park building

It is feasible that the chord tie then failed in tension, thus allowing the support beams to move apart and the collapse of the hollow core units to progress into the floor. Under such a loading regime it is possible that a deep block rotation would have occurred in the floor section, similar to the block rotations associated with flexural zones of deep beam sections. As already mentioned, the topping mesh appears to have provided little resistance against the actions which resulted in failure.

### 1.5.2 WELL ENGINEERED STRUCTURES

The relevance of observed failures in poorly designed and constructed buildings to well engineered buildings will depend on the comparative levels of demand placed on critical elements within the respective buildings. For example, in the Armenian and Northridge structures it is evident that certain details may have been present, but they were inadequate details for the localised levels of demand. In well engineered ductile structures, it is expected
that very high levels of demand will be placed on localised zones of key structural elements during a severe earthquake; this is the very basis of capacity design.

As such, demand may be roughly divided into two areas, strength demand and ductility demand. An idealistically safe seismic resisting structure would have a full compliment of both strength and ductility, however, alleviations in one of these qualities is usually permitted in design codes on the proviso that the other quality remains infallible.

It is perceived as being uneconomic to design structures which contain a full array of design features based on optimal strength and ductility, and this is justified in most cases. The general understanding is that the geometry of a structure will largely determine whether strength or ductility should be the governing criterion. However, the fact remains that individual designers often exercise differing views on acceptable building performance, meaning that a concession of strength in favour of ductility and vice versa is sometimes a fundamental topic of debate. The upshot is that many designers have preferences for particular building systems, such as the structural wall, dual systems and structural frames as discussed in the previous section.

Structures of dubious design and construction fail because the strength and/or ductility demand is exceeded. The obvious question with regards to floor construction joints is whether starter bar details provide adequate demand capacity in terms of strength and ductility to meet a very high local demand in the ductile moment resisting support structure.
1.6 AIMS OF THIS RESEARCH

The broad aim of this research is to provide additional knowledge relating to support tie details for hollow core flooring, and in such a manner that this knowledge may be interpreted by practising engineers for the betterment of seismic resisting construction. Consequently, the experiments conducted are of a rudimentary nature in that they are designed to establish basic facts which will reflect relatively straightforward but admissible assumptions.

Some analysis and theory is offered, and as always, it is to the disposition, experience, and discretion of those who interpret such workings as to their applicability and merit. It is the author's own belief that attempts should be made to present some form of theoretical development or analytical models. The reason is that, although the results of differing theoretical developments or analyses may vary, provided that they are based on reasonable assumptions and concur with the findings of others, they help to define the levels of confidence with which practising civil engineers may administer an otherwise approximate science.

The fundamental aims of this research are as follows:

- To examine the effects of dilation type loadings on support tie details employed in hollow core flooring construction joints.

- To examine the effect of dilation type loadings on the structural integrity of hollow core sections.

- To isolate the aspects of tie detail design which either mitigate or enhance their effectiveness as a support tie detail, under both monotonic and cyclic loading.

- To recommend tie details based on the findings which will withstand dilation type loadings and fully maintain the integrity of hollow core flooring systems.

- To examine related aspects such as moment continuity developed by ties, buckling of tie reinforcement under cyclic loading and bar bond development in topping slabs.
Diaphragm Analysis

2.1 GENERAL

In order to define the nature of interactive forces within flooring members that make up structural diaphragms, it is important to be able to predicate certain critical design actions. It is also important to be able to isolate the various components that may become crucial to the performance of flooring units in this role. In the introductory chapter, a number of fairly generalised concepts have been put forward, such as frame elongation, flooring participation in design seismic design actions and the likelihood of associated carry-over effects on flooring units. It is the purpose of this chapter to present analytical evidence of key structural actions that may be derived from the response of a tall building to earthquake ground motion. The chain of cause and effect is followed from base motion to the interaction between support and flooring members to diaphragm forces and the performance of individual materials within the diaphragm element.

2.2 RESPONSE BEHAVIOUR OF TALL BUILDINGS

2.2.1 BUILDING DEFORMATION AND PLASTIC HINGES

2.2.1.1 General

The development of plastic hinges in beams and walls is almost certain to have a direct influence on adjacent pretensioned flooring units, and probably vice versa depending on the structural configuration at floor level. The patterns of plastic hinge formation in a building will be largely influenced by structural geometry and the relative stiffness of primary seismic resisting members (see Section 1.4.2.). In order to design for such actions, methods acceptable to the engineering profession are employed, which generally involve recognised computer software packages (e.g., Response Spectrum Analysis and the ETABS family of programs). An alternative approach to the estimation of structural response has been derived, and is based on an equation of equilibrium. As a tool for precursory analysis, this approach has the advantage of directly defining structural displacement as a function of ground motion, distribution of storey mass and/or inter-storey stiffness, and the scheme is not necessarily limited to elastic assumptions. Once displaced shapes are ascertained, various important aspects of diaphragm behaviour can be determined by further considering structural interactions within the flooring support region.

Chapter 2: Diaphragm Analysis: General
2.2.1.2 Frame Displacement Model

The basis of Modal Response Spectrum Analysis is the dynamic response of a multistory shear building, for which the equilibrium of an applied force, inertial mass, acceleration and vertical stiffness effects are considered at each floor level (Fig. 2.1). By equating equilibrium at respective levels, and noting that typical seismic response only involves free vibration (i.e., no applied force terms), the well-known system of linear equations for an undamped structure may be formulated:

\[
[M]{y''} + [K]{y} = \{0\}
\]  

where \([M]\) and \([K]\) are the respective mass and stiffness matrices, \({y''}\) and \({y}\) are the respective horizontal acceleration and displacement vectors at storey height.

![Single column model representation of a shear building](Fig. 2.1)

This model can be elaborated to give the discrete system for the equilibrium of forces in a single bay of an arbitrary floor level (j) of a multi-storey building (Fig. 2.2).

\[
K_j(y_i - y_j) = M_j y''_j = K_U(y_i - y_j)
\]

- \(K\) = Interstorey stiffness
- \(M\) = Storey mass
- \(y\) = Relative storey displacement
- \(y''\) = Acceleration

![Equilibrium of forces at storey j in single bay of multi-storey building](Fig. 2.2)
For this model, the global horizontal storey displacements are denoted by \( \delta_i \), \( \delta_j \) and \( \delta_k \) for the respective levels \( i \), \( j \) and \( k \), and \( \delta''_j \) is the horizontal acceleration at level \( j \). For most well conditioned seismic resisting multi-storey buildings, it is reasonable to assume that the initial stiffness \( K \) of primary resistance members will be reasonably constant over the building height. Hence, by ignoring equivalent viscous damping (the effects of which can be approximated later) the requirements for equilibrium are that:

\[
M_j \delta''_j + K \left( \delta_j - \delta_i \right) = K \left( \delta_k - \delta_j \right)
\]  
(2.2)

Noting that the terms in brackets represent interstorey displacements, this may be written as:

\[
M_j \delta''_j + K \left( \Delta_{ij} \right) = K \left( \Delta_{kj} \right)
\]  
(2.3)

which gives:

\[
M_j \delta''_j = K \left( \Delta_{kj} - \Delta_{ij} \right)
\]  
(2.4)

This is an equilibrium model for discrete values, however, if the above was to be defined by a continuous function over the building height then the mass and displacements terms would need to be written in terms of the building height. By considering a distributed building mass \( m(h) \) as a function of height and in units of mass per unit length, Equation 2.4 may be written in differential form as:

\[
m(h) \frac{d^2 \delta}{dt^2} = K \frac{\partial \delta}{\partial h} \frac{dh}{dh}
\]  
(2.5)

This derivation may also be verified by assuming a differential element of height (Fig. 2.3). Hence, the partial differential equation for the approximation of building displacements as a function of time and height \((\delta = \delta(t,h))\) is:

\[
\frac{\partial \delta}{\partial h} = \frac{m(h)}{K} \frac{\partial^2 \delta}{\partial t^2}
\]  
(2.6)

\[
\begin{align*}
\text{V} & = \frac{\partial V}{\partial h} \text{dh} \\
\text{dh} & \rightarrow \\
m.dh.y'' & \rightarrow \\
\text{V} & = \text{Shear force} = K.y \\
m & = \text{Mass per unit height} \\
h & = \text{Height}
\end{align*}
\]

Fig 2.3  Equilibrium of forces for a differential element of height

---

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Incidentally, Equation 2.6 is of very similar form to the one-dimensional diffusion equation, for which many solutions exist. However, special consideration needs to be given to applying relevant initial conditions and boundary conditions.

The initial condition will essentially describe the initial or base ground motion (a displacement function) which will subsequently influence the general response of the structure. For brevity, it may be shown that a sinusoidal base motion of maximum amplitude $\delta_b$ will afford a simple solution and may also be regarded as consistent with general assumptions on seismic ground motion, therefore:

$$\delta(t,0) = \delta_b \sin \left( \frac{2\pi t}{T} \right) = \delta_b \sin \omega t$$  \hspace{1cm} (2.7)

Of the boundary conditions, the two important constraints are determined by the generic nature of seismic resisting building structures. Assuming (i) that the building is regular and vertical with no externally applied horizontal forces, and (ii) that the vertical resistance members will form plastic hinges at a rigid base:

$$\delta_{(0,0)} = 0$$
$$\frac{\partial \delta}{\partial h(t,0)} = 0$$  \hspace{1cm} (2.8)

The building mass per unit height $m(h)$ may be determined from the slope of the accumulated mass diagram for levels $0 \rightarrow n$ (Fig. 2.4). Since a constant initial stiffness has been assumed over the building height, the envelope of cumulative mass over height becomes an important parameter in the analysis. For well-conditioned seismic resisting structures, it is feasible that variations in inertial mass over the height of a building may be a more important design parameter than variations in initial stiffness (especially where significant superimposed floor loads are involved). Some forethought should always be given to possible variations in the distribution of occupancy loads in seismic resisting multi-storey buildings, but as a design principle, stiffness variations are avoided where possible. Nonetheless, it is evident that initial stiffness variations could have been treated similarly, with mass held constant over unit height.

An envelope for the cumulative mass $M_{(h)}$ from 0 at foundation level to the $j$th storey level may be expressed as:

$$\sum_{h=0}^{j} M_{(h)} = \sum_{h=0}^{H} M \left( \frac{h}{H} \right)^{n}$$  \hspace{1cm} (2.9)

where $n$ is a shape factor that is determined by fitting a power curve to the ordinates of the cumulative mass diagram (see Fig. 2.4).

It can be seen that $n = 1$ is the shape factor for a building with a uniform distribution of mass over total vertical height $H$, with $n > 1$ for top-heavy distributions and $n < 1$ for bottom-heavy distributions of mass.
Fig. 2.4  Cumulative mass over building height for determination of shape factor $\eta$

Hence, the mass per unit height may be calculated as the derivative of the cumulative mass model:

$$m_{(b)} = \frac{d}{dh} \sum_{h=0}^{\infty} M_{(b)} = \sum_{h=0}^{H} \frac{\eta h^{\eta-1}}{H^{\eta}}$$  \hspace{1cm} (2.10)

and the relationship of $m_{(b)}/K$ in Equation 2.6 becomes:

$$\frac{m_{(b)}}{K} = \sum_{h=0}^{H} M \frac{\eta h^{\eta-1}}{KH^{\eta}} = \alpha^2 h^{\eta-1}$$  \hspace{1cm} (2.11)

Solution of the PDE (Equation 2.6) is approached using the separation of variables method [Greenberg, 1988], which will yield two ordinary differential equations, one in time ($T$) and the other in height ($H$):

$$T'' + \kappa^2 T = 0$$
$$H' + \kappa^2 \alpha^2 h^{\eta-1} H = 0$$  \hspace{1cm} (2.12)

where $\kappa^2$ is the separation constant, which is not yet determined.

Respective general solutions to the ODEs in Equation 2.12 are:

$$T = A \cos \kappa t + B \sin \kappa t$$
$$H = C e^{-\kappa (\alpha^2 h^{\eta-1}) h^{\eta}} = C e^{-\kappa^2 \alpha^2 h^{\eta}}$$  \hspace{1cm} (2.13)

where $A$, $B$ and $C$ are arbitrary constants. By considering the cases for $\kappa = 0$ and $\kappa \neq 0$, and because the governing PDE is linear, the solutions may be summed to give:
\[ \delta_{(t,h)} = G + H t + (I \cos \kappa t + J \sin \kappa t) e^{-\frac{\alpha^2 \omega^2 h^2}{\eta}} \] (2.14)

where again, \(G, H, I\) and \(J\) are arbitrary constants.

Applying the boundary conditions given by Equation 2.8 results in the form of the equation for displacement as a function of time and height. To satisfy the boundary conditions without restricting the solution to the null result of \(J = 0\), \(\kappa\) is made equal to \(n\pi/T\) which gives:

\[ \delta_{(t,h)} = H t + J \sin \frac{n \pi t}{T} e^{-\frac{\pi^2 \alpha^2 h^2}{\eta}} \] (2.15)

In dealing with the arbitrary constant \(J\), it can be shown [Greenberg, 1988] that:

\[ \delta_{(t,h)} = H t + \sum_{n=1}^{N} J_n \sin \frac{n \pi t}{T} e^{-\frac{\pi^2 \alpha^2 h^2}{\eta}} \] (2.16)

As mentioned, the initial condition (Equation 2.7) is a reasonable approximation to base displacement during an earthquake that also easily satisfies Equation 2.16. However, more complex initial conditions could have been applied by constructing a Fourier series. Hence, Equation 2.16 becomes:

\[ \delta_{(t,h)} = H t + \delta_b \sin (\omega t) e^{-\frac{\alpha^2 \omega^2 h^2}{\eta}} \] (2.17)

In order to establish the velocity component \(H\), Equation 2.17 is differentiated with respect to time and suitable boundary and initial conditions are applied. The boundary condition for velocity at zero height (\(h = 0\)) is that the structure velocity is equal to the base velocity. Thus, the arbitrary constant \(H\) must equal zero at \(h = 0\). By considering the velocity component at the initial condition of \(t = 0\), the following is applicable:

\[ \frac{d \delta}{dt} \left( \delta_{(t,h)} \right) = \omega \delta_b \cos (0) = H + \omega \delta_b \cos (0) e^{-\frac{\alpha^2 \omega^2 h^2}{\eta}} \] (2.18)

\[ \therefore H = \omega \delta_b \cos \omega t \left( 1 - e^{-\frac{\alpha^2 \omega^2 h^2}{\eta}} \right) \]

When combined into Equation 2.17, the characteristic solution for horizontal structure displacement as a function of time and building height becomes:

\[ \delta_{(t,h)} = \delta_b \left( \sin \omega t - \omega t \cos \omega t \right) e^{-\frac{\alpha^2 \omega^2 h^2}{\eta}} + \omega t \cos \omega t \] (2.19)

Thus, Equation 2.19 describes the undamped motion of a structure subjected to a sinusoidal ground motion, starting from an at-rest initial condition at \(t = 0\). Undamped deformation

*Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings*
response according to Equation 2.19 is shown (Table 2.1 and Fig.2.5) for a hypothetical ten storey frame with an estimated natural period of $T = 1$ second and total mass of 3200 tonnes. The frame is subjected to a constant sinusoidal base motion with peak acceleration of 0.3g. The interstorey height is 3m and interstorey stiffness $K_o$ is constant at 500 MN/m. The deformed response is shown for three different cumulative distributions of building mass of $\eta = 0.8$, 1.0 and 1.25. A uniformly distributed mass of 320 tonnes per level applies when $\eta = 1.0$.

Table 2.1 Undamped displacement response at each level of a hypothetical 10 storey shear building subjected to a sinusoidal base motion with peak acceleration of 0.3g

<table>
<thead>
<tr>
<th>storey level</th>
<th>$\eta = 0.8$</th>
<th>$\eta = 1.0$</th>
<th>$\eta = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(time elapsed: $t-t_0$)</td>
<td>(time elapsed: $t-t_0$)</td>
<td>(time elapsed: $t-t_0$)</td>
</tr>
<tr>
<td></td>
<td>1 sec.</td>
<td>5 sec.</td>
<td>9 sec.</td>
</tr>
<tr>
<td>Base</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>18</td>
<td>92</td>
<td>166</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>159</td>
<td>286</td>
</tr>
<tr>
<td>3</td>
<td>43</td>
<td>217</td>
<td>390</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
<td>269</td>
<td>485</td>
</tr>
<tr>
<td>5</td>
<td>64</td>
<td>318</td>
<td>573</td>
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<td>73</td>
<td>364</td>
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<td>82</td>
<td>408</td>
<td>734</td>
</tr>
<tr>
<td>8</td>
<td>90</td>
<td>449</td>
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</tr>
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<td>9</td>
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<td>488</td>
<td>879</td>
</tr>
<tr>
<td>10</td>
<td>105</td>
<td>526</td>
<td>947</td>
</tr>
</tbody>
</table>

*Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings*
Fig. 2.5  Displacement response, as described by equation 2.19, for a hypothetical 10 storey building (see Table 2.1)

With regard to estimating the effects of damping, an overview is required of the terms in Equation 2.19. If the velocity based terms in Equation 2.19 are removed (i.e., $\omega t \cos \omega t = 0$ for all t), the residual Equation 2.20 satisfies the equilibrium of forces as prescribed by Equation 2.6 and describes the motion of a structure that effectively has nil resonant response, thus:

$$\delta_n(t, s) = \delta_b \sin \omega t e^{-\xi \alpha_1 h n}$$  \hspace{1cm} (2.20)

This may be regarded as the steady-state response for this model. With the cosine terms included, dynamic amplification occurs and Equation 2.19 will no longer satisfy the dynamic equilibrium requirements of Equation 2.6. However, since the response of the undamped structure will tend towards infinity, an imbalance of forces is anticipated because the system does not approach a state of equilibrium (which is consistent with harmonic response at or near resonance). Furthermore, the model assumes that the structure will participate regardless of the particular acceleration imparted by the selected base motion, and that the degree of participation depends on the absolute acceleration associated with the base motion (note that $\omega$ in the foregoing equations applies to the actual frequency of base motion, it does not infer the natural frequency of the superstructure). Consequently, the model requires that a plausible estimate is made of the shortest fundamental period of vibration that might be applicable to the structure, as this becomes the critical base shear parameter.
It is well understood that real structures cannot achieve the levels of resonant response implied by theoretical considerations of elastic dynamic response models. The reasons for this are partly analytical, such as the predictable yielding of materials, and partly empirical, such as the attributed levels of equivalent viscous damping within the structure. In the above model, the yielding of material can be allowed for by reducing the stiffness coefficient. This modification will not affect the validity of the steady state solution (Equation 2.20) and is therefore seen as applicable to Equation 2.19. However, degrading stiffness will also result in reduced resonant response, and this can be applied to the model semi-empirically by considering the reduction in initial stiffness (i.e., the stiffness at elastic levels of displacement) as a function of load cycles. A general observation of tests on elements of building frames displaced inelastically to large percentages of interstorey drift, is that the initial stiffness of a load cycle decreases at a decreasing rate until an almost constant residual value is reached (see Fig. 2.6). Applying reduced stiffness to response analysis has been advocated as a realistic method for modelling the effects of damping [Paulay and Priestley, 1992] and a general observation suggests that a suitable model could take the form of an exponential decay function. The reduction in initial stiffness can be taken as the ratio of the tangents of the angles relating load to displacement after a series of load cycles. Figure 2.6 indicates that the initial stiffness may decrease from peak to a near static value by a factor of about 1/15 after about seven post-elastic load cycles.

![Graph](image)

Fig. 2.6  Lateral load displacement response of ductile frame element [Restrepo, 1993]

Hence, if the a stiffness reduction coefficient $\zeta$ is adopted and applied to the resonance terms of Equation 2.19, a progressive diminishing will occur in the amplitude of response as time (and the corresponding number of load cycles) increases. For the development of such a coefficient (see Fig. 2.7) values of initial stiffness $K_0$, eventual stiffness $K_N$ and the corresponding number of damaging load cycles $N$ to achieve the eventual stiffness are required. The coefficient also represents the level of force activity in the structure, with greater forces resulting in greater

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
degradation in the plastic hinge zones. As a result, the rate of change of the stiffness reduction coefficient after \( n \) cycles is also a function of the coefficient, hence:

\[
\zeta - \left( \zeta + \frac{d\zeta}{dn} \right) = \zeta \frac{(K_0 - K_N)}{NK_0} \cdot dn
\]  

(2.21)

Integrating over the interval \( n \) and applying the initial condition of \( \zeta = 1.0 \) at \( n = 0 \) gives:

\[
\zeta(n) = e^{-NK_0 n}
\]

(2.22)

A further assumption is made that the fundamental mode of vibration is critical, with \( n = t/T \):

\[
\zeta(t) = e^{-NK_0 \frac{t}{T}}
\]

(2.23)

Fig. 2.7 Parameters utilised in the development of stiffness reduction coefficient \( \zeta(t) \)

In application of the stiffness reduction coefficient, Equation 2.19 is rewritten as:

\[
\delta(t,n) = \delta_b \left( \sin \omega t - \zeta \omega t \cos \omega t \right) e^{-\alpha \frac{t^3}{N}} + \zeta \omega t \cos \omega t
\]

(2.24)

The effect of the reduction coefficient on resonant response behaviour is shown (Fig. 2.8) for a hypothetical ten storey frame with an estimated natural period of \( T = 1 \) second that supports 320 tonnes at each storey and is subjected to a constant sinusoidal base motion with peak acceleration of 0.3g. The initial interstorey stiffness \( K_0 \) is taken as 500 MN/m, which degrades to \( K_N = 50 \) MN/m over \( N \) cycles. \( N \) has been varied to show the effect of differing rates of stiffness degradation; a larger value of \( N \) corresponds with a prolonged elastic response:
Fig. 2.8 Influence of stiffness reduction coefficient \( \zeta \) (according to Equations 2.23 and 2.24) on first mode response at differing rates of degrading stiffness

Because Equation 2.24 describes the displaced shape of the responding structure, from this we can obtain an estimate of building slope at given storey levels through differentiation, thus:

\[
\frac{d\delta}{d\tau} = \theta_{(b,\alpha)} = \delta_b \omega^2 \alpha^2 \frac{h^n}{h} \left( \sin \omega t - \zeta \cos \omega t \right) e^{-\omega^2 t^2 h^n} \tag{2.25}
\]

For the conceptual ten storey frame, variation in slope with height is shown (Fig. 2.9) at peak response for three differing configurations of distributed mass (i.e., \( \eta = 0.8, \eta = 1.0, \eta = 1.25 \)) (see Fig. 2.4) at an assumed stiffness degradation rate that corresponds to \( N = 7 \) (see Fig. 2.7):

Fig. 2.9 Building slope \( \theta_{(b,\alpha)} \) from vertical at storeys one through ten, showing the influence of differing vertical distributions of building mass

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
With regard to diaphragm analysis, the building slope at storey level (angle of drift) gives an apparent indication of the local rotational demand on support members. As discussed in section 1.4.2.2(c), consideration may then be given to the advent of flexural strength contributions from prestressed flooring units as a result of participation in plastic hinge rotations. Moreover, an estimate of the interstorey drift angle allows estimates to be made as to the potential effects of frame elongation, as described in section 1.4.2.2(d).

In order to relate the angle of drift at storey level with the bending strength development of prestressed flooring units, inelastic structural displacements need to be associated with the curvature ductility ratio demands in plastic hinge zones. The initial approach is to geometrically relate the rotation of vertical members to the average magnitude of rotation in support member plastic hinge zones (Fig. 2.10). Although the determination of curvature ductility ratios and member elongation may initially be separate issues in routine design, the two are fundamentally linked and this geometric model of rotation may be assumed as equally relevant to each. Unidirectional plastic hinges are not examined because of the reasons stated in section 1.4.2.2(d) regarding beam members spanning parallel to self-supporting pretensioned flooring units.

Fig. 2.10 Geometric model relating plastic hinge rotations within a frame assembly [after Restrepo, 1993]

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
2.2.1.3 Strain Energy and Equivalent Flexural Rigidity

For ductile beam members in seismic resisting structures detailed to code specified dimensions and reinforcement configurations [Standards New Zealand, 1995], the far greatest proportion of member rotation will result from deformations within plastic hinge zones. As such, detailed analysis of elastic bending deformations is not considered critical in establishing rotational magnitudes of seismic resisting members; the relationship between plastic and elastic portions of a member may be generally accounted for by use of correctly modified member stiffness. Likewise, it is usually sufficient to assume an effective plastic hinge length in order that curvature ductility ratios can be calculated. However, it is also a reasonable expectation that pretensioned flooring sections incorporated in plastic hinge rotations will exert unique influences on the characteristics of plastic hinge development. Although the consideration of overstrength capacity is an underlying statement in capacity design, specific analysis of effects arising from beam and pretensioned floor member interaction (see section 1.4.2.2(c)) does not appear in design literature. Thus, it is important to determine what variations may occur in frame displacement characteristics as a result of floor slab participation. The initial consideration has been to demonstrate that increased moments (and the associated shears) are fundamental in this process. The following examines the likely influence of a prestressed component on beam plastic hinge functions that subsequently effect displacement capacity and effective frame stiffness:

By considering an approach in which internal strain energy is equated, it can initially be shown for any beam section subjected to bending that:

\[
U_b = \int_{L} M_{\phi}(x) \varphi(x) \, dx
\]

(2.26)

where \(U_b\) is the total bending strain energy, \(M\) is bending moment and \(\varphi\) is curvature, and both are functions of length \((x)\). In the case of a reinforced concrete beam, this may be written in terms of the constituent materials, where \(u_c\) and \(u_s\) are the respective strain energy densities per unit volume of concrete and reinforcing steel:

\[
U_b = \int_{L} M_{\phi}(x) \varphi(x) \, dx = \int_{V} u_c \, dV + \int_{V} u_s \, dV
\]

(2.27)

The volume element \(dV\) is equal to \(dA \cdot dx\); \(f_c\) and \(f_s\) are the concrete and steel stresses that correspond with respective strains \(\varepsilon_c\) and \(\varepsilon_s\) to give strain energy densities. These energy density quantities are most commonly denoted as the areas under characteristic material stress-strain curves (see Fig. 2.11):

\[
U_b = \int_{L} M_{\phi}(x) \varphi(x) \, dx = \int_{L} \int_{\varepsilon_c} f_c \, A_c \, d\varepsilon_c \, dx + \int_{L} \int_{\varepsilon_s} f_s \, A_s \, d\varepsilon_s \, dx
\]

(2.28)

Equation 2.28 is a general summation. Because the strain energy stored in an effective length of plastic hinge may be taken as approximately constant over the hinge length, the equation may

\[Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings\]
be simplified to consider only the strain energy per unit length within the beam plastic hinge zone. Also, since the plastic hinge zones of seismic resisting beam members should maintain a constant area of principal reinforcement, the strain energy per unit length \( \Psi^* \) due to bending becomes:

\[
\frac{\partial U_b}{\partial x} = \Psi^* = \int_{\varepsilon_b}^{\varepsilon_u} A_e \, d\varepsilon_e + A_t \int_{\varepsilon_t}^{\varepsilon_u} d\varepsilon_s
\]  
(2.29)

Equation 2.29 can be easily verified for a homogeneous prismatic section such as an uncracked plain concrete beam that is subjected to a bending moment \( M \), as strain energy per unit length is directly proportional to applied moment (Fig. 2.12). Hence, if stress is proportional to strain and elastic modulus \( f = \varepsilon \cdot E \), strain is proportional to curvature and distance \( \varepsilon = \varphi \cdot y, \, d\varepsilon = \varphi \cdot dy \) and area \( dA = b \cdot dy \). The elastic bending strain energy per unit volume is generally expressed as:

\[
U_{(y)} = \frac{\sigma_{(y)}^2}{2E}
\]  
(2.30)

then for a section of total depth \( h \):

\[
\frac{\partial U_b}{\partial x} = M \varphi = 2 \int_0^{\frac{h}{2}} \! U_{(y)} \, bdy = E b \varphi \int_0^{\frac{h}{2}} \! y^2 \, dy
\]  
(2.31)

which gives the elastic bending strain energy relationship of:

\[
M \varphi = E b \varphi \left[ \frac{y^3}{3} \right]_0^{\frac{h}{2}} = \varphi^2 E \frac{b d^3}{24} \text{ or } \frac{\varphi^2 EI}{2}
\]  
(2.32)

or the more familiar form (since in elastic bending \( \varphi = M/ED \)):

\[
M \varphi = \frac{M_{(y)}}{2EI}
\]  
(2.33)

Equation 2.31 can be used for evaluation of cracked sections provided that the actual (or characteristic) material stress-strain relationships are used. Thus, as is typical to plastic bending analysis of reinforced concrete members, strain is generally considered proportional to curvature and distance but stress is not necessarily proportional to strain, etc. Hence, as bending stresses exceed the material proportional limits, strain energy per unit length is no longer directly proportional to bending moment and therefore the symbol \( \Psi^* \) has been adopted, instead of the product \( M \varphi \), to represent strain energy per unit length.
Fig. 2.11  Strain energy densities for concrete and steel as a function of one dimensional (axial) strain

Fig. 2.12  Elastic bending assumptions of a prismatic section

If a simple parabolic stress-strain model is assumed for the concrete component, Equation 2.31 may be written as:

\[ u_{(c)} = \int_0^c f_{(c)} \, \text{d}x \quad \text{or} \quad u_{(c)} = \int_0^c f_{(c)} \, \text{d}y \quad (2.34) \]

where, for the concrete stress block:

\[ u_{(c)} = f_{(c)} \int_0^c \left[ \frac{2 \varphi y}{\varepsilon_0} - \left( \frac{\varphi y}{\varepsilon_0} \right)^2 \right] \, \text{d}y \quad (2.35) \]

Integrating over the element of concrete area \( \text{d}A = b \, \text{d}y \) between the limits of zero and the neutral axis depth \( c \), and combining with the steel component gives:

\[ \Psi^* = b f_{(c)} \int_0^c \left[ \frac{\varphi y}{\varepsilon_0} \right]^2 \, \text{d}y + A_s \int_0^c f_s \, \text{d}x \quad (2.36) \]

and because \( \varphi = \varepsilon_c / c \), after integration and substitution this can be reduced to:

---

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\[
\psi^* = \frac{b f_c \varepsilon_c^3}{\varphi \varepsilon_o} \left( \frac{1}{3} - \frac{\varepsilon_c}{12 \varepsilon_o} \right) + A_s \int f_s \, d\varepsilon_s \tag{2.37}
\]

For practical application, the second term in Equation 2.37 is treated as a discrete summation of individual reinforcing steel (including prestressing steel) components. Equation 2.37 simply expresses a summation of strain energy per unit length of composite materials, and for the preliminary analysis of beam plastic hinges, two typical cases emerge (see Fig. 2.11):

(a) compression steel has not yielded:

\[
\psi^* = \frac{b f_c \varepsilon_c^3}{\varphi \varepsilon_o} \left( \frac{1}{3} - \frac{\varepsilon_c}{12 \varepsilon_o} \right) + \sum_i A_{h_i} E_t \left( \varepsilon_e - \varphi d_i \right) + \sum_i A_{h_i} f_y \left( \varphi d_i - \varepsilon_e - \frac{\varepsilon_y}{2} \right) \tag{2.38}
\]

(b) compression steel has yielded:

\[
\psi^* = \frac{b f_c \varepsilon_c^3}{\varphi \varepsilon_o} \left( \frac{1}{3} - \frac{\varepsilon_c}{12 \varepsilon_o} \right) + \sum_i A_{h_i} f_y \left( \varepsilon_e - \frac{\varepsilon_f}{2} - \varphi d_i \right) + \sum_i A_{h_i} f_y \left( \varphi d_i - \varepsilon_e - \frac{\varepsilon_y}{2} \right) \tag{2.39}
\]

In Equations 2.38 and 2.39, the first term relates to concrete, the second to compression steel and the third term to tension steel. It should be noted that Equations 2.38 and 2.39 do not allow for the effects of strain hardening and maximum steel strain is taken as equal to or less than strain at strain hardening. However, it is considered that the dimensional characteristics of plastic hinges will have developed before the onset of strain hardening. Strain hardening will effectively allow more strain energy to be stored in a given length of plastic hinge, resulting in a concentration of plastic hinge rotation under increased bending moment. This is a significant effect with regard to the participation of flooring units in plastic hinge rotations, however, the principal field of rotation (plastic hinge zone) will already be evident in the section.

It is proposed that the above model and summation may be utilised to establish the equivalent flexural rigidity (EI value) of a concrete section at various stages of flexural response. The equivalent flexural rigidity, denoted EI*, may be derived by considering the general summation provided by Equation 2.37, and re-arranging the terms of Equation 2.32 so that:

\[
EI^* = \frac{2 \psi^*}{\varphi^2} \tag{2.40}
\]

Verification of the foregoing equation is made by examination of a simple reinforced concrete structural wall element that is subjected to combinations of axial force and out-of-plane bending moment. The wall is 120 mm thick and reinforced by HD12 bars at 250 mm centres (Fig. 2.13), and the following assumptions are made with regard to materials (Table 2.2). Material stress-strain relationships are assumed to behave in accordance with Figure 2.11. The customary
parabolic stress-strain relationship for unconfined concrete has been adopted, as depicted by Equation 2.35.

Table 2.2 Material parameters for derivation of equivalent flexural rigidity of wall element

<table>
<thead>
<tr>
<th>$f_c$ (MPa)</th>
<th>$f_{cr}$ (MPa)</th>
<th>$\varepsilon_0$</th>
<th>$E_c$ (GPa)</th>
<th>$f_y$ (MPa)</th>
<th>$E_a$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.0</td>
<td>0.00213</td>
<td>23.5</td>
<td>430</td>
<td>200</td>
</tr>
</tbody>
</table>

![Diagram](image)

Fig. 2.13 Singly reinforced wall element subjected to combinations of axial force and bending moment

The equivalent flexural rigidity $EI^*$ per meter length is derived for three axial force regimes: (i) 50 kN axial tension, (ii) zero axial load and (iii) 50 kN axial compression. An elastic phase is exhibited between zero bending moment and first cracking moment $M_{cr}$. At first cracking moment, the equivalent flexural rigidity is compared with the same property calculated by elastic cracked section methods. Beyond first cracking, the equivalent rigidity is traced up to the ideal flexural capacity $M_i$ at an ultimate concrete strain of 0.003.

(a) Flexural Rigidity at First Cracking

The cracked elastic section properties are calculated by initially locating the neutral axis of the cracked section. Hence, first moments of area must be taken, and for a singly reinforced element, this will effectively balance the area of concrete in compression by the transformed area of reinforcement $nA_s$. The resulting formulation for neutral axis depth $y$ may be written as:

---

*Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings*
\[ y = \frac{i}{b} \left( \sqrt{\frac{nA_s}{nA_s + 2bd - nA_s}} \right) \]  

(2.41)

and the second moment of inertia about the neutral axis:

\[ I_{cr} = \frac{by^3}{3} + nA_s(d - y)^2 \]  

(2.42)

For the given configuration of section and materials (see Fig. 2.13 and Table 2.2), it follows that:

\[ y = 17.98 \text{ mm} \]
\[ EI_{cr} = 205.15 \times 10^9 \text{ Nmm}^2 \]

The equivalent section rigidity is obtained by consideration of Equation 2.37, and observing that the solution by Equations 2.41 and 2.42 is not sensitive to concurrence: levels of bending curvature. It is also evident that the solution in accordance with Equation 2.40 may only converge with the traditional section analysis, since curvature cannot equal zero. This is not to say that the equivalent flexural rigidity is less accurate. In fact, the contrary is true because the \( EI^* \) approach allows for the actual response of materials to be modelled through all stages of loading, and bending curvature will always be present. Likewise, the \( EI^* \) method will allow section analysis for situations where reinforcement has yielded and/or where prestressing steel is present. However, section analysis based on cracked elastic properties is often an approximation because, as typified by the given wall example, concrete compressive stress increased from 3.0 MPa to around 15 MPa at the instant of cracking. At this level of stress (i.e., 60% of crushing strength), there is certainly a departure from elastic behaviour within the concrete compression block.

Hence, for comparison with elastic analysis a nominally small level of concrete strain is adopted so that departure from linear elastic behaviour is negligible. As such, the same neutral axis depth \( y \) used in elastic analysis is applicable:

\[ \varepsilon_e = 1 \times 10^{-10} \quad \text{and} \quad y = 17.98 \text{ mm} \]
\[ \varphi = 5.562 \times 10^{-12} \text{ } \theta / \text{mm} \]

The summation of strain energy per unit length will involve the concrete compression block and unyielded tension reinforcement:

\[ \Psi^* = \Psi_{e} + \Psi_{se} = \frac{bf_{c}^2 e_{e}^2}{\varphi e_{o}} \left( \frac{1}{3} - \frac{\varepsilon_e}{12 \varepsilon_o} \right) + \frac{f_{s}^2}{2E_s} A_s \]  

(2.43)

which gives:

\[ 7.0422 \times 10^{-13} + 2.4687 \times 10^{-12} = 3.1729 \times 10^{-12} \text{ Nmm/mm} \]
and the equivalent flexural rigidity is calculated as:

\[
EI^* = \frac{2 \times 3.1729 \times 10^{-12}}{(5.5617 \times 10^{-12})^2} = 205.15 \times 10^9 \text{ Nmm}^2
\]

Closer examination will show that at the selected level of concrete strain, the two solutions converge to within an accuracy of 8 decimal places, i.e.:

\[
EI_{cr} = EI^* = 2.05149397 \times 10^{11} \text{ Nmm}^2
\]

(b) Flexural Rigidity in the Plastic State

Subsequently, the EI* value has been calculated (Equations 2.40 and 2.44) for the wall element subjected to each of the three axial force regimes (see Tables 2.3, 2.4 and 2.5):

\[
\Psi^* = \Psi_e^* + \Psi_{te}^* + \Psi_{tp}^* = \frac{bf_e^2}{\varphi \varepsilon_o} \left( \frac{1}{3} - \frac{\varepsilon_c}{12 \varepsilon_o} \right) + A_f \varepsilon_o^2 + A_f f_t \left( \varepsilon_s - \frac{f_t}{E_s} \right) \quad (2.44)
\]

<table>
<thead>
<tr>
<th>Table 2.3</th>
<th>Equivalent flexural rigidity EI* of wall element resisting 50 kN axial tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_c = )</td>
<td>0.00067</td>
</tr>
<tr>
<td>(M \text{ (kNm)})</td>
<td>6.209</td>
</tr>
<tr>
<td>(\varphi \text{ ((^\circ/\text{mm})})</td>
<td>4.141e-5</td>
</tr>
<tr>
<td>(\Psi_e \text{ (N)})</td>
<td>26</td>
</tr>
<tr>
<td>(\Psi_{te} \text{ (N)})</td>
<td>149</td>
</tr>
<tr>
<td>(\Psi_{sp} \text{ (N)})</td>
<td>NA</td>
</tr>
<tr>
<td>(\Psi^* \text{ (N)})</td>
<td>175</td>
</tr>
<tr>
<td>(EI^* \text{ (Nmm}^2)</td>
<td>204.1e9</td>
</tr>
<tr>
<td>(EI^*/EI_{cr})</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Table 2.4  Equivalent flexural rigidity $EI^*$ of wall element without axial load

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_c = 0.00068$</th>
<th>0.001</th>
<th>0.0015</th>
<th>0.002</th>
<th>0.0025</th>
<th>0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (kNm)</td>
<td>7.196</td>
<td>10.18</td>
<td>10.66</td>
<td>10.79</td>
<td>10.84</td>
<td>10.84</td>
</tr>
<tr>
<td>$\varphi$ (°/mm)</td>
<td>3.611e-5</td>
<td>5.189e-5</td>
<td>1.041e-4</td>
<td>1.660e-4</td>
<td>2.298e-4</td>
<td>2.885e-4</td>
</tr>
<tr>
<td>$\Psi_c$ (N)</td>
<td>31</td>
<td>67</td>
<td>105</td>
<td>143</td>
<td>184</td>
<td>240</td>
</tr>
<tr>
<td>$\Psi_{se}$ (N)</td>
<td>100</td>
<td>202</td>
<td>209</td>
<td>209</td>
<td>209</td>
<td>209</td>
</tr>
<tr>
<td>$\Psi_{sp}$ (N)</td>
<td>NA</td>
<td>NA</td>
<td>504</td>
<td>1130</td>
<td>1780</td>
<td>2360</td>
</tr>
<tr>
<td>$\Psi^*$ (N)</td>
<td>131</td>
<td>269</td>
<td>818</td>
<td>1482</td>
<td>2173</td>
<td>2809</td>
</tr>
<tr>
<td>$EI^*$ (Nmm$^2$)</td>
<td>201.3e9</td>
<td>199.4e9</td>
<td>151.0e9</td>
<td>107.6e9</td>
<td>82.3e9</td>
<td>67.5e9</td>
</tr>
<tr>
<td>$EI^*/EI_{cr}$</td>
<td>1.000</td>
<td>0.991</td>
<td>0.750</td>
<td>0.535</td>
<td>0.469</td>
<td>0.335</td>
</tr>
</tbody>
</table>

Table 2.5  Equivalent flexural rigidity $EI^*$ of wall element under 50 kN axial compression

<table>
<thead>
<tr>
<th></th>
<th>$\varepsilon_c = 0.00068$</th>
<th>0.001</th>
<th>0.0015</th>
<th>0.002</th>
<th>0.0025</th>
<th>0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (kNm)</td>
<td>8.218</td>
<td>11.16</td>
<td>13.07</td>
<td>13.29</td>
<td>13.37</td>
<td>13.36</td>
</tr>
<tr>
<td>$\varphi$ (°/mm)</td>
<td>3.099e-5</td>
<td>4.662e-5</td>
<td>8.278e-5</td>
<td>1.320e-4</td>
<td>1.829e-4</td>
<td>2.294e-4</td>
</tr>
<tr>
<td>$\Psi_c$ (N)</td>
<td>36</td>
<td>74</td>
<td>131</td>
<td>181</td>
<td>235</td>
<td>302</td>
</tr>
<tr>
<td>$\Psi_{se}$ (N)</td>
<td>63</td>
<td>146</td>
<td>209</td>
<td>209</td>
<td>209</td>
<td>209</td>
</tr>
<tr>
<td>$\Psi_{sp}$ (N)</td>
<td>NA</td>
<td>NA</td>
<td>256</td>
<td>733</td>
<td>1230</td>
<td>1670</td>
</tr>
<tr>
<td>$\Psi^*$ (N)</td>
<td>99</td>
<td>220</td>
<td>596</td>
<td>1123</td>
<td>1674</td>
<td>2181</td>
</tr>
<tr>
<td>$EI^*$ (Nmm$^2$)</td>
<td>207.0e9</td>
<td>202.5e9</td>
<td>174.1e9</td>
<td>128.9e9</td>
<td>100.1e9</td>
<td>82.9e9</td>
</tr>
<tr>
<td>$EI^*/EI_{cr}$</td>
<td>1.000</td>
<td>0.978</td>
<td>0.841</td>
<td>0.622</td>
<td>0.483</td>
<td>0.400</td>
</tr>
</tbody>
</table>

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Fig. 2.14 Equivalent flexural rigidity $EI^*$ of a singly reinforced wall (see Fig. 2.13) in response to increasing bending curvature, as described by Equations 2.40 and 2.44

Fig. 2.15 Ratio of $M/EI^*$ to the actual curvature $\phi$ for a singly reinforced wall with varying levels of axial load and increasing bending curvature

For further comparison, a more flexurally efficient section is examined. Figure 2.16 shows a pretensioned 300 mm deep double tee section without composite topping. The tee is prestressed by eight 12.7mm diameter supergrade strands, giving a prestressing steel area of 800 $\text{mm}^2$ centred at 75 mm above the leg soffit. The prestress force after losses is 967 kN. The flexural rigidity of the gross section is $EI_g = 55.84 \times 10^{12} \text{Nm}^2$. 

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Table 2.6  Material parameters for derivation of equivalent flexural rigidity of double tee

<table>
<thead>
<tr>
<th>$f_c$ (MPa)</th>
<th>$f_{cr}$ (MPa)</th>
<th>$\varepsilon_0$</th>
<th>$E_c$ (GPa)</th>
<th>$f_{pu}$ (MPa)</th>
<th>$E_{ps}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>3.8</td>
<td>0.00287</td>
<td>27.9</td>
<td>1900</td>
<td>195</td>
</tr>
</tbody>
</table>

Fig. 2.16  Pretensioned double tee section

Table 2.7  Equivalent flexural rigidity $EI^*$ of a pretensioned double tee section in response to increasing bending curvature, as described by Equations 2.40 and 2.44

<table>
<thead>
<tr>
<th>300 mm deep Pretensioned Double Tee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_c = 0.00026$</td>
</tr>
<tr>
<td>$M$ (kNm)</td>
</tr>
<tr>
<td>$\varphi$ (θ/mm)</td>
</tr>
<tr>
<td>$\Psi_c$ (N)</td>
</tr>
<tr>
<td>$\Psi_{ps}$ (N)</td>
</tr>
<tr>
<td>$\Psi^*$ (N)</td>
</tr>
<tr>
<td>$EI^*$ (Nm²)</td>
</tr>
<tr>
<td>$EI^*/EI_{eq}$</td>
</tr>
</tbody>
</table>

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Fig. 2.17  Relationship between $\text{EI}^*$ and the gross section rigidity $\text{EI}_g$ for a pretensioned double tee section (see Fig. 2.16) subjected to increasing bending curvature

![Diagram of a pretensioned double tee section with labeled dimensions and material properties]

Fig. 2.18  600 mm x 300 mm column section

Table 2.8  Material parameters for derivation of equivalent flexural rigidity of a column

<table>
<thead>
<tr>
<th>$f_c$ (MPa)</th>
<th>$f_{cr}$ (MPa)</th>
<th>$\varepsilon_0$</th>
<th>$E_c$ (GPa)</th>
<th>$f_y$ (MPa)</th>
<th>$E_{ps}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3.3</td>
<td>0.00239</td>
<td>25.1</td>
<td>430</td>
<td>200</td>
</tr>
</tbody>
</table>

The equivalent flexural rigidity $\text{EI}^*$ of the column element (see Fig. 2.18) is analysed for three axial force regimes: (i) 250 kN axial tension, (ii) zero load and (iii) 1000 kN axial compression:

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Table 2.9  Equivalent flexural rigidity $EI^*$ of a column section resisting 250 kN axial tension

<table>
<thead>
<tr>
<th>$\varepsilon_c = $</th>
<th>0.0005</th>
<th>0.001</th>
<th>0.0015</th>
<th>0.002</th>
<th>0.0025</th>
<th>0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (kNm)</td>
<td>201.7</td>
<td>234.9</td>
<td>236.7</td>
<td>237.9</td>
<td>238.7</td>
<td>239.0</td>
</tr>
<tr>
<td>$\varphi$ (°/mm)</td>
<td>4.421e-6</td>
<td>1.333e-5</td>
<td>2.513e-5</td>
<td>3.704e-5</td>
<td>4.864e-5</td>
<td>5.976e-5</td>
</tr>
<tr>
<td>$\Psi_e$ (N)</td>
<td>34</td>
<td>84</td>
<td>142</td>
<td>212</td>
<td>293</td>
<td>393</td>
</tr>
<tr>
<td>$\Psi_{sy}$ (N)</td>
<td>516</td>
<td>642</td>
<td>635</td>
<td>630</td>
<td>627</td>
<td>627</td>
</tr>
<tr>
<td>$\Psi_{sp}$ (N)</td>
<td>NA</td>
<td>2439</td>
<td>5929</td>
<td>9458</td>
<td>12890</td>
<td>16160</td>
</tr>
<tr>
<td>$\Psi^*$ (N)</td>
<td>550</td>
<td>3165</td>
<td>6706</td>
<td>10300</td>
<td>13810</td>
<td>17180</td>
</tr>
<tr>
<td>$EI^*$ (Nmm²)</td>
<td>56.29e12</td>
<td>35.61e12</td>
<td>21.25e12</td>
<td>15.02e12</td>
<td>11.68 e12</td>
<td>9.62e12</td>
</tr>
<tr>
<td>$EI^*/EI_g$</td>
<td>0.42</td>
<td>0.26</td>
<td>0.16</td>
<td>0.11</td>
<td>0.09</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 2.10  Equivalent flexural rigidity $EI^*$ of a column section with zero axial load

<table>
<thead>
<tr>
<th>$\varepsilon_c = $</th>
<th>0.0005</th>
<th>0.001</th>
<th>0.0015</th>
<th>0.002</th>
<th>0.0025</th>
<th>0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (kNm)</td>
<td>177.7</td>
<td>293.8</td>
<td>298.8</td>
<td>300.7</td>
<td>301.7</td>
<td>302.0</td>
</tr>
<tr>
<td>$\varphi$ (°/mm)</td>
<td>1.001e-6</td>
<td>7.764e-6</td>
<td>1.665e-5</td>
<td>2.685e-5</td>
<td>3.731e-5</td>
<td>4.754e-5</td>
</tr>
<tr>
<td>$\Psi_e$ (N)</td>
<td>4</td>
<td>144</td>
<td>214</td>
<td>297</td>
<td>384</td>
<td>491</td>
</tr>
<tr>
<td>$\Psi_{sy}$ (N)</td>
<td>24</td>
<td>678</td>
<td>687</td>
<td>685</td>
<td>681</td>
<td>679</td>
</tr>
<tr>
<td>$\Psi_{sp}$ (N)</td>
<td>NA</td>
<td>653</td>
<td>3211</td>
<td>6189</td>
<td>9255</td>
<td>12240</td>
</tr>
<tr>
<td>$\Psi^*$ (N)</td>
<td>28</td>
<td>1475</td>
<td>4112</td>
<td>7171</td>
<td>10320</td>
<td>13410</td>
</tr>
<tr>
<td>$EI^*$ (Nmm²)</td>
<td>54.44e12</td>
<td>48.95e12</td>
<td>29.67e12</td>
<td>19.90e12</td>
<td>14.83 e12</td>
<td>11.87e12</td>
</tr>
<tr>
<td>$EI^*/EI_g$</td>
<td>0.40</td>
<td>0.36</td>
<td>0.22</td>
<td>0.15</td>
<td>0.11</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Table 2.11  Equivalent flexural rigidity $EI^*$ of a column section resisting 1000 kN axial compression

<table>
<thead>
<tr>
<th>$\varepsilon_c$</th>
<th>0.0005</th>
<th>0.001</th>
<th>0.0015</th>
<th>0.002</th>
<th>0.0025</th>
<th>0.003</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (kNm)</td>
<td>146.8</td>
<td>302.9</td>
<td>437.6</td>
<td>508.2</td>
<td>522.4</td>
<td>529.6</td>
</tr>
<tr>
<td>$\varphi$ (°/mm)</td>
<td>$9.843e^{-7}$</td>
<td>$3.272e^{-6}$</td>
<td>$5.869e^{-6}$</td>
<td>$9.302e^{-6}$</td>
<td>$1.400e^{-5}$</td>
<td>$1.914e^{-5}$</td>
</tr>
<tr>
<td>$\Psi_c$ (N)</td>
<td>151</td>
<td>343</td>
<td>608</td>
<td>854</td>
<td>1034</td>
<td>1214</td>
</tr>
<tr>
<td>$\Psi_{sy}$ (N)</td>
<td>28</td>
<td>182</td>
<td>602</td>
<td>946</td>
<td>1066</td>
<td>1193</td>
</tr>
<tr>
<td>$\Psi_{sp}$ (N)</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>563</td>
<td>1778</td>
<td>3137</td>
</tr>
<tr>
<td>$\Psi^*$ (N)</td>
<td>179</td>
<td>525</td>
<td>1210</td>
<td>2363</td>
<td>3878</td>
<td>5544</td>
</tr>
<tr>
<td>$EI^*$ (Nm²)</td>
<td>$369.0e^{12}$</td>
<td>$98.03e^{12}$</td>
<td>$70.29e^{12}$</td>
<td>$54.61e^{12}$</td>
<td>$39.58e^{12}$</td>
<td>$30.25e^{12}$</td>
</tr>
<tr>
<td>$EI^*/EI_g$</td>
<td>2.72</td>
<td>0.72</td>
<td>0.52</td>
<td>0.40</td>
<td>0.29</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Fig. 2.19  Relationship between $EI^*$, gross section rigidity $EI_g$ and cracked section rigidity $EI_{cr}$ for a column section (see Fig. 2.18) subjected to increasing bending curvature

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
2.2.1.4 Plastic Hinge Rotations and Compatibility

The plastic hinge regions of typical beam sections that incorporate a proportion of pretensioned floor section will naturally exhibit different moment capacities and rotational characteristics under alternating positive and negative bending actions. In order to rationally deduce the likely magnitude of beam plastic hinge rotations, strain energy quantities obtained through the preceding equations may be applied on the basis that internal strain energy will equal the work done by an external force moving through a distance (i.e., \( M \theta = P \delta \)). As discussed, beams spanning parallel to pretensioned flooring members usually support little more than self weight. Therefore, the predominant bending moment and shear force envelopes are easily defined in relation to the point of contraflexure for frame elements subjected to horizontal sway actions (Fig. 2.20).

The displacement \( \delta \) (Fig. 2.20) is comprised of respective elastic and inelastic components, and from the distribution of bending curvature may be calculated as:

\[
\delta = \frac{\varphi \psi^2}{3} + \left( \varphi_p - \varphi_y \right) \ell_p \left( \frac{z - \ell_p}{2} \right)
\]  
(2.45)

Equating the external work done to the quantity of internally stored strain energy, the following may be written in terms of respective elastic and inelastic components (see Fig. 2.20):

\[
V \delta = \int M_y \varphi_y \, dx + \Psi_p \ell_p
\]  
(2.46)

By expanding this equation, noting that \( V = M_f / z \) for the frame sway model and simplifying so that (see Fig. 2.20):

\[
\varphi_\Delta = \varphi_p - \varphi_y
\]  
(2.47)

then:

\[
M_p \left[ \frac{\varphi y z}{3} + \varphi_\Delta \ell_p \left( 1 - \frac{\ell_p}{2z} \right) \right] = \frac{M_y^2 z}{6E_y I_y} + \Psi_p \ell_p
\]  
(2.48)

In the first term on the right hand side of Equation 2.48, which relates to nominally elastic strain energy, the full length of beam segment has been used without subtracting the plastic hinge length. This simplifies the equation and semi-empirically accounts for additional internal strain energy due to strain penetration into the section beyond the plastic hinge zone, as well as shear displacements. From the above, it can be shown that:

\[
\frac{\ell_p^2}{2z} + \left( \frac{\Psi_p}{M_p \varphi_\Delta} - 1 \right) \ell_p - \frac{\varphi y z}{M_p \varphi_\Delta} \left( \frac{M_p}{3} - \frac{M_y}{6} \right) = 0
\]  
(2.49)

and solving for the equivalent length of plastic hinge required to meet the constraints of Equations 2.45 and 2.46 gives the exact solution of:
\[
\ell_p \frac{1}{z} \left( \frac{3 \psi_p^2}{\psi_\Delta} - \psi_\Delta \left[ 6 \psi_p - 3 \psi_\Delta - \phi_y \left( 2 M_p - M_y \right) \right] \right) + \psi_\Delta - \psi_p
\]

(2.50)

where:
\[
\psi_\Delta = M_p \phi_\Delta
\]

(2.51)

By considering that in general the plastic hinge moment \( M_p \) is not significantly greater than first yield moment \( M_y \), Equation 2.50 can usually be simplified with negligible error to:

\[
\ell_p \frac{1}{z} \left( \frac{\psi_p}{\psi_\Delta} - 1 \right)^2 + \frac{M_p}{3 M_y \left( \frac{\phi_p}{\phi_y} - 1 \right)} - \left( \frac{\psi_p}{\psi_\Delta} - 1 \right)
\]

(2.52)

\[\text{Fig. 2.20} \quad \text{Bending moment and displacement model of frame element under sway action}\]

Equation 2.52 would be seen as applicable to most ordinarily reinforced sections. However, where prestressing steel and possible overstrength is involved, the ratio of \( M_p \) to \( M_y \) can become significant and (especially where \( \phi_y \) is relatively small) Equation 2.50 may be (but is not necessarily) more appropriate. For applications involving typical sections, Equation 2.52 will give only a marginally larger estimate of equivalent plastic hinge length than Equation 2.50.

In order to assess the influence of plastic hinge development as determined by the foregoing equations, further reference is made of the hypothetical perimeter beam element, Figure 1.57 of Section 1.4.2.2(c). Using this beam model for dimension and reinforcement, estimates of effective plastic hinge lengths and subsequent plastic hinge rotations (as a proportion of flexural length \( z \)) are calculated for alternate positive and negative bending (see Table 2.12, Figs 2.21 and 2.22). For each bending case, the effective plastic hinge lengths and hinge rotations have been plotted against concrete strain in the extreme compression fibre. In keeping with Figs 1.58 and 1.59 of Section 1.4.2.2(c), the characteristic plastic hinge lengths correspond to varying numbers of participating 12.7 mm Supergrade strands (from \( n = 0 \) to \( n = 4 \)) over the strain interval.

\[\text{Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings}\]
Table 2.12  Calculation (Equation 2.50) of effective plastic hinge lengths under negative (\(b = 300 \text{ mm}\)) and positive (\(b = 1500 \text{ mm}\)) bending moments for a hypothetical perimeter beam in a ductile moment resisting frame with pretensioned hollow core flooring (see Fig. 1.57 and Section 1.4.2.2 (c))

<table>
<thead>
<tr>
<th>Strands</th>
<th>(\varepsilon_c)</th>
<th>Bending</th>
<th>(M_p)</th>
<th>(M_y)</th>
<th>(\theta_p)</th>
<th>(\theta_y)</th>
<th>(\Psi_p)</th>
<th>(\varepsilon I^*/EI_x)</th>
<th>(I_p/z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n)</td>
<td></td>
<td>(kNm)</td>
<td>(kNm)</td>
<td>(°/m)</td>
<td>(°/m)</td>
<td>(kNm/m)</td>
<td></td>
<td></td>
<td>(m/m)</td>
</tr>
<tr>
<td>0</td>
<td>0.002</td>
<td>-ve</td>
<td>301</td>
<td>289</td>
<td>0.0267</td>
<td>0.00545</td>
<td>7.116</td>
<td>0.081</td>
<td>0.206</td>
</tr>
<tr>
<td>2</td>
<td>0.002</td>
<td>-ve</td>
<td>420</td>
<td>372</td>
<td>0.0170</td>
<td>0.00591</td>
<td>6.289</td>
<td>0.176</td>
<td>0.217</td>
</tr>
<tr>
<td>4</td>
<td>0.002</td>
<td>-ve</td>
<td>506</td>
<td>449</td>
<td>0.0124</td>
<td>0.00635</td>
<td>5.830</td>
<td>0.304</td>
<td>0.197</td>
</tr>
<tr>
<td>0</td>
<td>0.002</td>
<td>+ve</td>
<td>325</td>
<td>307</td>
<td>0.0625</td>
<td>0.00463</td>
<td>19.060</td>
<td>0.039</td>
<td>0.155</td>
</tr>
<tr>
<td>2</td>
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<td>+ve</td>
<td>391</td>
<td>348</td>
<td>0.0500</td>
<td>0.00479</td>
<td>18.640</td>
<td>0.060</td>
<td>0.152</td>
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<tr>
<td>4</td>
<td>0.002</td>
<td>+ve</td>
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<td>387</td>
<td>0.0408</td>
<td>0.00491</td>
<td>17.940</td>
<td>0.087</td>
<td>0.157</td>
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<tr>
<td>0</td>
<td>0.003</td>
<td>-ve</td>
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<td>289</td>
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<td>0.00545</td>
<td>13.430</td>
<td>0.048</td>
<td>0.165</td>
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<tr>
<td>2</td>
<td>0.003</td>
<td>-ve</td>
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<td>372</td>
<td>0.0333</td>
<td>0.00591</td>
<td>13.230</td>
<td>0.096</td>
<td>0.182</td>
</tr>
<tr>
<td>4</td>
<td>0.003</td>
<td>-ve</td>
<td>541</td>
<td>449</td>
<td>0.0236</td>
<td>0.00635</td>
<td>11.800</td>
<td>0.170</td>
<td>0.203</td>
</tr>
<tr>
<td>0</td>
<td>0.003</td>
<td>+ve</td>
<td>331</td>
<td>307</td>
<td>0.0938</td>
<td>0.00463</td>
<td>29.310</td>
<td>0.027</td>
<td>0.143</td>
</tr>
<tr>
<td>2</td>
<td>0.003</td>
<td>+ve</td>
<td>396</td>
<td>348</td>
<td>0.0811</td>
<td>0.00479</td>
<td>30.890</td>
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</tr>
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<td>+ve</td>
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<td>387</td>
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<td>0.00491</td>
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</tr>
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<td>-ve</td>
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<td>449</td>
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<td>307</td>
<td>0.1210</td>
<td>0.00463</td>
<td>38.410</td>
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<td>0.124</td>
</tr>
<tr>
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<td>396</td>
<td>348</td>
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<td>0.00479</td>
<td>40.480</td>
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<td>0.119</td>
</tr>
<tr>
<td>4</td>
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<td>460</td>
<td>387</td>
<td>0.0909</td>
<td>0.00491</td>
<td>41.000</td>
<td>0.040</td>
<td>0.118</td>
</tr>
</tbody>
</table>

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Fig. 2.21(a) Ratio of negative bending equivalent plastic length $l_p$ to contraflexure distance $z$ as a function of concrete compressive strain and the number of strands ($n$) contributing as flexural tension reinforcement.

Fig 2.21(b) Ratio of positive bending equivalent plastic length $l_p$ to contraflexure distance $z$ as a function of concrete compressive strain and the number of strands ($n$) contributing as flexural tension reinforcement.

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Fig. 2.22(a) Ratio of negative bending plastic hinge rotation \( \theta \) to contraflexure distance \( z \) (metres) as a function of concrete compressive strain and the number of strands (n) contributing as flexural tension reinforcement.

Fig. 2.22(b) Ratio of positive bending plastic hinge rotation \( \theta \) to contraflexure distance \( z \) (metres) as a function of concrete compressive strain and the number of strands (n) contributing as flexural tension reinforcement.
In order to determine the deformation capacity of an element of building frame under sway actions, we may observe (see Fig 2.20) that there is almost constant shear force on the beam member. Hence, for practical purposes there is a direct proportion between the magnitude of plastic hinge bending moments \( M_{p1} \) and \( M_{p2} \), and their respective distances to the points of contraflexure, \( z_1 \) and \( z_2 \). Also, because the angle of beam rotation \( \theta' \) is equally applicable to both beam-end plastic hinges (Fig. 2.20) then the relationship may be written:

\[
\frac{M_{p1}}{z_1} \theta' = \frac{M_{p2}}{z_2} \theta'
\]  

(2.53)

Subsequently, by multiplying the plastic hinge rotations derived in Figures 2.22(a) and 2.22(b) by their corresponding plastic hinge bending moments, a direct reckoning can be made between concrete compressive strain in opposing plastic hinges and a prescribed level of beam rotation \( \theta' \) (see Figs 2.23). Thus, it is possible to estimate the ability of a beam to meet sway compatibility requirements in relation to building slope \( \theta_{ch} \) without exceeding a limiting parameter of concrete compressive strain:

The total strain energy required of two opposing beam plastic hinges in a frame at a prescribed rotation is:

\[
U^* = \left| M_{p1} + M_{p2} \right| \theta'
\]  

(2.54)

Directly calculated from Table 2.12 (as the product of \( M_p \phi \frac{1}{p} \frac{1}{z} \)), Figures 2.23 show quantities of strain energy per unit length as influenced by varying numbers of participating prestressing strands. These are given for positive and negative bending moments at various levels of concrete strain. Typical of plastic moment-curvature relationships, it may also be noted from Table 2.12 that variation in moment capacity under positive and negative bending is small (and often imperceptible) over the plastic bending curvature interval.

Hence, the limiting value of strain energy per unit length may be applied as the basis for estimating a minimum compatible beam length for a given rotation:

\[
I_{\text{min},z} = \left| \frac{M_{p1} + M_{p2}}{z} \right| \theta'
\]  

(2.55)

Based on Table 2.12, the exact data for calculation of minimum compatible beam lengths has been tabulated in Table 2.13.
Fig. 2.23(a)  Ratio of negative bending strain energy $M_p \theta$ (kNm) to contraflexure distance $z$ (meters) as a function of concrete compressive strain and the number of strands ($n$) contributing as flexural tension reinforcement.

Fig. 2.23(b)  Ratio of positive bending strain energy $M_p \theta$ (kNm) to contraflexure distance $z$ (meters) as a function of concrete compressive strain and the number of strands ($n$) contributing as flexural tension reinforcement.

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Table 2.13 Strain energy per unit length $M_p\theta/z$ under negative ($b = 300$ mm) and positive ($b = 1500$ mm) bending moments (see Fig. 1.57 and Section 1.4.2.2(c)) for use with Equation 2.55 to estimate minimum compatible beam lengths at a limiting concrete strain and prescribed rotation

<table>
<thead>
<tr>
<th>Strands ($n$)</th>
<th>$\varepsilon_c$</th>
<th>Bending</th>
<th>$M_p$ (kNm)</th>
<th>$\varphi_p$ ($°/m$)</th>
<th>$l_p/z$ (m/m)</th>
<th>$M_p\theta/z$ (kNm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.002</td>
<td>-ve</td>
<td>301</td>
<td>0.0267</td>
<td>0.206</td>
<td>1.650</td>
</tr>
<tr>
<td>2</td>
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<td>-ve</td>
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<td>0.0170</td>
<td>0.217</td>
<td>1.550</td>
</tr>
<tr>
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<td>-ve</td>
<td>506</td>
<td>0.0124</td>
<td>0.197</td>
<td>1.237</td>
</tr>
<tr>
<td>0</td>
<td>0.002</td>
<td>+ve</td>
<td>325</td>
<td>0.0625</td>
<td>0.155</td>
<td>3.152</td>
</tr>
<tr>
<td>2</td>
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<td>+ve</td>
<td>391</td>
<td>0.0500</td>
<td>0.152</td>
<td>2.973</td>
</tr>
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<td>+ve</td>
<td>457</td>
<td>0.0408</td>
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<td>2.924</td>
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<td>2.365</td>
</tr>
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<td>-ve</td>
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<td>+ve</td>
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<td>0.0938</td>
<td>0.143</td>
<td>4.430</td>
</tr>
<tr>
<td>2</td>
<td>0.003</td>
<td>+ve</td>
<td>396</td>
<td>0.0811</td>
<td>0.134</td>
<td>4.287</td>
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<td>0.0909</td>
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</tbody>
</table>
The following is an example of the compatibility application which gives comparisons of ductile beams with varying numbers of participating prestressing strands (i.e., \( n = 0, 2, 4 \)):

Referring to Figure 2.9, the building slope at level 1 with \( \eta = 1 \) is given as \( \theta = 0.011 \). With the ratio of column spacing to effective beam length taken as \( l/l' = 1.25 \), then rotation \( \theta' = 0.2l/l' \approx 0.014 \), which is constant for the given configuration of building slope and effective beam length. Therefore, both sides of Equation 2.53 may be divided by this value without prejudice to the equality inferred by the expression.

It is assumed that the limiting concrete compression strain is \( \varepsilon_c = 0.003 \), and that this will first occur under negative bending. Hence, the corresponding positive bending (flange) compression strain in the opposing plastic hinge will be somewhat less. However, the exact determination of flange compression strain is not necessary, and may be assumed to be in the order of \( \varepsilon_c = 0.002 \) with little variation in bending moment capacity.

If it is considered that two prestressing strands of the adjacent floor section will participate in flexural actions, then Table 2.13 is entered at \( \varepsilon_c = 0.003 \), \( n = 2 \) strands and negative (-ve) bending. The corresponding moment is \( M_p = 425 \text{ kNm} \) and strain energy per unit length is \( M_p\theta/z = 2.572 \). The opposing moment at \( n = 2, \varepsilon_c = 0.002 \) and positive (+ve) bending is \( M_p = 391 \text{ kNm} \) and strain energy per unit length of \( M_p\theta/z = 2.973 \). According to Equation 2.55, compatibility between plastic hinge rotation capacity and the limiting value of compression strain will not be exceeded in the span, provided that the effective beam length is no less than:

\[
\frac{425 + 391}{2.572} \times 0.014 = 4.45 \text{ m}
\]

Alternatively, if it was considered that flange compression strain should not exceed \( \varepsilon_c = 0.002 \) and that the beam soffit strain could approach spalling, then the appropriate SE per unit length is substituted to give a reduced minimum length of:

\[
\frac{425 + 391}{2.973} \times 0.014 = 3.85 \text{ m}
\]

Likewise, if the number of participating strands is increased to \( n = 4 \), and the same regime of limiting strains are applied, the minimum effective lengths become:

\[
\frac{541 + 457}{2.588} \times 0.014 = 5.40 \text{ m}
\]

and with flange compression critical at \( \varepsilon_c = 0.002 \) as in the above case (b):

\[
\frac{541 + 457}{2.924} \times 0.014 = 4.80 \text{ m}
\]

Based on the above, variations in compatible beam lengths are shown in Figure 2.24.
Fig. 2.24 Minimum compatible effective beam lengths between opposing plastic hinges under a prescribed rotation of \( \theta' = 0.014 \); at limiting values of concrete compressive strain \( (\varepsilon_c = 0.002, 0.003 \) and \( 0.004 \)) and numbers \( (n = 0, 2 \) and \( 4 \)) of participating prestressing strands.

2.2.2 PLASTIC HINGE LOCATION AND DILATION EFFECTS

2.2.2.1 General

In Section 1.4.2.2(d), frame dilation due to beam plastic hinge rotations was discussed in a general way and mostly reflected the considerations of elongation effects as given in current New Zealand design guidelines [NZCS-NZNSEE, 1991]. In addition to these guidelines there are other aspects that need to be examined, especially with regard to the floor unit and beam interactions. In the preceding Sections, it has been shown that plastic hinges incorporating a sufficient length of pretensioned flooring strands are likely to exhibit significantly greater bending moment and less plastic hinge rotation capacity at a limiting value of concrete compressive strain. Hence, it is important to determine what factors may influence the formation of principal flexural cracks in potential regions of plastic hinging. It is also important to appreciate that the floor component forms a large portion of the elastic section properties. Therefore, it is conceivable that in the initial stages of frame sway, the state of stress within the floor element itself may determine where and when the principal cracks occur.

2.2.2.2 State of Stress in a Hollow Core Flooring Unit

The state of stress in the support region of a pretensioned hollow core flooring unit is likely to have a direct influence on principal crack formation when subjected to negative bending moment (i.e., flange in tension). This is because a disparity exists in the stress states of the pretensioned member and the adjacent beam member. Unless very shallow hollow core sections...
are involved, the centre of prestress force in hollow core sections is usually near or outside the middle third kern. Therefore, due to prestress eccentricity, the end regions of hollow core units are often designed to have a limited amount of top fibre tension stress in the end region. When incorporated into the composite section, the residual state of stress in the pretensioned hollow core component may exert some influence on the general propagation of flexural cracks (see Fig. 2.25).

![Diagram showing stress states in hollow core sections](image)

**Fig. 2.25** Stress states at a short distance from end support of a beam and pretensioned hollow core unit (both simply supported at construction), plus composite effect.

### 2.2.2.3 Bar Curtailment and Plastic Hinge Development

Additional to a conducive state of stress in the floor section, the tendency for cracks to originate in the floor is very likely to be augmented by the curtailment of topping starter reinforcement. As such, this portion of the composite member may be susceptible to tension shift effects, and it is feasible that topping cracks will have already formed at the starter curtailment point due to shrinkage induced tension at the discontinuity of topping reinforcement.

By contrast, the formation of cracks in the same composite member subjected to positive bending (i.e., flange in compression) will be more strongly influenced by the discontinuity at the beam-column intersection. Hence, under reversing bending moments, the resulting origins of cracks will facilitate the development of inclined flexure-shear cracks in a plastic hinge zone (see Fig. 2.26). The implication of plastic hinge elongation is that much of the dilation effect could occur at a location that is remote from the actual floor unit end seating. From an analysis point of view, this introduces some complexity into the required design approach. However, it also suggests that some options may be examined for providing an effective means of control within the broad context of plastic hinge actions.

Hence, when developing comprehensive support tie details, consideration should also be given to the effects of curtailment. In particular, the bond relationship between the pretensioned floor unit, composite topping and beam element becomes an important aspect of the overall design.
For example, Equation 1.22 in Section 1.4.2.2(c) relates the development of pretensioned strand force (through strand bond capacity within the extruded section) to the shear friction force developed in the joint between the flooring unit and adjacent beam. The basis of this equation is re-emphasised (Fig. 2.27) by the likelihood that the location of principal cracks will be influenced by bar curtailment in the topping slab. Thus, if a point of curtailment is provided within the domain of a beam plastic hinge and relatively close to the end seating, it may be possible to mitigate the unfavourable effect of developing a large pretensioned strand force through strand bonding. Consequently, this would reduce the likelihood of strands in the pretensioned section causing problems with overly strong beam members (see Figs 1.58 and 1.59). Alternatively, if continuous topping reinforcement is extended well beyond the potential plastic hinge zones, it may be possible to gain more control of composite topping behaviour and reduce the occurrence of premature cracking.

Where the actual beams have been detailed to form plastic hinges sufficiently far from the column face (Fig. 1.56), it would appear difficult to avoid the full potential contribution of prestressing strands to the flexural capacity of the composite member.

2.2.2.4 Maintaining a Composite Section

What is strongly implied by the above is that the curtailment of topping reinforcement (starters and/or continuity bars) should occur either well within the plastic hinge zone, or well past the plastic hinge zone. In addition to the reasons already given, there is an important issue of detailing. If a principal crack occurs through the topping under negative bending, then this crack will almost certainly extend down into the hollow core section due to the conducive state of stress (Fig. 2.25). Because the large concurrent shear force acting on the section produces a rotation such as indicated by Figure 2.26, tie capacity is also required between the topping and floor unit to avoid the potential of delamination or tearing. Hence, it is desirable that an embedded tie force can be developed between the pretensioned floor unit and the cast in place topping slab over a distance that is no shorter than the beam plastic hinge zone, and preferably somewhat greater.

![Diagram](Image)

**Fig. 2.26** Crack pattern induced by concurrent flexure and shear in side-sway members

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*Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings*
2.3 Forces Entering a Diaphragm

2.3.1 Reactions at Columns and Walls

2.3.1.1 General

It is widely recognised that for forces entering building diaphragms, a reliable mechanism is required to achieve force transfer between the floor plate and members of the vertical resistance system. In the case of diaphragms of structures that may need to resist severe seismic actions, the ability to effect reliable force transfer between these structural members is imperative.

The way in which forces arise in relation to particular diaphragms has been discussed in Section 1.3, along with generalised procedures for the treatment of diaphragms in analysis and design. As noted (Section 1.3.3.8), design for force transfer by strut and tie methods has become an increasingly accepted method for establishing viable load paths within diaphragms. Based on established principles, the method revolves around constructing a system of nodes that will facilitate the equilibrium of tensile and compressive axial forces in a manner similar to simple truss analysis.

Chapter 2: Diaphragm Analysis: Response Behaviour of Tall Buildings
Regardless of whether critical design actions for diaphragms are based on strut and tie models or the more traditional notion of a uniform shear stress [Kolston and Buchanan, 1980], starter type reinforcement must be provided so that reactions can be developed through shear friction between the floor plate and support members. It is a feature of all well developed design standards and codes of practice that starter type reinforcement should be considered for this purpose. The European design recommendations give an actual minimum value of tensile force capacity per unit length of building in the transverse and longitudinal directions [FIP, 1988].

With regard to capacity design procedures, it can be reasoned that the provision of starter reinforcement should be based on an approach that is consistent with associated overstrength actions. From earlier deliberations (see Section 1.4.2.2), it was established that plastic hinge mechanisms may generally impose the most critical forces on the support regions of simple diaphragms during a severe earthquake. It follows that an in-plane reaction force at floor level will form one component of the bending moment couple generated by plastic hinges. In this Section, the components of forces entering diaphragms are considered from the perspective of immediate reactions at node points (which coincide with column locations and walls) and the flow of forces away from node points into the floor plate in response to diaphragm inertia.

2.3.1.2 Reactions at Node Points

To determine the components that result from plastic hinge zones of ductile beams subjected to oblique frame actions, the vector sum of moments may be considered (see Fig. 1.47 and Fig. 2.28). The magnitude of coupling force resisting the vector sum of overstrength moment $M_{oxy}$ is the resulting component of in-plane reaction, $N_{oxy}$. The directional sum of this force component is perpendicular to the plane of the maximum bending moment. Hence, starter reinforcement may be proportioned in accordance with the maximum force component as it acts in relation to the orientation of beams. At the intersection of beams, it can be shown that the maximum and minimum components of force resisted by shear friction along the interface of beam and floor are:

$$V_{tr(max)} = \frac{N_{oxy}}{2} \left( \frac{\cos \psi}{\cos \beta} + \frac{\sin \psi}{\sin \beta} \right), \quad V_{tr(min)} = \frac{N_{oxy}}{2} \left( \frac{\cos \psi}{\cos \beta} - \frac{\sin \psi}{\sin \beta} \right)$$

(2.56)

where $\beta$ is the bisection angle and $\pm \psi$ is the angle made between the force $N_{oxy}$ and the bisection angle (see Fig. 2.29). The force $V_{tr}$ can be related to shear friction capacity per unit length by:

$$\frac{V_{tr}}{\ell} = q_{tr} = \frac{A_{sf} f_y \mu \phi}{s}$$

(2.57)

from which the required spacing of starter reinforcement may be calculated as:

$$s_{x,y} = \frac{2 A_{sf} f_y \mu \phi \ell_{x,y}}{N_{oxy} \left( \frac{\cos \psi}{\cos \beta} \pm \frac{\sin \psi}{\sin \beta} \right)}$$

(2.58)

Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm
The length $l_{x,y}$ is the effective length over which shear friction may be developed in the applicable $x$ or $y$ direction, and should not be taken larger than the actual beam length.

![Diagram](Image)

**Fig. 2.28** Vector sum of overstrength moments in beam plastic hinges near a corner column

![Diagram](Image)

**Fig. 2.29** Model application of Equations 2.56 to 2.58

*Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm*
Equation 2.58 may be implemented for any configuration where beams intersect at a column. In accordance with Equation 2.56, the shear force $V_{tr}$ is shown as a proportion of the resultant $N_{oxy}$ at different beam intersections (Figs 2.30). The indicated beams have equal bending moment capacity, hence, the deviation angle $\nu = 0^\circ$ and $V_{tr(\text{max})} = V_{tr(\text{min})}$.

Figs. 2.30 Forces $Q$ (according to Equation 2.56) as a proportion of resultant $N_{oxy}$, resisted in shear friction by starters at the interface of respective beams and floor slab.
Considering the case where a corner column is intersected by two beams at right angles (see Fig. 2.31). The beams concurrently develop respective plastic hinge moments of \( M_{ox} = 400 \) kNm and \( M_{oy} = 300 \) kNm with \( j_d = 0.5 \) m. Hence, the vector sum of moments is \( M_{oxy} = 500 \) kNm, the couple component of force \( N_{oxy} = 500/0.5 = 1000 \) kN and acts through the floor plane at approximately 37 degrees to the x-axis. Thus, the bisection angle \( \beta = 90/2 = 45^\circ \) and the deviation angle \( \nu = 45 - 37 = 8^\circ \). If HD10 starters (Grade 430) are selected and the construction joint is cast monolithically, then the required spacing in millimetres in accordance with Equation 2.58 and based on 6 metre beam lengths in the x and y directions is:

\[
\frac{s_{x,y}}{s_x} = \frac{2 \times 79 \times 430 \times 1.4 \times 0.75 \times 6}{1000 \left( \frac{\cos 8}{\cos 45} \pm \frac{\sin 8}{\sin 45} \right)}
\]

\[
s_x = \frac{428}{1.2} \approx 350 \text{ mm}
\]

\[
s_y = \frac{428}{1.6} \approx 250 \text{ mm}
\]

**Fig. 2.31** Resistance to plastic hinge bending moments at the intersection of beams through the shear friction developed by starters along construction joints

---

*Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm*
2.3.2 COMPRESSION FIELD IN A SIMPLE DIAPHRAGM

2.3.2.1 General

As discussed in Section 1.3.2.2, all floor diaphragms may be subjected to a degree of simple diaphragm action and possibly transfer actions. With regard to simple diaphragm actions, there are two forms to consider; those associated with externally applied forces (e.g., wind) and those that may be associated with body forces (e.g., seismically induced inertia forces). For design purposes, the actual mechanism of force development in simple diaphragms is generally considered to be negligible. Thus, for the design of diaphragms it is usually considered sufficient to apply floor level inertia forces as uniformly distributed loads acting along one side of the structure and determine bending moments and shear forces from simple beam theory. In this way, inertia forces are treated identically to a uniform wind load applied to the windward face of a structure (Fig. 2.32). The above assumption would appear very reasonable for routine design applications. For instance, the calculation of maximum bending moments is unlikely to be much affected if inertia forces are treated as externally applied loads.

However, as it stands, a simplified model of bending and shear is not of great assistance if we are to envisage the likely path of principal strut action. Furthermore, on the assumption that a strut force of appreciable magnitude does occur, it should be consistent with boundary conditions of shear and bending moment. The simplified model would indicate that tension cord reinforcement is not critical in regions of lower bending moment, which does not agree with the formation of principal strut and tie nodes at corner columns. Consequently, a model of strut action is examined, based on development of a strut force that remains compatible with concurrent bending moment and shear.

![Diagram of diaphragm forces](image)

**Fig. 2.32** Calculation of diaphragm actions in a cast-in-place topping slab [NZCS, 1983]
2.3.2.2 Diaphragm Strut Model

The object of the model is to obtain an effective strut force in response to concurrent bending moment and shear forces. Thus, the component of this principal strut must satisfy the concurrent conditions of bending and shear. The principal strut force is envisaged as a compression field that exists within the diaphragm plate under the effects of seismically induced inertia. The horizontal component of the strut force couples with the tension cord forces to produce bending moment equilibrium, the vertical component of the strut force produces shear equilibrium (see Fig. 2.33).

![Diagram](image)

**Fig. 2.33** Model used for development of an equivalent principal strut

It is reasonable to assume that bending and shear equilibrium can be achieved through the action of an equivalent principal strut field. For example, the tension forces in the tension cord will form a moment couple with the nearest component of compressive reaction from where an equal and opposite force is obtainable. With the strut force taken as N, the coupling moment becomes (see Fig. 2.33):

\[ M(s) = N \cos \theta \cdot y(s) \]  \hspace{1cm} (2.59)

and the corresponding shear force \( V \) is:

\[ V(s) = N \sin \theta \]  \hspace{1cm} (2.60)

Rearranged, the above may be written as:

\[ \frac{\sin \theta}{\cos \theta} = \tan \theta(s) = \begin{bmatrix} V \\ y \\ M \end{bmatrix} \] \hspace{1cm} (2.61)

*Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm*
The underlying assumptions to this equation are quite straightforward, and the solution for distance ordinate $y$ involves a first order differential equation that is easily solved:

$$\frac{dy}{dx} - \left[ \frac{V}{M} \right]_{(1)} = 0$$  \hspace{1cm} (2.62)

and may be written in terms of separated variables so that:

$$\frac{dy}{y} = \left[ \frac{V}{M} \right]_{(1)} dx$$  \hspace{1cm} (2.63)

Hence, the position and inclination of the principal strut field may be defined according to the relationship between shear and bending moment, written as a function of the distance variable, $x$. In the general analysis of seismic resisting floor diaphragms there are only a few important flexural cases to consider, of which the two most fundamental cases are shown (Figs 2.34). The relationship between shears and bending moments for these cases, when applied in Equation 2.63, produce elementary parabolic stress fields.

(a) Uniformly Loaded Simple Beam

$$\frac{dy}{y} = 2 \left( \frac{L}{2} - x \right) \frac{dx}{Lx - x^2} \Rightarrow \ln(y) = \ln(Lx - x^2) + C$$  \hspace{1cm} (2.64)

The constant $C$ is also treated as logarithmic, therefore $C = \ln(\chi)$ and the general solution is:

$$y = \chi(Lx - x^2)$$  \hspace{1cm} (2.65)

To obtain a characteristic solution, a boundary condition is applied. The obvious boundary condition concerns the stress field location at mid-span, which is generally taken as the width between tension and compression cords, $B$. Hence, the characteristic solution is:

$$y = \frac{4Bx}{L^2} (L - x), \text{ and } \tan\theta = \frac{4B}{L^2} (L - 2x)$$  \hspace{1cm} (2.66)

(b) Uniformly Loaded Cantilever Beam

$$\frac{dy}{y} = 2 \frac{dx}{x} \Rightarrow \ln(y) = 2 \ln(x) + C$$  \hspace{1cm} (2.67)

$$y = \chi x^2$$  \hspace{1cm} (2.68)

Again assuming that the coupling distance is the width of diaphragm, $B$:

$$y = \frac{Bx^2}{L^2}, \text{ and } \tan\theta = \frac{2Bx}{L^2}$$  \hspace{1cm} (2.69)

---

*Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm*
Figs. 2.34  Fundamental load cases for routine analysis of seismic resisting diaphragms

In order to assess diaphragm forces under simple response actions, information is required of inertia forces at the given floor level. Inertia force is produced by the mass-acceleration relationship at floor level, and therefore, both an estimate of effective weight and peak floor accelerations are required. The hypothetical 10-storey building configuration (see Section 2.2.1.2) has a total occupied building weight of 32000 kN. The corresponding floor accelerations $a_{(c,b)}$ result from the second derivative of Equation 2.24:

$$a_{(c,b)} = \delta_c \omega^2 \left[ \zeta \omega t \cos \omega t \left( e^{-\omega^2 t \frac{B^2}{\eta}} - 1 \right) + \sin \omega t \left( (2 \zeta - 1) e^{-\omega^2 t \frac{B^2}{\eta}} - 2 \zeta \right) \right]$$  \hspace{1cm} (2.70)

For the case where the number of load cycles to residual frame stiffness is $N = 7$, and the building is assumed to have a uniform weight of 3200 kN at each floor level ($\eta = 1$) (also see Fig. 2.8), the corresponding storey accelerations are shown in Table 2.14 and Fig. 2.35.

If the building plan is assumed to have plan dimensions of $L = 25m$ and $B = 12.5m$ (see Fig. 2.36), then it follows that the maximum simply supported bending moment (at level 10) resulting from the storey acceleration is $1.21 \times 3200 \times 25/8 = 12100$ kNm. The maximum simply supported shear force is therefore $1.21 \times 3200/2 = 1936$ kN. From Equation 2.66, the initial strut angle at $x = 0$ is $\tan \theta = 2.0 = 63.4^\circ$. Since the model requires that the cord tie force is constant along the tension cord, then the maximum strut force is $12100 \text{ kNm}/(12.5 \text{ m} \times \cos \theta) = 2165$ kN at $x = 0$. The vertical component at $x = 0$ is $2165 \text{ kN} \times \sin \theta = 1936$ kN, which equals the simply supported shear force, etc.

Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm
<table>
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<td>9.74</td>
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<td>0.88g</td>
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<tr>
<td>6</td>
<td>7.49</td>
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</table>

Table 2.14 & Fig. 2.35 Storey accelerations \(a_{(t,h)}\) according to Equation 2.70, for hypothetical 10-storey building described in Section 2.2.1.2 with \(N = 7\) and \(\eta = 1\).

Fig. 2.36 Simple diaphragm actions at level 10 of a hypothetical 10-storey frame building.

Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm
2.3.3 OPENINGS IN DIAPHRAGMS

2.3.3.1 General

One of the recognised advantages of using strut and tie methods is the treatment of openings in diaphragms. Strut and tie models allow the designer to establish viable load paths, and can be adapted to situations that involve load paths where openings occur and alternate actions may apply (see Section 1.3.2.2 and Figs 1.39 and 1.40). However, there are instances where the predominant strut or tie action is of less significance than the immediate distribution of stresses produced by concurrent bending moment and shear force. One such situation is cantilevered portions of diaphragms, described as case (b) in the preceding Figure 2.34. As shown, (Fig. 2.36) openings may occur in regions of high shear and bending without interrupting the viability of a simple principal strut field for resisting the associated forces.

2.3.3.2 Elastic Design of Beams with Openings

Since diaphragms are initially designed as elastic beam elements, it is consistent to adopt methods for elastic beams with openings. A recognised method for the analysis of slotted elastic beams is based on combined stresses with an allowance for coupling and shear effects [Young, 1989]. Referring to Figure 2.37, extreme fibre stresses are given as:

\[ \sigma_a = \frac{M_A}{Z} - \frac{V_A x I_1/(I_1 + I_2)}{Z_1} \quad \text{(compression)} \]
\[ \sigma_b = \frac{M_A}{Z} + \frac{V_A x I_2/(I_1 + I_2)}{Z_2} \quad \text{(tension)} \]  

(2.71)

In these formulas, it is assumed that all forces acting to the left of line A may be replaced by an equivalent couple \( M_A \) and shear \( V_A \) at line A. The couple \( M_A \) produces stress due to bending moment in the net beam section, as provided by the first term in the equations. The shear force \( V_A \) is proportioned to parts 1 and 2 in accordance with their respective stiffness, as given by second moments of inertia, \( I_1 \) and \( I_2 \). Thus, the bending moment resulting from the shear \( V_A \cdot x \) produces an increment or decrement in stress at points a and b in accordance with the respective section moduli, \( Z_1 \) and \( Z_2 \).

![Fig. 2.37 Model of coupling and shear for derivation of Equations 2.71 [Young, 1989]](image-url)
Fig. 2.38  Principal strut actions unaffected by an opening in cantilever diaphragm

The increment of extreme fibre stress caused by the above configuration is calculated in accordance with Equation 2.71. The seismic weight and storey accelerations are as assumed in the simply supported diaphragm example of Section 2.3.2.2. Likewise, the overall plan dimensions of the diaphragm are length \( L = 25 \text{ m} \) and width \( B = 12.5 \text{ m} \). The various requisite section properties are shown in Table 2.15:

Table 2.15  Member properties of the slotted diaphragm portion in Fig. 2.38. The effective thickness of 200 hollow core plus 65mm topping is taken as 165mm

<table>
<thead>
<tr>
<th>( I_1 ) (mm(^4))</th>
<th>( Z_1 ) (mm(^3))</th>
<th>( I_2 ) (mm(^4))</th>
<th>( Z_2 ) (mm(^3))</th>
<th>( x ) (mm)</th>
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<tr>
<td>32.2e12</td>
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<td>6250</td>
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The weight over the slotted portion is reduced in proportion to the area of flooring removed. Therefore, the seismic weight per unit length of building is calculated as \( 1.21 \times (1 - 2.4/12.5) \times (3200 \text{ kN/25m}) = 125 \text{ kN/m} \). If the slot is ignored and simple cantilever bending assumed, the bending stress at point b (Fig. 2.38) will be:

\[
\sigma_b = \frac{M_b}{Z} = \frac{125 \times 12500^2}{2 \times 5.13 \times 10^9} = 1.9 \text{ MPa}
\]

Incorporating the slot effect in accordance with Equation 2.71, the stress at point b is:

\[
\sigma_b = \frac{125 \times 6250^2}{2 \times 5.15 \times 10^9} + \frac{125 \times 6250 \times 6250 \times 1/2}{938 \times 10^9} = 3.1 \text{ MPa}
\]

Hence, in an elastic diaphragm, the slot effect increases bending stresses by more than 60% and approaches the concrete modulus of rupture at point b.

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Chapter 2: Diaphragm Analysis: Forces Entering a Diaphragm
2.4 DISCUSSION

2.4.1 GENERAL

In this chapter, a number of analytical methods have been developed to assist with the rationalised design of structural frames and floor diaphragms. With the developed models, emphasis has been placed on relatively tangible concepts that recognise plasticity effects on building seismic response and member compatibility functions. As such, some exercise has been given to the subject of elastic-plastic transition and plastic hinge development in ordinary reinforced and prestressed concrete members.

2.4.2 BUILDING DEFORMATION

The solution to a partial differential equation describes the fundamental mode of deformed shape of a building frame as a function of time and building height (Equation 2.19). Essentially, the derived structural deformations are the diffused shapes of a prescribed base motion, which is displacement as a function of time. For both simplicity and relative correctness, the selected base motion has been taken as sinusoidal in the examples (Equation 2.7).

The effects of plasticity are applied to the model by use of a semi-empirical stiffness reduction coefficient based on the observed stiffness degradation of beam-column elements under cyclic loading (Equations 2.23 and 2.24). Consistent with increasing plasticity and reduced stiffness in the structure, the coefficient effectively dampens elastic response by reducing the activity of resonance terms in the transient solution. In this way, the nil resonance or steady-state response deformation of an inelastic structure is approached (Equation 2.20).

The displaced shape is described by a continuous function in both time and height. Thus, the respective velocities and accelerations (Equation 2.70) are obtained by differentiation with respect to time. Likewise, differentiating with respect to height (Equation 2.25) approximates the beam-column rotation at storey level; noting that the displaced shape describes the relative displacements between storey levels and not the curvature of columns.

The natural period of structure does not feature in the analysis, since response is derived from inertia forces that react against directly imputed ground motion. Consequently, the geotechnical aspect of site-specific input motion is more relevant to the model than an assumed natural period of vibration. Yet, although from a differing perspective, the outcomes generally concur with those of traditional dynamic (harmonic) analysis. As a preliminary design tool, it is considered that the model would best be applied to compare response at both the upper (critical displacement) and lower (critical acceleration) bounds of natural period that characterise a structure (i.e., a response period envelope).

For example: considering two sinusoidal input ground motions of equal accelerations, one of short period and small amplitude motion, the other of long period and large amplitude. It is assumed that periods, displacement amplitudes and acceleration are interrelated by the expression: \( \delta = a \left( \frac{T}{2\pi} \right)^2 \). The following observations result from the various equations:

---

*Chapter 2: Diaphragm Analysis: Discussion*
- At equal input accelerations; the response accelerations resulting from short period ground motion are significantly higher than for long period ground motion, but the corresponding structural displacements are small. Hence, the model suggests that a structure subjected to distinct short period ground motion (generally mitigated in real structures by damping) is more susceptible to shear failure at foundation level than structural damage at higher levels.

- Long period ground motion results in lower response accelerations but larger structural displacements, suggesting that long period ground motion is synonymous with larger inter-storey deflections and ductility demands.

With regards to variations in initial structural stiffness and stiffness degradation due to plastic deformation, the following are observed from the model for structures of equal mass:

- Structures with a large initial stiffness tend towards small response accelerations and displacements. Hence, rigid ground block motion is approximated by very stiff structures.

- Initially stiff structures are influenced to a lesser extent by stiffness degradation than flexible structures.

- Flexible structures may be concurrently subjected to significant response accelerations and displacements, especially if stiffness degradation requires an appreciable number of load cycles.

- For effective damping of resonant elastic response by plasticity mechanisms, the early occurrence of stiffness degradation is significantly more effective than the actual degree of degradation. Hence, judicious and well-distributed placement of ductile fuse mechanisms would appear beneficial in flexible structures.

General conditions that are implicit in the model:

- The fundamental pattern of displacement response is governed by the vector components (i.e., direction and amplitude) of input ground motion.

- Resonant response is governed by the velocity components of input ground motion.

2.4.3 EQUIVALENT FLEXURAL RIGIDITY (EI*)

The equation for equivalent flexural rigidity (Equation 2.40) is a rigorous derivation of the flexural stiffness response of concrete members through the elastic-plastic transition. Hence, the equation provides reasonable estimates of rigidity values for plastically deformed members under varying load conditions. As such, the stiffness variations of critical members in a typical elastic analysis may be modified on a rational basis.
For members subjected to simply supported bending moments, the variation in equivalent flexural rigidities of a prestressed tee and reinforced beam are indicated by Figures 2.39 and 2.40 respectively.

**Fig. 2.39**  Relationship between simply supported bending moment and flexural rigidities $E_1g$ (gross section) and $E_1^*$ (Equation 2.40) for prestressed tee beam (see Fig. 2.16) at a mid-span concrete compressive strain of $\varepsilon_c = 0.003$.

**Fig. 2.40**  Relationship between simply supported bending moment and flexural rigidities $E_1g$ (gross section) and $E_1^*$ (Equation 2.40) for reinforced beam (see Fig. 2.18) at a mid-span concrete compressive strain of $\varepsilon_c = 0.003$. 

*Chapter 2: Diaphragm Analysis: Discussion*
2.4.4 **PLASTIC HINGE ROTATIONS AND COMPATIBILITY**

An equation for equivalent plastic hinge length (Equation 2.52) has been derived for the purpose of estimating compatible rotations in sway members. For ductile beam members that incorporate boundary elements of pretensioned flooring, the compatibility equations (Equations 2.53 to 2.55) indicate that increased effective beam lengths are required to achieve prescribed levels of rotation at limiting values of concrete compressive strain.

Based on the above, reasonable estimates can be made regarding the compatibility of beam-column joint rotations with the development of beam plastic hinges within an effective span length. Hence, preliminary design may be approached from the point of view of maintaining beam rotation compatibility with a prescribed displaced shape of structure.

2.4.5 **PLASTIC HINGE LOCATION AND DILATION**

Practical aspects are discussed, relating to the likely effects of flooring member configuration and local reinforcement detailing on the development of beam plastic hinges. Further consideration is given to the ability of details to maintain composite action.

2.4.6 **DIAPHRAGM FORCES**

Equations are presented for practical detailing of diaphragm boundary elements to resist the various forces that may develop during a severe earthquake. The equations provide consideration of:

- The distribution of starter reinforcement along floor construction joints to resist, in shear-friction, the reactions derived from couple forces in the floor plane. The couple forces are those derived from concurrent bending actions at beam-column joints (nodes). (Equation 2.58).

- The geometric developments of principal strut actions in common diaphragm elements (Equations 2.66 and 2.69) that are compatible with boundary assumptions. The described strut geometry is a means of determining the viability of force transfer.

2.4.7 **DIAPHRAGM OPENINGS**

A calculation method is presented (Equation 2.71) for estimating the effects of openings on flexural stresses generated at extreme tension fibres of elastic diaphragms.

*Chapter 2: Diaphragm Analysis: Discussion*
3

Loss Of Support (LOS) Tests

3.1 GENERAL

One of the fundamental aims of this research project is to establish the ability of support tie
details to withstand displacements caused by the effects of frame dilation. Hence, an important
initial criterion is to determine the performance of contemporary support details. Without doubt,
the most common form of support tie detail used when composite topping is placed involves
simple starter bars. This type of detail was discussed in Sections 1.2.3.3 and 1.2.3.4, where it
was also noted that there is no documented evidence that starter bar details can provide ductile
tie capacity. Hence, the inaugural part of the experimental programme has been mainly directed
at ordinary starter bar details subjected to dilation effects.

3.2 TEST METHODOLOGY

3.2.1 GENERAL PROCEDURE AND SUMMARY OF TEST SPECIMENS

To remain consistent with the earlier experiments of Mejia-McMaster and Park, the same basic
methodology of testing was adopted. As such, the general emphasis of testing is to apply an
axial tension force across the support interface so to extract the flooring unit from its seating. In
this way, an effective force causing elongation is applied directly to the flooring unit and
resistance to this force is provided by reinforcement details in the support region.

As discussed in Section 1.4.2.2(d), it is considered feasible that there may be sufficient
accumulated elongation of plastic hinges to cause physical loss of support. Thus, in accordance
with the earlier experimentation, a total axial displacement of 55 mm was adopted. This value is
sufficient to give a clear loss of support (support length is generally taken as 50 mm minimum)
without the beneficial effects provided to shear resistance through aggregate interlock.

On the assumption that support tie details will sustain an axial displacement of 55 mm, a
vertical force can then be applied to examine residual support capacity. Perhaps the singularly
most important criterion for the success of such details is residual tie capacity after the physical
loss of support.

The details tested, LOS 1, LOS 2, LOS 3 and LOS 4 are summarised as follows:
Test LOS 1:
Involved a 200 mm hollow core unit with 65 mm composite topping. The support detail featured a typical configuration of 4-HD12 starter bars plus 665 hard-drawn wire mesh. The hollow core edge keys were fully integrated with the support block concrete. A superimposed load of 3.25 kPa was applied at testing.

Test LOS 2:
Involved a 200 mm hollow core unit with 65 mm composite topping. The support detail featured a special tie bar configuration of 6 HD10 hairpin ties, one grouted into each of the hollow core voids. No superimposed loads were applied at testing.

Test LOS 3 and LOS 4:
Involved a 200 mm hollow core unit with 65 mm composite topping. The support detail featured the typical configuration of starters and mesh used in LOS 1. The hollow core edge keys were debonded from the support block concrete. No superimposed loads were applied at testing.

3.2.2 DESCRIPTION OF TEST EQUIPMENT

3.2.2.1 General

Test equipment, divided into three major categories, was comprised of:

- Test specimens and associated structural support elements, which generally involves full scale concrete specimens and reinforcement to represent an actual portion of building construction.

- Actuating system that involves hydraulic rams, pumps, heavy steel reaction members and positive methods of attachment to the concrete test members.

- Electronic and mechanical componentry for the general measurement and logging of forces and displacements as derived from testing.

Of these three basic categories, the first may vary in principal with different tests. However, the equipment used for the general application of forces and the logging of results essentially remains unchanged throughout the LOS test programme. Hence, the description of structural mechanisms governing particular tests is deferred to the actual descriptions of individual tests. The basic support beam, actuating equipment and measuring apparatus is described as follows:

3.2.2.2 Precast Support Beams

A precast concrete support beam was constructed for each test (see Figs. 3.1). The fundamental beam design remained unchanged throughout the test programme. The actual reinforcement configuration of the support beam (other then attached starter bars) is not considered to have any influence on test results. The beam was designed to essentially remain uncracked.
throughout the tests. Any cracks migrating into the beam section from the floor support region were intercepted by sufficient principal and shear reinforcement to prevent them from becoming significant.

Provision was made for locating sole plates attached to 250 UC 73 steel reaction members by bolting through the beam section. To achieve cyclic loading capacity (tests LOS5 and LOS6), this detail was enhanced by adding two extra bolts to each sole plate and matching anchor plates on the opposite face of the precast support beam.

![Fig. 3.1(a)](image)  
**Fig. 3.1(a)** Detail of precast concrete support beam and seating blocks

![Fig. 3.1(b)](image)  
**Fig. 3.1(b)** Reinforcement details of precast support beams

*Chapter 3: Loss of Support (LOS) Tests: Test Methodology*
3.2.2.3 Horizontal Displacement

(a) Actuating System

Horizontal displacement was applied to the test specimens by two parallel acting hydraulic rams, each rated at 43 tonnes (see Figs 3.2). These rams were attached directly to the support beam blocks, were self-equilibrating and had no reliance on external reactions. At the other end, the rams were each connected to a hinging carriage that permitted both horizontal and rotational displacements. In each case, hydraulic pressure was applied to the rams by two identical hand operated pumps acting in unison.

The hollow core flooring unit was secured onto the support carriages by two 310 UC 97 beams placed above and below the precast section, and bolted to it by eight 24 mm diameter threaded rods. The threaded rods were incorporated into the section by filling the adjacent hollow core voids with topping concrete, over a distance of 600 mm from the end of the member. Additional reinforcement was placed at the topping interface, sufficient to develop the resulting horizontal reaction force through shear friction.

(b) Measurements

Principal horizontal displacements were measured by placing two 100 mm linear displacement potentiometers near each support carriage, which connected between target plates mounted on the hollow core unit and rigidly braced uprights bolted to the laboratory strong-floor. Potential slip between the topping and precast concretes was monitored by two sets of potentiometers placed on isolated stands that were epoxied into the top flange hollow core unit (see Fig. 3.4). Further potentiometers were attached to the concrete support block to monitor movement of the test rig in relation to the floor. Force measurements were accorded by 44 tonne load cells positioned between the respective hydraulic rams and support carriages.

3.2.2.4 Vertical Force

(a) Actuating System

The provision of vertical force was made available by a vertical reaction frame supporting a hydraulic ram with 130 tonnes capacity (see Fig. 3.3). The ram acted into the floor section via a series of spreader beams. The spreader beams were each seated onto a bedding layer of either plaster of Paris or cement-sand mortar. Hydraulic pressure was provided by a hand operated pump.

(b) Measurements

Two 300 mm linear displacement potentiometers were connected to the hollow core unit near the support to measure vertical displacements. A 100 tonne load cell was placed between the vertical ram and spreader beams for vertical force measurement.
Fig. 3.2(a)  Method for applying horizontal displacements

Fig. 3.2(b)  End elevation of loading carriage and hollow core section

Chapter 3: Loss of Support (LOS) Tests: Test Methodology
(a) Set-up immediately prior to test LOS 1

(b) Method for applying vertical force

Fig. 3.3 Test rig assembly

Chapter 3: Loss of Support (LOS) Tests: Test Methodology
Horizontal Displacements

Vertical Displacements

Linear Potentiometers & Load Cells

Fig. 3.4 Methods for obtaining measurements of forces and displacements

Chapter 3: Loss of Support (LOS) Tests: Test Methodology
3.2.2.5 Reinforcing Bar Strain Gauges

Electrical resistance strain gauges were connected to principal reinforcement to establish bar strain characteristics. Two types of electrical resistance strain gauges were considered for these tests. In the locations where principal cracking was expected to occur, 20% extension gauges were attached to the bar. These gauges were supplied by the Tokyo Sokki Kenkyujo Co. and were gauge type YL-5, with 120Ω resistance and 5 mm gauge length. Because of the large strain involved, a suitable adhesive is required to secure the gauge to the reinforcing bar. A recommended adhesive for use with large extensions is Armstrong A-12 epoxy. This adhesive is applied as a bedding for the gauge, and requires oven drying to achieve strength in a reasonable time frame. Previous tests at the University of Canterbury showed that 19% extensions could be attained with the YL-5 gauge [Mejia-McMaster and Park, 1994].

At bar positions away from the expected critical section, ordinary 3% extension strain gauges were used. These were supplied by Tokyo Sokki Kenkyujo Co. gauge type FLA-5-11, with 120Ω resistance and 5 mm gauge length.

Surface preparations and the method of fixing electrical resistance strain gauges was carried out in accordance with the departmental guidelines for these procedures [Hill, 1992].

3.2.2.6 Data Logger Unit

The load cells, potentiometers and strain gauges were all connected to the Metabyte logger which converted voltage changes caused by linear displacement into digital values. These values were recorded against respective scan numbers that were manually taken throughout the tests. At the conclusion of a test, the logged information was converted to an ASCII file that was then imported into Excel (spreadsheet program) for subsequent editing and data extraction.

3.2.3 MATERIALS AND CONSTRUCTION

3.2.3.1 General

The emphasis of this experimental programme was to reflect the general performance of pretensioned floor construction. As such, no special efforts were made to embellish or unduly influence the characteristics of materials provided. As may be assumed in regular commercial construction practice, a competent manufacturer supplied the pretensioned hollow core flooring and a Special Grade batch plant supplied the topping concrete.

Subsequently, there was general avoidance of the idealised concrete construction that can result in a laboratory placing and curing environment. With regard to construction practice, it is easy to appreciate that a single precast hollow core unit set in a loading frame inside a laboratory will automatically receive more detailed attention than perhaps 50 such units as part of a typical floor layout. Hence, the construction practice adopted for the preparation of LOS tests specimens was respectful, though not of a flattering nature.
3.2.3.2 Precast Pretensioned Hollow Core Units

The hollow core units were supplied by Firth Strescrete Ltd out of their Auckland Hollow Core Factory at Papakura. All units were standard 200 mm deep by 1197 mm wide extruded Dycore/Partek profile hollow core sections (Fig. 3.5). It should be noted that the section profile was not a criterion for selection and it is considered that the actual section profile will not have had a measurable influence on subsequent test results. The only dimension stipulated to the precasting factory was for 200 mm deep extruded pretensioned hollow core flooring units of varying lengths with clean but unroughened top flange surfaces.

Variations in prestress force was an additional requirement, namely, units with four, five and seven 12.7 mm diameter Supergrade strands. In all cases, the centre of prestress was 45 mm above the unit soffit. Inspection of the actual strand centres indicated positions that were sufficiently close to the design value so that the variation was negligible.

The hollow core units were inspected upon arrival and it was evident that they were all of good quality. The concrete matrix was dense and uniform with no indication of the type of irregularities that may result from incorrect mix design or insufficient vibration. The average strand slippage was in the order of three millimetres, which is comfortably within the acceptance criteria for the development of flexural bond strength [Anderson and Anderson, 1976] (see Section 1.2.2.1).

From data provided by the precasting factory, the hollow core units were released from the casting palle: at a concrete compressive stress of around 35 MPa (as indicated by impact hammer) and had a design concrete compressive stress of 45 MPa.

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**Fig. 3.5** Section profile of hollow core of units incorporated in the LOS test programme

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*Chapter 3: Loss of Support (LOS) Tests: Test Methodology*
3.2.3.3 Prestressing Strand

The hollow core units were pretensioned with 12.7 mm diameter Grade 1360 strands supplied from Australia by BHP Industries. Strand of the LRSS type (i.e., low relaxation, stress relieved, super-grade) directly complies with the requirements of the New Zealand standard [SAA, 1987], and is probably the most common source of pretensioning strand used in New Zealand at present.

The mechanical characteristics of strand used in the precasting factory at the time the hollow core units were manufactured is as follows (Table 3.1 and Fig. 3.6):

<table>
<thead>
<tr>
<th>Coil Number</th>
<th>Cross section area (mm²)</th>
<th>Proof Load (0.2%) (kN)</th>
<th>Ultimate tensile strength (kN)</th>
<th>Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3506</td>
<td>98.4</td>
<td>190</td>
<td>200</td>
<td>5.5</td>
</tr>
<tr>
<td>3532</td>
<td>99.2</td>
<td>189</td>
<td>198</td>
<td>5.0</td>
</tr>
<tr>
<td>3533</td>
<td>99.2</td>
<td>189</td>
<td>198</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Average E = 196 GPa

![Graph](image)

**Fig. 3.6** Typical stress-strain characteristics of prestressing strand employed in the manufacture of hollow core units that were subsequently used in the LOS test programme.

*Chapter 3: Loss of Support (LOS) Tests: Test Methodology*