3.2.3.4 Cast-In-Place Topping Concrete

Because the LOS series of tests would involve direct tension applied to a built-up floor section, some care needed to be exercised in relation to specifying concrete compressive strength within a testing time frame. The most important consideration was that concrete tensile strength will generally develop at a slower rate than compressive strength. Hence, it was concluded that concrete with a target compressive strength higher than may normally be used should be applied as cast-in-place topping. This measure was adopted to ensure that a reasonable magnitude of topping tensile strength would be exhibited at testing.

Obtaining the actual direct tensile strength of concrete at testing was not considered because of the difficulty in establishing this value with certainty. Hence, the result would be meaningless in the context of general floor construction. Furthermore, it was expected that topping cracks would occur during testing and that micro-cracks would have already developed at an earlier stage due to typical shrinkage effects.

In accordance with recommended construction practice, the top of the hollow core unit was lightly dampened before the topping was placed. In order to ensure sufficient moist curing of the topping concrete, wetted hessian cloth was applied to the slab area for at least five days after casting. For each specimen, a minimum of three representative concrete test cylinders were cured in the same environment, adjacent to the topping slab.

The actual descriptions of topping concrete that was specified and tested are deferred to the reporting of individual LOS tests.

3.2.3.5 Reinforcing Steel

(a) Starter and Tie Bars

12 mm diameter Grade 430 (HD12) starter bars and 10 mm diameter Grade 300 (D10) tie bars complying with the appropriate New Zealand standard [Standards New Zealand, 1989] were supplied cut and bent from recognised reinforcing steel merchants. Specifically manufactured for use in ductile seismic resisting structures, the New Zealand standard specifies that these steels must exhibit minimum elongation capacities of 15% and 20% respectively before fracture. Subsequent tensile tests indicated that all reinforcement met with these requirements (see Fig. 3.7). The actual tensile characteristics of starter and tie reinforcement are deferred to individual LOS tests.

(b) Welded Wire Fabric (Mesh)

665 mesh (5.3 mm diameter) complying with the appropriate New Zealand standard [Standards New Zealand, 1975] was supplied by recognised reinforcing steel merchants. A minimum elongation capacity for hard drawn wire mesh has not been a feature of the New Zealand standard. Earlier tests conducted on 665 mesh [Mejia-McMaster and Park, 1994] indicated that
hard drawn wire mesh exhibited very poor elongation capacity, with an average of only 2.7% elongation over a 50 mm gauge length before fracture.

Tests were conducted on mesh used in the LOS programme with elongations based on 100 mm gauge lengths (see Table 3.2 and Fig. 3.7). Although the extensions so obtained were greater than those obtained by Mejia-McMaster and Park, they are still very small when placed in context with the requirements of ductile design. It is immediately evident that hard drawn wire mesh will exhibit no more than one third the elongation capacity of ductile reinforcing bars which are manufactured to the New Zealand standard.

![Graph: Stress-strain relationship](image)

**Fig. 3.7** Typical stress-strain relationship of 12 mm diameter Grade 430 bars employed as starters in the LOS test programme

**Table 3.2** Characteristics of 5.3 mm diameter (5 gauge) hard drawn wire comprising the 665 mesh used in the LOS test programme

<table>
<thead>
<tr>
<th>665 Mesh Specimen</th>
<th>Proportional Limit (MPa)</th>
<th>Ultimate tensile strength (MPa)</th>
<th>Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>529</td>
<td>657</td>
<td>6.4</td>
</tr>
<tr>
<td>2</td>
<td>532</td>
<td>671</td>
<td>4.1</td>
</tr>
<tr>
<td>3</td>
<td>523</td>
<td>648</td>
<td>5.6</td>
</tr>
<tr>
<td>average:</td>
<td>528</td>
<td>660</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Average E = 196 GPa

*Chapter 3: Loss of Support (LOS) Tests: Test Methodology*
3.3 RESULTS OF EXPERIMENTS

3.3.1 LOS 1

3.3.1.1 General

The initial LOS test was directed at the typical starter bar detail (Figs 3.9 and 3.10). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit that was pretensioned with five 12.7 mm diameter Supergrade strands. A total of four 12 mm diameter Grade 430 (HD12) starter bars were placed at 300 mm centres, and extended 600 mm into the topping slab. 665 mesh was placed over the hollow core unit, and was continued beyond the end of the seating length, thus contributing to the tensile strength across the construction joint (assumed to be the critical section).

The mechanics of this detail clearly depend on the development of horizontal shear strength between the precast unit and composite topping concretes. For this detail to successfully perform under dilation type loading, the shear strength developed at the interface of concrete surfaces must at least match the full tensile strength of starter bar reinforcement.

The initial assumption with regard to composite bond capacity was based on the composite bond strength of chapter eight (ie., Composite Concrete Flexural Members) of the then current (June 1994) New Zealand design standard [SANZ, 1982]. Section 8.4.1.4 of this document states that “When ties are not provided, but the contact surfaces are clean and intentionally roughened, permissible $v_h = 0.55$ MPa. Allowing an overstrength factor of 1.25 for reinforcement, and considering a development length $l_d$ that is 100 mm shorter than the embedment length (ie., 500 mm), the design horizontal shear stress corresponding to one starter was calculated as:

---

**Chapter 3: Loss of Support (LOS) Tests: Test Methodology**
Based on the above properties and dimensions, the calculated horizontal shear stress becomes:

\[ v_{th} = \frac{f_{su} A_s}{\phi s f_d} \text{ and } v_h \geq v_{th} \]  

(3.1)

Fig. 3.9 Support configuration of LOS tests that involved typical starter bar details

Fig 3.10 Typical LOS test set-up before the addition of topping concrete
3.3.1.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 3.2.2 (also Fig. 3.4).

(b) Reinforcement

Both 20% and 3% extension electrical resistance strain gauges (as described in Section 3.2.2) were employed on two individual items of starter reinforcement. These gauges were configured so that the 20% extension varieties were situated directly over the expected plane of cracking, at the end of the hollow core section and 50 mm into the support block. The 3% strain gauges were set at 100 mm centres about the expected cracking zone (see Fig. 3.11).

![Diagram of strain gauge positions](image)

Fig. 3.11 Strain gauge positions on starter bars used in the LOS test series

3.3.1.3 Cast-In-Place Topping Concrete

Some difficulty surrounded the supply of concrete at the specified slump of 100 mm. The initial delivery was measured at 160 mm, which was outside the snatch sample upper tolerance value of 140 mm. The replacement delivery was measured at 60 mm slump, and was accepted as just inside the minimum tolerance level of 60 mm (see Table 3.3). Although this slump was just within tolerance and was accepted for casting over a single specimen, it is unlikely that this slump could have been used on a construction site without the addition of a workability agent.

Hence, because of the small volume of material involved and the high labour input it was possible to cast and compact this concrete to an adequate standard at 60 mm slump

Table 3.3 Characteristics of cast-in-place topping concrete for test LOS 1

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 at 28 days</td>
<td>13</td>
<td>100</td>
<td>60</td>
<td>38 at 23 days</td>
</tr>
</tbody>
</table>

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
3.3.1.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2.

(b) HD12 Starters

Tensile tests were performed on three specimens of 12 mm diameter Grade 430 bars, and the following was recorded (Table 3.4):

<table>
<thead>
<tr>
<th>Average yield strength (MPa)</th>
<th>Average ultimate tensile strength (MPa)</th>
<th>Average strain at $\varepsilon_{th}$ (%)</th>
<th>Average strain at ultimate tensile strength (%)</th>
<th>Average strain at fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>445</td>
<td>613</td>
<td>1.35</td>
<td>12.74</td>
<td>20.90</td>
</tr>
</tbody>
</table>

Average $E = 205$ GPa

3.3.1.5 Results of Testing

(a) Initial Response

The initial response to horizontal loading showed a very stiff system (Fig. 3.12) that allowed only 0.27 mm of horizontal displacement at the peak load of 352 kN. The first cracks appeared across the topping above the end of the hollow core unit at 250 kN, with a maximum crack width of around 0.3 mm occurring in this region at a force of 350 kN.

(b) Fracture

At 352 kN a sudden and resonant fracture occurred, resulting in a fracture through the hollow core section along the support line, and a crack of 0.6 mm width through the topping slab at an average distance of 650 mm from the face of the support. This topping fracture was located just beyond the curtailment point of the starter reinforcement.
Fig. 3.12  Force-displacement response of test LOS 1

Fig. 3.13  Force-displacement diagram divided into regions of significance in terms of resistance mechanisms

From this point onwards, the whole nature of the test changed. The horizontal force dropped to 130 kN, and reached a post-fracture maximum of 178 kN at 5 mm displacement. Horizontal
restraint capacity progressively diminished, out to 18 mm displacement. Throughout this phase there was intermittent tensile fracturing of topping mesh wires. Once the restraint provided by mesh wires was completely overcome, only residual sliding friction was observed.

Although all eight mesh wires were fractured during the test, not all the wires failed along the principal crack that had developed through the topping. Three wires fractured a short distance inside the topping, away from the crack face. Assisted by the largely unaffected state of the transverse wires, this reinforcement was able to develop a degree of catenary action at the latter stages of the test. This support action tended to peel the topping slab away from the hollow unit on the mid-span side of the principal crack.

(c) Displacements

There was an instantaneous loss of bond between the precast and topping concretes at the initial point of fracture. At 18 mm displacement, this had progressed to a vertical separation of 4 mm between these elements. At 25 mm displacement, the hollow core section had dropped to 7 mm below the projection of the topping slab as a result of support concrete spalling. At 32 mm displacement, this distance had increased to 12 mm. The test terminated when the hollow core section slipped from the support at a displacement of 43 mm.

3.3.1.6 Analysis of Test Results

In order to analyse the response of test LOS 1, the force-displacement diagram has been divided into the four distinct regions that were reflected in the test outcome (Fig. 3.13). Each of these regions (i.e., stages) indicates a basic change in the resistance mechanism against imposed horizontal displacements. As a whole, the diagram clearly indicates the differing contributions of respective steel reinforcement and concrete components.

(Stage I) Peak Load and Fracture

The initial response showed an average axial stiffness of 1825 kN/mm, with resistance provided by the entire composite section. At 300 kN (i.e., 85% of peak load), only 0.14 mm of horizontal displacement was registered, with topping cracks appearing across the entire unit above the interface between the hollow core unit and the support beam.

Initial cracking had appeared at about 250 kN and just under 0.1 mm horizontal displacement. This introduced a predictable tensile response in the starter reinforcement (Figs 3.17 and 3.18). As displacement increased beyond 0.1 mm towards fracture at 0.27 mm, the tensile contribution of the reinforcement continued to increase, but at a sharply decreasing rate. This is significant, and indicates that the reinforcing steel would not reach a particularly high degree of tension at the point of fracture. At peak load, reinforcement contributed to only 28% of the total measured horizontal reaction (Fig. 3.14).
Fig. 3.14  Proportion of total horizontal force resisted by starter bars and mesh in the support region, up to sudden fracture at 0.27 mm displacement

Fig 3.15  Total force and reinforcing steel force (starters and mesh) across the support region, before and immediately after sudden fracture

With reference to Figure 3.15, the force resisted by concrete alone is the difference between the total force and the force resisted by steel reinforcement. It can be deduced that an almost constant force of 250 kN was carried by the concrete section from around 0.1 mm horizontal

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
displacement up to fracture at 0.27 mm (see Fig 3.16). This implies that the concrete section was mobilised in a plastic manner, showing no increase in resistance under increased displacement. However, the small displacements involved may not have been sufficient to fully overcome the post-cracking tensile strength of infill concrete around the hollow core voids. It is certain that the influence of edge effects augmented as fracture displacement was approached, in particular, resistance from wedging between the sides of the hollow core unit and the support block. Analysis of Table 3.5 indicates that starter reinforcement was not the element providing critical restraint in the support region at the pre-fracture stage.

![Graph](image)

**Fig 3.16**  
Horizontal force resisted before fracture by the concrete section alone

**Table 3.5**  
The average measured tensile stresses in the starter bars and mesh over the support immediately before and after fracture

<table>
<thead>
<tr>
<th>Reinforcing Element</th>
<th>Average stress before fracture (MPa)</th>
<th>Proportion of total reaction (%)</th>
<th>Average stress after fracture (MPa)</th>
<th>Proportion of total reaction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD12 starters</td>
<td>134</td>
<td>17</td>
<td>253</td>
<td>58</td>
</tr>
<tr>
<td>665 mesh</td>
<td>216</td>
<td>11</td>
<td>350</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>$\Sigma$ 28%</td>
<td></td>
<td>$\Sigma$ 89%</td>
<td></td>
</tr>
</tbody>
</table>

The maximum starter bar stress at this stage of the test was 59% of the nominal yield stress of 430 MPa. The corresponding maximum stress in the mesh was 73% of the nominal proportional limit. The respective distributions of tensile stress in the HD12 starter bars and 665 mesh are shown in Figures 3.17 and 3.18, and indicate that yield stress was not approached in either the starter bar or mesh reinforcement during the decisive first stage of this test.

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1*
Figure 3.17 indicates a sudden increase in starter bar tension at gauge station number 9 after the section had fractured. This increase was due to a sizeable crack that developed through the topping slab. It is apparent that this crack was forming prior to the section fracture, as implied by the small preceding increase in bar strain.
Inspection of Figure 3.17 suggests that starter bar bond efficiency was greater in the cast-in-place support beam region than in the topping slab over the hollow core unit. This is indicated by the respective slopes of bar strain distributions on either side of the pivotal gauge station number 5.

The strength development of starter bar and mesh elements was not in proportion to their respective areas. The area of HD12 starters was 452 mm² (i.e., 72% of total topping steel area) and the area of mesh was 176 mm². In the pre-fractured section, the meshed developed 40% of the total steel reaction force, decreasing to 35% of the total steel reaction after fracture. It is evident that the mesh developed proportionally greater tensile stresses than the starters, and this difference could be due to the characteristic bond efficiencies of each of these materials at small displacements.

The sudden fracture of the concrete section resulted in two principal cracks through the hollow core unit. These cracks extended through the entire hollow core section and were generally focused along the face of the support beam. The cracks started near the termination point of the concrete support beam at the edges of the hollow core unit and propagated back towards the centre-line of the hollow core section. Hence, this pattern of cracking further suggests that shear stresses were developed between the edge of the hollow core unit and the cast-in-place infill concretes (Fig. 3.19). The crack width along the line of the support was about 1.0 mm, and corresponded with the measured post-fracture horizontal test displacement of almost 1.1 mm.

---

**Fig. 3.19**  Planes of fracture through the topping slab and hollow core section in the vicinity of the support region, as influenced by cast-in-place mortar joint
(Stage II) Post-Fracture to Post-Fracture Peak Force

From a static position at post-fracture, further horizontal displacement resulted in an increased reaction up to the post-fracture peak force of 178 kN, one half of the peak pre-fracture force. Throughout this stage of the test, resistance was provided by a complex combination of mechanisms. Although it was clear that complete separation had occurred between the hollow core unit and the support, prestressing strand stubs were still embedded in the support beam near the edges of the unit. The maximum embedment length of strand stub was 60 mm, and although pullout effects may have initiated, it is considered that the embedded stubs may have developed an appreciable tensile resistance. An amount of resistance was also developed through aggregate interlock and friction between the precast unit and the support, however, it is very difficult to estimate the magnitudes of these constituent forces.

At the end of this loading phase, the maximum developed force was mostly dependant on the integrity of mesh wires which bridged the principal topping crack, 650 mm from the face of the support (see Fig. 3.19). Assuming that the mesh wires were near their maximum tensile capacity of 660 MPa, the tensile contribution of reinforcing steel was about 116 kN, or 65% of the post-fracture peak force.

The maximum starter bar stress recorded during test LOS 1 was 292 MPa, which is two-thirds of the nominal yield stress of reinforcement. The maximum stress occurred over the support line of the hollow core unit (gauge station 5), and concurrently with the peak post-fracture force at 5.5 mm horizontal displacement (Fig. 3.20). The maximum mesh wire stress in the support region was 338 MPa, a reduction from the 375 MPa that was recorded at this location during Stage I of the test.

![Stress distribution graph](image)

**Fig 3.20** Average stress distributions along HD12 starter bars at the indicated horizontal displacements during Stage II of test LOS 1

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1*
(Stage III) Post-Fracture Peak Load to Full Loss of Steel Reaction

Once the peak load displacement of 5.5 mm was exceeded, a steady loss of reaction resulted from the serial tensile fracture of mesh wires. Likewise, the initially significant effects of residual concrete bond and friction were completely diminished during this stage of loading. Because the mesh wires were the only form of reinforcement offering resistance across the principal topping crack (see Fig. 3.19), the entire contribution of reinforcement was lost with fracture of the last wire at 18 mm displacement. Beyond 18 mm displacement it was clear that the only resistance provided was due to sliding friction between the soffit of the hollow core unit and the seating ledge.

Sliding resistance between the hollow core unit and its seating caused the support ledge to spall, resulting in a 4.0 mm downward dislocation of the hollow core section by the end of this stage. Visible separation had occurred between the precast and topping concretes prior to loading Stage III. Therefore, it is considered that significant frictional resistance could not have been developed along the precast-to-topping interface over this stage.

(Stage IV) Loss of Reaction to Collapse

For the reasons discussed earlier, the final stage of the test registered almost negligible reaction force. The final stage of the test involved displacing the unit until the eventual collapse occurred. Of particular interest was the amount of downward movement displayed by the precast section as a result of support spalling (Fig. 3.21). During the later stage of the test, the unit was supported primarily by dowel action provided by the prestressing strand stubs (as referred to in Stage II). At 43 mm displacement, the unit effectively slipped off the support.

![Graph showing vertical dislocation of the hollow core section](image)

**Fig. 3.21** Vertical dislocation of the hollow core section due to spalling of the support ledge, measured in relation to horizontal displacement up to collapse

---

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1*
Fig. 3.22  Soffit of hollow core unit showing fracture at the support

Fig. 3.23  Separation of the principal topping crack immediately prior to collapse

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
Fig. 3.24  Topping separation caused by downward dislocation of the hollow core unit

Fig. 3.25  View of support at the end of testing, showing intact plastic dams in outer voids and spalling along the support ledge

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
Fig. 3.26 Fractured hollow core section after removal, showing intact plastic dams in two inner voids

3.3.1.7 Specific Analysis

(a) Fracture of the Hollow Core unit at the Support

Before the section fractured, substantial topping cracks had established over the support region. Therefore, the tensile contribution of the topping slab would have been largely depleted at that stage. Assuming the reinforcing and precast section alone resisted the total horizontal reaction at fracture, the proportions of resistance were approximately as follows (Table 3.6):

Table 3.6 Reaction provided by reinforcement and precast section immediately prior to sudden fracture

<table>
<thead>
<tr>
<th>Element</th>
<th>Horiz. Reaction (kN)</th>
<th>Proportion of Total (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>665 mesh</td>
<td>38</td>
<td>11</td>
</tr>
<tr>
<td>HD12 starters</td>
<td>61</td>
<td>17</td>
</tr>
<tr>
<td>hollow core section</td>
<td>253</td>
<td>72</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>352</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
Hence, the average axial tensile stress in the hollow core unit at fracture was \( f_{tx} = P_{pc}/A_{pc} = 253 \) kN/0.119 m², which gives \( f_{tx} \) as 2.1 MPa. Considering the effect of shear stresses acting on the two key edges of the hollow core section (see Fig. 3.19), it can be shown that the principal tensile strength of concrete was reached at fracture.

The shear capacity of uncracked mortar joints may be estimated by Equation 1.20. Since there is no apparent normal component of stress, the maximum shear that may be developed as such is 0.05\( f'_{c} \). Referring to Table 3.3, this would indicate a potential resisting shear stress value at the time of testing of 0.05 \times 38 \text{ MPa} = 1.9 \text{ MPa}.

For an element with combined mono-axial tensile stress and shear stress \( \tau \), the equation for the principal tensile stress under brittle failure criterion is given as:

\[
f'_{t} = \frac{f_{tx}}{2} + \sqrt{\left(\frac{f_{tx}}{2}\right)^2 + \tau^2}
\]  

(3.6)

and the principal stress plane \( \theta_{p} \) is located in relation to the plane of axial stress:

\[
\theta_{p} = \frac{1}{2} \tan^{-1}\left(\frac{2\tau}{f_{tx}}\right)
\]  

(3.7)

The ACI recommended value for the principal tensile capacity of concrete is 0.33 \( \sqrt{f'_{c}} \) [Lin and Burns, 1982], hence, Equations 3.6 and 3.7 may be rearranged and combined to give the principal stress plane as:

\[
\theta_{p} = \frac{1}{2} \tan^{-1}\left(\frac{2\sqrt{\left(0.33\sqrt{f'_{c}} - \frac{f_{tx}}{2}\right)^2 - \left(\frac{f_{tx}}{2}\right)^2}}{f_{tx}}\right)
\]  

(3.8)

Substituting the appropriate values of \( f'_{c} = 45 \text{ MPa} \) and \( f_{tx} = 2.1 \text{ MPa} \) into Equation 3.8 yields a principal tensile fracture plane at 12.7° relative to the support line.

The actual fracture planes of the test unit are shown in Figure 3.25. The inclinations of the left and right side fracture planes, as caused by combined shear and tension, were respectively about 13° and 9° relative to the cross section of the unit. Fracture occurred through the entire hollow core section, with the exception of a small portion of web which measured 30 mm (web width) by 40 mm high.

(b) Vertical Reaction at the Support

A static superimposed load of 3.25 kPa was applied to 60% of the hollow core unit plan area by adding two layers of 40 kg cement bags. The superimposed load in combination with the unit self weight and various items of rigging resulted in a support reaction of 9.8 kN. This is not a
particularly large reaction for a hollow core flooring unit, and may be only about one third of the service load reaction of a typical floor member.

Fig. 3.27  End view of hollow core unit, showing inclined planes of fracture caused by combined shear and tension at the support

It is evident, however, that a greater magnitude of vertical support reaction would have had little influence on the fundamental result of test LOS 1. It is presumed that a greater vertical reaction could have accelerated the vertical dislocation of the hollow core unit as collapse displacement was approached (see Fig. 3.22). Therefore, the test configuration may be considered as producing non-conservative test results in terms of applied loading and apparent behaviour.

(c) Bar Bond in the Support Region

Bar bond stresses may be calculated by considering stress distributions along the bar, as shown in Figure 3.18. Fitted curves of stress distribution are shown in Figures 3.28(a) and 3.28(b) for gauge stations 1 to 5 (over support block) and stations 5 to 9 (in topping slab). It is evident that the fitted parabolas provide strong correlations with measured stresses.

Hence, this would indicate a linear variation in bond stress alone the starter bars at the given magnitudes of stress. Equations 3.1 and 3.2 are the fitted equations for support block and topping slab bar stresses, in megapascals as functions of distance x (where 0 ≤ x ≤ 400 mm).
Fig. 3.28(a)  Fitted curve of starter bar stress distribution in the support beam at post-fracture peak force (5.5 mm horizontal displacement)

Fig. 3.28(b)  Fitted curve of starter bar stress distribution into the topping slab at post-fracture peak force (5.5 mm horizontal displacement)
Support block (gauges 1 to 5): \[ f_s = 0.0025x^2 - 0.33x + 6 \] (3.1)

Topping slab (gauges 5 to 9): \[ f_s = 0.0023x^2 - 1.3x + 280 \] (3.2)

For round bars, the concrete bond stress \( u \) may be expressed as:

\[ u = \frac{d_b \cdot df_s}{4 \cdot dx} \] (3.3)

hence, for 12 mm diameter reinforcement the respective bar bond stresses are:

Support block (gauges 1 to 5): \[ u = 0.015x - 1 \] (3.4)

Topping slab (gauges 5 to 9): \[ u = 0.014x - 4 \] (3.5)

From these Equations 3.4 and 3.5, maximum bond stresses may be calculated adjacent to the initial support-to-precast interface crack located at gauge station number 5 (see Fig. 3.18). Therefore, substituting \( x = 400 \) mm into Equation 3.4 yields bond stress of 5.0 MPa. Likewise, substituting \( x = 0 \) into Equation 3.5 yields –4.0 MPa, the negative sign indicating the direction at which the bond stress acts. These values indicate that adequate starter bond stresses were developed under the given conditions of bar stress and adjacent concrete cover [Park and Paulay, 1975].

(d) Work Done

The performance of ductile tie connections is characterised by the measured quantity of work done by the connection detail in the form of internal strain energy (see Figs 1.22 and 1.23). In establishing this value, it is assumed that the conservation of energy principles apply. Thus, the quantity of internal strain energy is equal to the sum of external work done, written as:

\[ U_i = U_e \] (3.9)

The external work done by a connection detail is simply the summation of area under the force-displacement curve produced by the test. The support detail exhibited primarily elastic behaviour up to the point of sudden fracture (see Fig. 3.12). Hence, the internal strain energy in this detail up to fracture would have been mostly conservative (ie., recoverable). Beyond the point of fracture, the detail exhibited non-conservative modes of energy dissipation through the plastic deformation of mesh wires and frictional forces. Energy dissipation through secondary effects is not a warranted consideration in practical concrete research.

The summation of external work may be found with sufficient accuracy by taking small increments of displacement:

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
\[ U_e = \sum \frac{1}{2} \left[ P_{n} + P_{n+1} \right] \left[ \delta_{i+1} - \delta_{i} \right] \] (3.10)

Referring to Figure 3.13, the summation of total work done through each loading stage (I to IV) is shown in Table 3.7:

<table>
<thead>
<tr>
<th>Stage</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Done (kN-mm)</td>
<td>277</td>
<td>307</td>
<td>1006</td>
<td>120</td>
<td>1710</td>
</tr>
<tr>
<td>Proportion (%)</td>
<td>16</td>
<td>18</td>
<td>59</td>
<td>7</td>
<td>100%</td>
</tr>
</tbody>
</table>

### (c) Internal Strain Energy

The nature of developed internal strain energy up to sudden fracture was characterised by the primary materials involved. There were three essential groups of materials to consider, namely steel reinforcement, precast concrete section and cast-in-place concrete section. It is apparent that these respective materials exhibited decreasing relative proportions of conservative strain energy at the point of sudden fracture.

It has been demonstrated that the steel reinforcement remained fully elastic over the support zone, and based on the magnitude of bond stresses, there would have been negligible bar slippage. However, with regard to the elasto-plastic behaviour of the concrete elements, it is difficult to fully deduce the individual pre-fracture contributions of the precast and cast-in-place portions. For prior analyses, it has been assumed as reasonable that the effective tensile contribution of topping was negligible at the point of sudden fracture. Hence, it is certain that the topping concrete portion would have at most exhibited plastic force-deformation characteristics at this point.

Figure 3.16 indicates that the combined concrete section became mobilised and exhibited plastic behaviour prior to sudden fracture. A transition phase from elastic to plastic deformations occurred between 0.1 mm and 0.14 mm displacement, and it is evident that fully plastic behaviour was maintained between 0.14 mm and sudden fracture at 0.27 mm displacement.

The total quantity of elastic strain energy stored in the combined elements immediately before sudden fracture must approximately equal the total energy change in the post-fracture system. Thus, large force at small displacement corresponds with lesser force at increased displacement, etc. The internal strain energy may be calculated and compared to the external work done up to sudden fracture, based on section properties, reasonable assumptions of material parameters and information derived through measurements. Written in terms of the primary reserves of internal
strain energy, respectively the composite section, starter bars and mesh, corresponding with external work done in accordance with Equation 3.9:

\[
\frac{P_{\text{max}}^2 L}{2A_e E_e} + \frac{A_s}{2E_s I_s} \int f_s^2 \text{dx} + \frac{A_{\text{sm}}}{2E_{\text{sm}} I_{\text{sm}}} \int f_{\text{sm}}^2 \text{dx} = \sum_{k=0}^{\delta} \frac{1}{2} \left[ F_{\text{sl}} + F_{\text{sl+1}} \right] \left[ \delta_{\text{sl+1}} - \delta_{\text{sl}} \right] - U_{\delta p} \quad (3.11)
\]

Immediately prior to sudden fracture, the fitted stress curves for starter bars and mesh expressed in megapascals as functions of distance \( x \) over the effective development lengths \( l_d \) were:

\[
\begin{align*}
    f_s &= 0.66x - 135 & 200 < l_d \leq 400\text{mm} \\
    f_s &= 0.0011x^2 - 0.72x + 132 & 0 < l_d \leq 400\text{mm} \\
    f_{\text{sm}} &= 1.4x + 6 & 0 < l_d \leq 150\text{mm} \\
    f_{\text{sm}} &= 1.2x + 40 & 0 < l_d \leq 150\text{mm}
\end{align*}
\]

Because of the described inelastic displacements, elastic displacement corresponding with the maximum horizontal force is less than the total displacement. This effect has been allowed for in Equation 3.11 by incorporating the subtractive term \( U_{\delta p} \) to account for plastic displacement. Thus, \( U_{\delta p} \) is the product of the average force carried by the concrete section over the extent of plastic displacement up to sudden fracture. It is evident from Figure 3.16 that the concrete section exhibited fully plastic behaviour from 0.14 mm displacement onward. Hence, the phase of fully plastic displacement has been taken as \( 0.27 - 0.14 = 0.13 \text{ mm} \). Associated with this plastic displacement is the average horizontal force resisted by the concrete section, which was 250 kN.

Taking the density of extruded concrete as 2400 kg/m³, the estimated moduli of elasticity of the hollow core section and topping slab are respectively 31 GPa and 27 GPa. (Section 3.8.1.2 of the Concrete Structures Standard) [Standards New Zealand, 1995]. Therefore, the modular ratio of the topping slab concrete to precast concrete is \( n = \frac{27}{31} = 0.87 \), and the effective elastic section area \( A_e \) becomes \( 119200 + (0.87 \times 1.160 \times 65) \approx 185000 \text{ mm}^2 \). The length of composite slab between the support block and the actuating frame was 3120 mm. Subsequently, the elastic internal strain energy evaluated in accordance with terms of Equation 3.11 is:

\[
U_i = \frac{1}{2} \left( \frac{352 \text{kN}^2 \times 3120}{185000 \times 31000} + \frac{452 \times 2.44e^6}{205e^3} + \frac{176 \times 5.26e^6}{196e^3} \right)
\]

\[
= 33.7 + 2.7 + 2.4 = 38.8 \text{ kN-mm}
\]

The corresponding elastic external work done is the total work done (see Table 3.7) less \( U_{\delta p} \):

\[
U_e = 69.0 - (250 \text{ kN} \times 0.13 \text{ mm}) = 36.5 \text{ kN-mm}
\]

Hence, this summation would indicate that on the basis of materials response, a comparison can be drawn between the conservative portions of internal strain energy and external work done.
Composite Topping Bond

At sudden fracture, the horizontal force was transferred to three planes of resistance. The resulting system involved two vertical fracture planes and one horizontal bond surface (Fig. 3.22). Of the vertical fracture planes, only the principal topping fracture contained fully bonded reinforcement. From available measurements, it is impossible to ascertain the proportions of horizontal force resisted by each of the three primary surfaces throughout the post-fracture phase of Stage I. However, the general test observations suggest that bond contributions from the precast-to-topping interface were negligible at the beginning of Stage II.

The sudden brittle nature of fracture and its radical effect on the entire configuration would strongly suggest that initial rupture occurred through the hollow core section along the support line (see Figs 3.19 and 3.22). Since 72% of the horizontal force was resisted by the hollow core section (Table 3.6), the majority of force was transferred to the available resistance mechanisms in an instant. It is certain that the dynamic aspect of this release diminished the prospect of a suitable mechanism developing through composite bond between the topping and precast concretes.

Assuming that the tensile contribution of the composite concrete section was transferred to a resistance mechanism in the form of composite bond, it is evident that the average static topping bond shear stress \( \tau_b \) required to maintain equilibrium would not have been very large. Taking the force resisted by the precast concrete element and dividing over the total bond interface area gives:

\[
\tau_b = \frac{P_{pe}}{A_b}
\]  

(3.12)

The topping bond shear stress required to resist a equivalent static horizontal force was:

\[
\tau_b = \frac{253 \text{kN}}{(1160 \times 660) \text{mm}^2} = 0.33 \text{MPa}
\]

However, dynamic considerations involve the ability of the bond surface to absorb elastic internal strain energy associated with the hollow core section at fracture. If the bond surface was unable to conserve this quantity of strain energy as an alternative elastic mechanism (neglecting secondary effect losses) then plastic deformation would result. In conclusion, plastic deformation would immediately imply a complete loss of bond strength due to the customarily brittle behaviour of unreinforced shear mechanisms.

The required peak bond stress capacity over the tributary bond surface area may be postulated by considering the elastic strain energy per unit volume of material deformed by shear stress. Since only the bond surface area \( A_b \) is known, an effective depth of material is required so as to obtain a deformable volume. Considering the hollow core surface as infinitely rigid, the distance between the topping-to-precast interface and the centre-line of the starter bars would
form a distance between opposing bond forces, thus forming a shear depth of \( t = \frac{h}{2} = 32 \text{ mm} \) (Fig. 3.29).

Fig. 3.29 Assumed mechanism for the development of elastic strain energy in shear

The proportion of elastic internal strain energy stored in the hollow core section alone is taken as a ratio of the section area of precast to the total transformed concrete area \( A'_c \) in Equation 3.11. Hence, the energy stored in the precast section alone was \( U_{pc} = (119200/185000) \times 33.7 \) kN-mm = 21.7 kN-mm.

The elastic strain energy density for shearing stress may be written as:

\[
u = \frac{s_b^2}{2G}
\]

Equating the stored strain energy in the precast section to the shear mechanism and rearranging gives the required peak shear stress of:

\[
\tau_b = \sqrt{\frac{2GU_{pc}}{A_h t}}
\]

Based on the prior established values and taking the concrete modulus of rigidity \( G \) as 10 GPa gives a required elastic peak bond shear stress of:

\[
\tau_b = \sqrt{\frac{2 \times 10000 \times 21700}{(1160 \times 660) \times 32}} = 4.2 \text{ MPa}
\]

Direct-shear (as opposed to shear flow) topping bond tests conducted as part of this study and reported in Chapter 6 confirm that shear stress of this magnitude will not be sustained by

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Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 1
ordinary bond surfaces. In fact, it was shown that relatively smooth bond surfaces, such as extruded hollow core, might not even develop the static bond stress as given by Equation 3.12. It is apparent that a bond shear mechanism cannot be considered as a reserve of elastic strain energy in response to sudden tensile fracture of a hollow core section.

3.3.2 LOS 2

3.3.2.1 General

Test LOS 2 was directed at a special support tie detail (Figs 3.30 and 3.31) that entails embedded D10 tie bars. The detail has been specifically developed by industry to overcome short seating problems that may occur with modular precast flooring. The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit that was pretensioned with five 12.7 mm diameter Supergrade strands. A 10 mm diameter Grade 300 (D10) tie bar, in the form of a hairpin detail, was placed in each of the six voids of the hollow core section. The hollow core flanges were removed from above the voids to allow grouting with cast-in-place topping concrete. 665 mesh was placed over the hollow core unit, and was continued beyond the end of the seating length, thus contributing to the tensile strength across the construction joint (assumed to be the critical section). There were no starter bars placed in the topping concrete.

For the serviceability limit state, the mechanics of this detail depend on the development of shear friction across the construction joint between the end of the precast unit and the support. With the proper implementation of unit layout (Fig. 3.32), this detail has been successfully employed in practice for many years. However, for the detail to perform successfully under dilation type loadings that may involve significant structural ductilities, tie reinforcement must have the fundamental capacity to endure the maximum predicted elongation. With increased displacement, the mechanics of support shift from purely shear friction to combined shear friction and the catenary support associated with bar kinking (Fig. 3.33) and eventually to purely catenary support. Because small diameter deformed bars are necessary for the given detail, there is a general concern regarding the elongation capacity of this reinforcement.

3.3.2.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 3.2.2 (also Fig. 3.4).

(b) Reinforcement

Both 20% and 3% extension electrical resistance strain gauges (as described in Section 3.2.2) were employed on two individual items of hairpin reinforcement. These gauges were arranged...
so that the 20% extension variety was situated directly over the expected plane of cracking, at the end of the hollow core section. The 3% strain gauges were set at 100 mm centres about the expected cracking zone (see Fig. 3.34).

![Diagram](image)

**Fig. 3.30** Support configuration of test LOS 2 involving a hairpin tie detail

![Image](image)

**Fig. 3.31** LOS 2 test set-up before the addition of topping concrete

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2*
Fig 3.32 End support detail where embedded hairpin ties are employed for units with inadequate seating [NZCS-NZNSEE, 1991]

Fig. 3.33 Shear transferred by interface friction across a narrow gap [Mejia-McMaster and Park, 1994]

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2
3.3.2.3 Cast-In-Place Topping and Infill Concrete

The pre-mixed concrete for this test was received at a slump that was well within tolerances for a snatch sample and would be described as a good workable mix (Table 3.8).

![Strain gauge positions on hairpin bars used in the LOS 2 test](image)

**Fig. 3.34** Strain gauge positions on hairpin bars used in the LOS 2 test

**Table 3.8** Characteristics of cast-in-place topping and infill concrete used in test LOS 2

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 at 28 days</td>
<td>13</td>
<td>100</td>
<td>85</td>
<td>31 at 11 days</td>
</tr>
</tbody>
</table>

3.3.2.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) D10 Hairpins

![Stress-strain relationship](image)

**Fig. 3.35** Typical stress-strain relationship of 10 mm diameter Grade 300 bars employed as hairpin ties in test LOS 2

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2*
Tensile tests were performed on three specimens of 10 mm diameter Grade 300 bars, and the following was recorded (Table 3.9):

<table>
<thead>
<tr>
<th>Average yield strength (MPa)</th>
<th>Average ultimate tensile strength (MPa)</th>
<th>Average strain at $\varepsilon_{uh}$ (%)</th>
<th>Average strain at ultimate tensile strength (%)</th>
<th>Average strain at fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>312</td>
<td>434</td>
<td>2.53</td>
<td>16.3</td>
<td>23.6</td>
</tr>
</tbody>
</table>

Average $E = 196$ GPa

3.3.2.5 Results of Testing

(a) Initial Response

The initial response to horizontal loading showed a stiff system (Fig. 3.36). The first cracks, measuring approximately 0.1 mm, appeared in the infill strip between the precast unit and support block at a force of 250 kN. At a force of 441 kN a small plateau occurred in the force displacement response, with an increment in displacement from 0.85 mm to 1.4 mm. This plateau signalled the loss of notable concrete tensile contribution. From this point the horizontal resistance increased at a steady rate up to peak reaction of 520 kN at 7.2 mm displacement.

(b) Fracture

Immediately beyond the peak horizontal reaction, mesh wires began to fracture in a serial fashion and this continued out to displacement of 21 mm. From 25 mm displacement a plateau occurred at which the D10 hairpins approached fracture elongation. Failure of the hairpins commenced at 38 mm displacement, and between 38 mm and 43 mm displacement, seven of the twelve hairpin legs had fractured. A second smaller plateau featured until 49 mm displacement when a further three legs fractured. The remaining two legs were still intact at the terminal displacement of 55 mm. However, these legs failed under the small additional weight of the technician who had climbed onto the test specimen to place the vertical ram load cell. Hence, complete fracture of the tie reinforcement was achieved without the addition of a significant vertical point load.

(c) Displacements

Section cracks were concentrated in the construction joint region and developed progressively under applied force. The principal crack was fully developed at around 200 kN and 0.4 mm displacement. The latter stages of the test were characterised by increasing downward vertical displacement at the support. The vertical dislocation measured 11 mm when horizontal loading was terminated at 55 mm displacement.
Fig. 3.36  Force-displacement response of test LOS 2

Fig. 3.37  Force-displacement diagram divided into regions of significance in terms of resistance mechanisms

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2
3.3.2.6 Analysis of Test Results

In order to analyse the response of test LOS 2, the force-displacement diagram has been divided into the five distinct regions that were reflected in the test outcome (Fig. 3.37). Each of these regions (Stages) indicates a basic change in the resistance mechanism against imposed horizontal displacements. As a whole, the diagram clearly indicates the differing contributions of respective steel reinforcement and concrete components.

(Stage I) Section Yield

Tensile stresses had begun to register in both the hairpin ties and 665 mesh between 0.1 mm and 0.15 mm displacement. At 0.4 mm displacement the D10 hairpins had typically reached yield stress of just over 300 MPa. Hence, the section response was essentially elastic up to the point where the D10 hairpins began to yield. Strain hardening had commenced in the D10 hairpins by the end of Stage I (Fig. 3.38). The 665 mesh had achieved 150 MPa of tensile stress at 0.2 mm displacement and 400 MPa at 0.4 mm displacement. The mesh reached the proportional limit just prior to the onset of plastic deformation near the end of Stage I. (Fig. 3.39).

From zero to 0.4 mm displacement, the section exhibited elastic properties with a stiffness of 1025 kN/mm. Immediately beyond 0.4 mm displacement, the tensile response rapidly softened to a stiffness of 90 kN/mm. At 0.85 mm displacement the section commenced a short plateau of plastic displacement. During this stage of the test, the 25 mm wide construction joint had fully ruptured and it was evident that the tensile contribution of the concrete section was lost (Fig. 3.40).

![Graph showing stress distributions](image)

**Fig. 3.38** Average stress distributions along D10 hairpin bars at the indicated horizontal displacements during Stage I of test LOS 2

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2*
Fig. 3.39  Average stress distributions along 665 mesh wires at the indicated horizontal displacements during Stage I of test LOS 2

Fig. 3.40  Proportion of force resisted by reinforcing steel during Stage I of test LOS 2

(Stage II)  Section Yield to Peak Force

From 1.4 mm displacement to peak horizontal force at 7.2 mm displacement, the 665 mesh provided its maximum contribution to tensile strength. At the end of Stage II, the mesh contributed 22% of the 520 kN total reaction force. The D10 hairpins had all reached strain hardening by 2.0 mm displacement, and by the end of Stage II had attained an average of 5.9% elongation at the critical section.

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2
A crack that had initiated late in Stage I became more pronounced during Stage II. Situated in the hollow core unit (Fig. 3.41), the crack opened at approximately the same rate as the principal crack in the 25 mm wide infill strip. During Stage II, the section crack reached a maximum of 5.0 mm opening at the top of the unit, and averaged 4.0 mm opening through the depth of the section.

It is evident that the placement of prestressing strand exerted some influence on the formation and behaviour of this crack. Since there were no pretensioned strands located at the edge of this particular hollow core unit, the crack propagated from the edge, starting 165 mm from the end of the unit at the side and extending toward the centre-line of the unit at an initial angle of 16°. It appears that the course of the crack was affected by the interception of the first strand located 220 mm from the side of the unit.

![Fig. 3.41 Crack through section of hollow core slab](image)

(Stage III) Peak Force to Fracture of Mesh

From peak force at 7.2 mm displacement through to 22 mm displacement, the reaction force diminished in two notable steps from 520 kN to 412 kN due to fracture of the topping mesh. Between 22 mm and 26 mm displacement, the reaction decreased gradually as the extension of D10 hairpins began to exceed the ultimate tensile strength plateau of the stress-strain relationship. The peak steel stress (UTS) elongation was first reached during this stage of the test at about 15 mm displacement. Toward the end of Stage III, bar slippage had occurred and significant strain penetration was apparent (Fig. 3.42).

---

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2*
Fig 3.42 Average stress distributions along D10 hairpin bars at the indicated horizontal displacements to the end of Stage III of test LOS 2

(Stage IV) Plateau Region to Onset of Hairpin Fracture

This region indicates a plateau between 26 mm and 37 mm displacement. The plateau was likely to have been symptomatic of the state of stress within the hairpins coupled with an increasing incidence of slippage and strain penetration. It is certain that the hairpin legs were extended into the range of the stress-strain curve between the peak tensile stress and fracture elongations. The net effect of bar slippage was an increasing gauge length that would offset the falling tensile resistance of reinforcement and resulted in a distinctive plateau over several millimetres of displacement. Inspection of the specimen clearly indicated that significant penetration into the section had occurred, with cleanly fractured pull-out cones measuring an average of 10 mm to 15 mm deep (i.e., 1.0 to 1.5d_b) along most of the bars.

(Stage V) Hairpin Fracture and End of Test

Starting at 37 mm displacement, the hairpins failed in quick succession. Seven of the twelve legs fracturing in almost a single volley. This occurred over an increased extension of 5.0 mm, in which the reaction fell from 375 kN to 155 kN. Between 42 mm and 48 mm displacement, a second brief plateau was sustained until a further three of the ties failed. The final small plateau went from 48 mm to 55 mm on the remaining two tie legs. At this point, the reaction had fallen to 68 kN and failure of these ties was imminent. The detail had survived just to the end of the prescribed 55 mm displacement, and failed under the small additional weight of the technician.

Immediately before failure, the end of the unit had dropped an average of 11 mm below the original support level and was apparently supported by the two remaining tie legs.
Fig. 3.43  Well defined crack opening through the support construction joint

Fig. 3.44  Fractured D10 support ties at end of test

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2
3.3.2.7 Specific Analysis

(a) Work Done

The quantity of work done by this detail over the prescribed 55 mm displacement totalled 18,700 kN-mm.

(b) Ductility

From Figure 3.42, it is evident that the tensile overstrength capacity of a straight length of well confined reinforcing bar can be developed in less than 20\(d_b\). At the hook end, the strength capacity would probably have developed in an even shorter distance. Hence, most of the strength development and slippage would have occurred between the critical section and the tangent of the hook return.

In effect, the gauge length of the hairpins would have changed continuously due to progressive strain penetration and bar slippage. However, a reasonable estimate of the effective gauge length would be to take the distance to bar yield on either side of the critical section at peak bar stress. The total extension capacity is the product of fracture strain and effective gauge length:

\[
\Delta L = \varepsilon_p L_o \tag{3.15}
\]

Referring to Figure 3.42 and Table 3.4, the respective lengths on either side of the critical section are 100 mm and 130 mm at 21 mm displacement, giving an extension capacity of \(\Delta L = 23.6\% \times 230 \text{ mm} = 54 \text{ mm}\).

(c) Bar Bond in the Support Region

In this detail, the straight embedment length grouted into the hollow cores, will determine the critical bond capacity of reinforcement. From Figure 3.42, peak bond stress was clearly developed in the bond region between gauge stations No. 3 and No. 4.

The average bar bond may be calculated by taking the difference between stress readings at successive gauge stations and dividing the net axial force over the bond surface area between stations:

\[
\frac{u}{\bar{u}} = \frac{A_t (f_{sl} - f_{sij})}{\pi d_b L} = \frac{d_b (f_{sl} - f_{sij})}{4L} \tag{3.16}
\]

Hence, with reference to Figure 3.45, the average peak bar bond stress is:

\[
\frac{u}{\bar{u}} = \frac{10 (407 - 91)}{4 \times 100} = 7.9 \text{ MPa}
\]

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 2
Fig. 3.45  Peak stress distribution along hairpin tie bars embedded in grouted hollow core voids (26 mm horizontal displacement)

(d)  **Dowel Action at the Support**

The support reaction on the test specimen was calculated to be 12.2 kN. At the end of testing the measured bar angle was $\theta_i = 8^\circ$ (see Fig. 3.33). Because there was no friction component at full extension, the shear force was transferred entirely by dowel action, given as:

$$V_n = A_v f_t \sin \theta_i$$  \hspace{1cm} (3.17)

The average ultimate tensile strength of reinforcement was measured at 434 MPa (Table 3.4). Immediately before collapse the end support was provided by two tie legs, giving a calculated vertical reaction of $V_n = 2 \times 79 \text{ mm}^2 \times 434 \text{ MPa} \times \sin 8^\circ = 9.5 \text{ kN}$.

The difference between the calculated dowel reaction of 9.5 kN and the abovementioned support reaction of 12.2 kN was due to interactions between the fractured hairpin legs and the face of the concrete section. As the unit deflected and rotated, a number of the fractured tie legs were effectively thrust into the concrete support face, thus providing a degree of shear resistance.
3.3.3 LOS 3

3.3.3.1 General

Test LOS 3 was the second of three tests directed at the typical starter bar detail (Figs 3.9 and 3.10). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit that was pretensioned with four 12.7 mm diameter Supergrade strands. Four 12 mm diameter Grade 430 (HD12) starter bars were placed at 300 mm centres, and extended 600 mm into the topping slab. 665 mesh was placed over the hollow core unit, and was continued beyond the end of the seating length, thus contributing to the tensile strength across the construction joint (assumed to be the critical section). Unlike the first test involving this detail (ie., LOS 1) a superimposed dead load was not applied to the test specimen.

3.3.3.2 Instrumentation

(a) Forces and Displacements

Refer to the methods described in Section 3.2.2 (also Fig. 3.4).

(b) Reinforcement

Electrical resistance strain gauges were employed as described in Section 3.3.1.2(b).

3.3.3.3 Cast-In-Place Topping Concrete

The concrete supplied for test LOS 3 was exactly on the target slump of 100 mm. This was a good consistent mix that showed ease of placement and good workability (Table 3.10).

Table 3.10 Characteristics of cast-in-place topping concrete for test LOS 3

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 at 28 days</td>
<td>13</td>
<td>100</td>
<td>100</td>
<td>25 at 17 days</td>
</tr>
</tbody>
</table>

3.3.3.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2.

(b) HD12 Starters

The characteristics of 12 mm diameter Grade 430 starter bars were as given in Table 3.4.

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3*
3.3.3.5 Results of Testing

(a) Initial Response

The initial response to horizontal loading showed a stiff system (Fig. 3.46) up to about 200 kN. At this stage a crack appeared through the topping slab at 620 mm from the support, just beyond the point of starter bar curtailment. Under increased force the topping crack widened and slippage occurred between the precast and topping concretes.

(b) Fracture

At just over 2.0 mm displacement and at a force of almost 300 kN, the hollow core section suddenly fractured. The resulting crack occurred across the section at 570 mm from the support, adjacent to the principal topping crack, and extended 135 mm downward into the hollow core section. The resistance of the section fell to 215 kN, and under continued displacement reached the post-fracture peak force of 231 kN at a displacement of 6.5 mm.

(c) Displacements

Slippage between the precast and topping concretes was observed in the very early stages of this test. The magnitude of net topping slippage closely matched the horizontal unit displacement throughout the pre-fracture stage, as recorded in Figure 3.46. The post-fracture stages were characterised by upward (i.e., hogging) displacement resulting from the moment couple formed between the prestressing strand reaction in the fractured hollow core section and the centroid of applied force at the support carriage. Eventually, this rotation began to affect the test procedure and the experiment was terminated at 15 mm horizontal displacement. At this stage the rotation had lifted the unit by about 11 mm at the position of the section fracture.

3.3.3.6 Analysis of Test Results

In order to analyse the response of test LOS 3, the force-displacement diagram has been divided into the four distinct regions that were reflected in the test outcome (Fig. 3.47). Each of these regions (Stages) indicates a basic change in the resistance mechanism against imposed horizontal displacements. As a whole, the diagram clearly indicates the differing contributions of respective steel reinforcement and concrete components.

(Stage I) Peak Load and Fracture

The initial response showed an average axial stiffness of 1980 kN/mm, with resistance provided by the entire composite section. At about 200 kN (i.e., 67% of peak load), the response softened markedly to an average of 71 kN/mm up to sudden fracture at just under 300 kN. The reduced stiffness is attributed to the propagation of cracks through the topping slab, especially at the point of starter bar curtailment and over the support region. The subsequent loss of topping
shear bond (see Fig. 3.48) and mobilisation of the precast section contributed to stiffness reduction.

Fig. 3.46  Force-displacement response of test LOS 3

Fig. 3.47  Force-displacement diagram divided into regions of significance in terms of resistance mechanisms

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3
Fig 3.48 Pre-fracture relationship between the unit and topping slippage displacements in the support region, indicating significant shear bond loss between precast and topping concretes

The decrease in stiffness between 200 kN and sudden fracture corresponded with an increase in starter bar and mesh strains. Reinforcement stresses generally increased in direct proportion to applied horizontal force. Analysis of reinforcement stresses indicates that reinforcement provided the majority of axial restraint in the immediate vicinity of the support (Fig. 3.49). Peak starter bar stresses also occurred over the support region, reaching first yield at about 1.3 mm horizontal displacement.

Fig. 3.49 Proportion of total horizontal force resisted by starter bars and mesh in the support region, up to sudden fracture at 2.2 mm displacement

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3
Fig 3.50 Total force and reinforcing steel force (starters and mesh) across the support region, up to sudden fracture of the hollow core section

Analysis of Figures 3.49 and 3.50 would suggest that the sudden fracture of the hollow core section might have been instigated by an increase in precast section tension in the support region. At the location of the hollow core fracture (i.e., 650 mm from the support region) the proportion of tension force resisted by the precast section would have been substantially greater than indicated by Figures 3.49 and 3.50. The section force can be estimated by subtracting the force resisted by topping mesh at the proportional limit from the total force, giving 300 kN—(528 MPa x 176 mm²) = 207 kN. Hence, a slight increase in direct restraint applied to the precast section at the support could have been sufficient to cause fracture at a location resisting a much greater proportion of the total force.

It is most likely that the increase in precast section restraint was due to wedging effects in the support region under increasing displacement. Unlike test LOS 1, the sides of the hollow core unit in test LOS 3 were deliberately debonded from the cast-in-place support block concrete so as to avoid the wedging restraint that contributed to the sudden fracture in test LOS 1. However, it was expected that significant wedging forces would develop between the displacing precast section and the typical aberrations that occur in topping slab and support block surfaces.

The distribution of starter bar stresses (Fig. 3.51) shows that bar bond efficiency was greater in the cast-in-place support block than in the topping slab, as indicated by the respective slopes on either side of gauge number 3.

The strength development of starter bar and mesh elements was almost in direct proportion to their respective areas (Fig. 3.52). The area of HD12 starters was 452 mm² (i.e., 72% of total topping steel area) and the area of mesh was 176 mm². Immediately before fracture, the starters had developed 70% of the total steel reaction force.
Fig. 3.51  Average pre-fracture stress distributions along HD12 starter bars at the indicated horizontal displacements during Stage I of test LOS 3

Fig. 3.52  Average pre-fracture restraint forces provided by HD12 starter bars and 665 mesh in the support region

The sudden fracture of the precast concrete section resulted in a principal crack through the entire width of the hollow core unit which extended about two thirds of the section depth. The sudden fracture was accompanied by a horizontal displacement of 0.6 mm, over which horizontal resistance force fell from the peak value of 298 kN to the post-fracture resistance of 215 kN at the end of loading stage I.

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3
It is feasible that the effect of eccentric prestress in the hollow core section (i.e., prestressed outside the middle third kern of the section) may have contributed to the precast section fracture by reducing tensile resistance in the section top fibre. The crack occurred immediately beyond the curtailment point of the topping starter bars, at a location of intensified section stresses.

(Stage II) Post-Fracture to Post-Fracture Peak Force

At the beginning of loading stage II the topping crack opening ranged from 1.2 mm to 1.5 mm, which was about one half of the total unit displacement. From a static position at post-fracture, further horizontal displacement resulted in an increased reaction up to the post-fracture peak force of 231 kN (78% of the peak pre-fracture force) with 6.5 mm total displacement. Throughout this stage of the test, resistance to horizontal force was manifested in a combination of tension and bending, with tensile reactions respectively provided by prestressing strand in the fractured precast section and mesh wire in the topping slab. The bending moment resulted from the force couple produced between applied force acting through the centroid of the elastic composite section at the support carriage, and the centroid of tensile reaction in the fractured section.

During this loading stage, the average starter bar and mesh stresses in the support region (see Fig. 3.53) generally behaved in relative proportion to the overall force displacement response (Fig. 3.46). This would suggest that although significant bond loss had occurred between the precast and topping concretes, the wedging mechanism described in the prior section was still apparent in the post-fracture stages.

![Average stress distributions along HD12 starter bars at the indicated horizontal displacements during Stage II of test LOS 3](image)

**Fig 3.53** Average stress distributions along HD12 starter bars at the indicated horizontal displacements during Stage II of test LOS 3

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3*
(Stage III) \hspace{1cm} \textbf{Post-Fracture Peak Load to Fracture of Mesh Wires}

An important feature of this loading stage was the upward rotation caused by the force couple. At the beginning of load stage III, the rotation had caused an upward deflection of 3.0 mm at the location of the section fracture. It is evident that this effect influenced the duration of mesh wire resistance to applied force. Between 6.5 mm and 7.0 mm horizontal displacement, the reaction fell slightly to 224 kN and was followed by the simultaneous fracture of seven out of the eight mesh wires that bridged the topping crack. Further applied force from the residual of 118 kN at 8.0 mm displacement caused fracture of the last mesh wire at 129 kN and 8.7 mm displacement.

(Stage IV) \hspace{1cm} \textbf{Fracture of Mesh Wires to Termination of Test}

Loading Stage IV was characterised by a modest increase in horizontal reaction and significant rotation. Since resistance to bending moment was lost with fracture of the mesh, the rate of upward deflection had accelerated. From the beginning of Stage IV to termination of the test at horizontal displacement of 15 mm, the upward deflection progressed from 8.0 mm to 11 mm. Hence, it became necessary to terminate the test because this magnitude of rotation began to impinge on test rig equipment (Fig. 3.54).

![Fig. 3.54 Accentuated crack opening and unit rotation at the end of testing](image)

\textit{Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3}
3.3.3.7 Specific Analysis

(a) Section Fracture

The combination of direct tension, eccentric prestress and small positive bending moment is likely to have had a direct influence on the formation of the hollow core section fracture. At the time of testing, the stress state in the hollow core extreme top fibre is estimated as follows (Fig. 3.55) (also see Fig. 3.2a).

![Diagram](image)

**Fig. 3.55** Loads and reactions on test specimen

Hence, bending moment at distance \( x \) from the support is equal to \( 5.62x - \frac{1}{2}wx^2 \) (kNm). With \( x = 0.65 \) m to the end of the starter bars and a weight per unit length of \( w = 4.67 \) kN/m (precast section plus topping concrete), the calculated bending moment \( M(x) \) at the point of starter curtailment is +2.7 kNm. This was the bending moment resisted by the precast section from the time that composite topping was placed, since there was no temporary shoring applied to the system at construction.

The effective prestress force \( P_f \) is taken to be the initial prestress \( P_i \) less 20% losses. As such, the assumed losses are above average to account for strand slippage incurred by relatively short members with saw-cut tendons. Hence, the estimated effective pretension force is \( P_f = (0.8 \times 0.72 \times 184 \text{ kN}) \times 4 = 424 \) kN. The prestress eccentricity is the normal distance between the neutral axis and the centroid of strands. Thus, \( e = \dot{y}_x - \dot{y}_s \), which equals \((100 - 45) = 55 \) mm.

Ignoring short-term effects, the section stress at the time of testing may be calculated under the usual format of combined stresses:

---

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3*
\[ f_c = \frac{P_r}{A_{pe}} \pm \frac{P_r \cdot e}{Z} \pm \frac{M_{(s)}}{Z} \]  \hspace{1cm} (3.18)

Thus, for the test specimen top fibre:

\[ f_c = \frac{-424 \text{kN}}{119300} + \frac{(424 \times 0.055) \text{kNm}}{6.4e^6} = \frac{4.8 \text{kNm}}{6.4e^6} = -0.66 \text{MPa} \]

Under direct axial tension during testing, a uniform tensile stress is considered to apply across the section. It was noted in the early stages of testing (see Stage I) that cracks had formed in the topping slab at the starter curtailment point, in the vicinity of the section fracture. Hence, it is certain that tensile resistance of the topping slab was significantly reduced at an early stage.

The variation in hollow core geometry precludes straightforward analysis of section stresses based on cracked elastic section properties. Consequently, the principles of equilibrium analysis have been used in an iterative calculation. In this procedure, the topping mesh reaction is treated as a constant force, equal to the characteristic yield strength of mesh (ie., 530 MPa) and independent of section curvature. Because of the small curvatures involved under predominantly axial load and the observed topping slab slippage, this is a reasonable assumption. The configuration of forces and moments and the effective cross section are shown in Figure 3.56.

![Diagram](image)

**Fig. 3.56** Internal forces with applied force P and moment M acting on section, and effective cross section assumed for hollow core unit

As previous, the peak axial force is taken as P = 300 kN and the concurrent section bending moment as \( M_{(s)} = +2.7 \text{kNm} \). With an initial pretension force of \( F_p = 425 \text{kN} \) and constant mesh tension of \( F_s = 93.3 \text{kN} \), section equilibrium is satisfied at a neutral axis depth of \( c = 156 \text{ mm} \) and corresponding curvature of \( \varphi = 1.0 \times 10^{-6} \text{ rad/mm} \).
With an assumed concrete elastic modulus of $E_c = 32000 \text{ MPa}$, the hollow core top fibre tension stress at fracture is calculated as 1.4 MPa from the relationship $f_c = E_c \cdot \varphi \cdot (H - c)$. Based on 45 MPa concrete, this is equal to a tensile rupture strength of $0.21 \sqrt{f'_c}$ under predominantly axial loading.

(b) Topping Mesh Fracture

At the fracture of topping mesh, strain-equilibrium principles are likewise applied. However, the mesh force is increased to fracture strength (i.e., 660 MPa) and the tensile contribution of hollow core section is ignored (Fig. 3.57).

![Diagram](image.png)

**Fig. 3.57** Internal forces with applied force $P$ and moment $M$ acting on the section

![Diagram](image.png)

**Fig. 3.58** Internal forces with applied force $P$ and moment $M$ acting on section

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3*
The peak post-fracture axial force is taken as $P = 230$ kN and the concurrent section bending moment as $M = 2.7$ kNm. With an initial pretension force of $F_p = 425$ kN and constant mesh tension of $F_s = 116.2$ kN, section equilibrium is satisfied at a neutral axis depth of $c = 213$ mm and corresponding curvature of $\varphi = 7.14 \times 10^{-7}$ rad/mm.

(c) End Of Test

At the end of test there is no tensile contribution from topping mesh and the tensile contribution of hollow core section is ignored (Fig. 3.58). The peak post-fracture axial force is taken as $P = 125$ kN and the concurrent section bending moment as $M = 2.7$ kNm. With an initial pretension force of $F_p = 425$ kN, section equilibrium is satisfied at a neutral axis depth (corresponding with strand height) of $c = 45$ mm and curvature of $\varphi = 8.33 \times 10^{-6}$ rad/mm.

3.3.4 LOS 4

3.3.4.1 General

Test LOS 4 was the third of three tests directed at the typical starter bar detail (Figs 3.9 and 3.10). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit that was pretensioned with four 12.7 mm diameter Supergrade strands. Four 12 mm diameter Grade 430 (HD12) starter bars were placed at 300 mm centres, and extended 600 mm into the topping slab. 665 mesh was placed over the hollow core unit, and was continued beyond the end of the seating length, thus contributing to the tensile strength across the construction joint (assumed to be the critical section). Similarly to the second test involving this detail (ie., LOS 2) a superimposed dead load was not applied to the test specimen.

3.3.4.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 3.2.2 (also Fig. 3.4).

(b) Reinforcement

Electrical resistance strain gauges were employed as described in Section 3.3.1.2(b).

3.3.4.3 Cast-In-Place Topping Concrete

The concrete supplied for test LOS 4 was considerably above the target slump of 100 mm. However, the concrete was accepted on the basis that it had good consistency and did not exhibit bleeding when worked (Table 3.11).

---

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 3*
Table 3.11 Characteristics of cast-in-place topping concrete for test LOS 4

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 at 28 days</td>
<td>13</td>
<td>100</td>
<td>165</td>
<td>30 at 25 days</td>
</tr>
</tbody>
</table>

3.3.4.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) HD12 Starters

The characteristics of 12 mm diameter Grade 430 starter bars were as given in Table 3.4

3.3.4.5 Results of Testing

(a) Initial Response

The initial response to horizontal loading showed a stiff system (Fig. 3.59) up to about 251 kN. At this stage, a crack of average 0.25 mm width had appeared through the topping slab across the line of the support. There was a correspondingly small loss of horizontal reaction, ranging from 251 kN to 230 kN.

(b) Fracture

At 230 kN and 1.4 mm displacement, a sudden fracture occurred through the topping slab at an average distance of 840 mm from the support line. This resulted in a significant loss of reaction from 230 kN to 150 kN, and 2.0 mm horizontal displacement. Under increased force the principal topping crack continued to widen, with pronounced slippage occurring between the precast and topping concretes. The post-fracture peak force of 172 kN was recorded at 3.9 mm displacement, with a plateau occurring up until the beginning of topping mesh fracture at around 7.0 mm horizontal displacement.
Fig. 3.59  Force-displacement response of test LOS 4

Fig. 3.60  Force-displacement diagram divided into regions of significance in terms of resistance mechanisms

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 4
Horizontal restraint capacity progressively diminished between 7.2 mm displacement and the completion of topping mesh fracture at 20 mm displacement. Throughout this phase there was intermittent tensile fracturing of mesh wires, similar to observations in test LOS 1. Once the restraint provided by mesh wires was fully overcome, only the component of residual sliding friction was recorded.

Drying shrinkage cracks had developed in the topping slab during the curing period, and it is certain that these cracks exerted an influence on unit behaviour during testing. Shrinkage cracks observed before testing are shown in Figure 3.64, with crack widths measuring between 0.1 mm and 0.2 mm. The principal topping crack that eventuated under applied load was propagated by an initial drying shrinkage crack situated close to the curtailment of starter bars (see Fig 3.61).

![Figure 3.61](image)

Fig. 3.61 Drying shrinkage cracks in topping slab immediately prior to testing. Under applied axial force, a shrinkage crack (indicated) developed into the principal topping fracture

(c) Displacements

A very small amount of slippage was observed between the precast and topping concretes in the early stages of this test, up to the point of topping fracture. Just prior to topping fracture (approximately 1.4 mm displacement) a considerable increase occurred in the rate of slippage. The post-fracture stages were characterised by continued and increasing slippage. Towards the final stages of the test, the only resistance to axial force was provided by residual sliding friction between the precast section and support block. As a result of support block spalling, the hollow core unit was effectively pulled off the support at a horizontal displacement of 45 mm.
3.3.4.6 Analysis of Test Results

In order to analyse the response of test LOS 4, the force-displacement diagram has been divided into the four distinct regions that were reflected in the test outcome (Fig. 3.60). Each of these regions (Stages) indicates a basic change in the resistance mechanism against imposed horizontal displacements. As a whole, the diagram clearly indicates the differing contributions of respective steel reinforcement and concrete components.

(Stage I) Peak Load and Fracture

The initial response showed an average axial stiffness of 1405 kN/mm, with resistance provided by the entire composite section. The initial stiff response endured until the peak horizontal force of 251 kN and 0.3 mm displacement. The drop in reaction directly beyond peak force was due to gradual crack opening (up to average 0.25 mm) in the support region, and was not caused by sudden topping fracture. However, sudden topping fracture resulted from the reapplication of axial force, immediately after crack opening in the support region.

Sudden loss of shear bond between precast and topping concretes was a feature of post-fracture in loading stage I. Prior to fracture, only a small amount of slippage was recorded between the precast and topping concretes. Figure 3.63 indicates sudden conspicuous slippage just prior to topping fracture, followed by an almost uniform (but slightly increasing) rate of slip in the post-fracture range.

![Slippage vs. Horizontal Displacement Graph]

**Fig. 3.62** Relationship of precast unit and topping slippage displacements in the support region, showing a step in slippage immediately prior to topping fracture at 1.4 mm displacement

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 4*
The appearance of topping cracks over the support region corresponded with a marked increase in starter bar and mesh strains. Reinforcement stresses generally increased in proportion to applied horizontal force up to the maximum starter stress of 448 MPa at 1.3 mm displacement, immediately prior to topping fracture. The stress gradients in Figure 3.63 indicate that bar bond stresses were generally more efficient in the topping slab region than in the support block.

![Stress Distribution Diagram](image)

**Fig. 3.63** Average pre-fracture stress distributions along HD12 starter bars at the indicated horizontal displacements during Stage I of test LOS 4

![Restraint Forces Diagram](image)

**Fig. 3.64** Average pre-fracture restraint forces provided by HD12 starter bars and 665 mesh in the support region

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 4*
(Stage II) Post-Fracture to Onset of Mesh Fracture

Beyond topping fracture and up to the onset of mesh fracture (i.e., between 1.4 mm and 7.2 mm displacement), starter bar strains in the support region maintained almost static levels of tensile stress under continued loading (see Fig. 3.63).

The post-fracture peak force of 172 kN was attained at 3.9 mm displacement, and gradually decreased to 163 kN prior to first mesh fracture.

(Stage III) Onset of Mesh Fracture to Full Loss of Steel Reaction

Once the peak load displacement of 7.2 mm was exceeded, a steady loss of reaction resulted from the serial tensile fracture of mesh wires. Likewise, the initially significant effects of residual concrete bond and friction were completely diminished during this stage of loading. Because the mesh wires were the only form of reinforcement offering resistance across the principal topping crack, the entire contribution from reinforcement was lost with fracture of the last wire at 20 mm displacement. Beyond 20 mm displacement it was clear that the only resistance provided was due to sliding friction between the sofit of the hollow core unit and the seating ledge.

(Stage IV) Loss of Reaction to Collapse

The final stage of the test registered almost negligible reaction force. The final stage of the test involved displacing the unit until the eventual collapse occurred at 45 mm displacement. A degree of downward movement of the precast section resulted from support spalling. Immediately prior to collapse, the unit had slipped to 6.0 mm below the original support level (Fig. 3.65).

![Graph](image)

**Fig. 3.65** Vertical dislocation of the hollow core section due to spalling of the support ledge, measured in relation to horizontal displacement up to collapse

*Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 4*
3.3.4.7 Specific Analysis

(a) Work Done

Referring to Figure 3.60, the summation of total work done through each loading Stage (I to IV) is shown in Table 3.12:

Table 3.12 Summation of external work done by the connection detail of test LOS 4 over each loading stage. Note that stage I exhibited mainly elastic deformation up to peak force, and likewise, plastic deformation beyond sudden fracture:

<table>
<thead>
<tr>
<th>Stage</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work Done</td>
<td>415</td>
<td>862</td>
<td>791</td>
<td>152</td>
<td>2220</td>
</tr>
<tr>
<td>(kN-mm)</td>
<td>(182 pre-fracture)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion</td>
<td>18</td>
<td>39</td>
<td>36</td>
<td>7</td>
<td>100%</td>
</tr>
</tbody>
</table>

Fig. 3.66 Crack through composite topping at 8mm horizontal displacement, corresponding with the onset of topping mesh fracture

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 4
Fig. 3.67  Side view at end of test, showing separation between topping and precast concretes

Fig. 3.68  View of support block after removal of hollow core section. Note smooth soffit of cantilevered topping slab

Chapter 3: Loss of Support (LOS) Tests: Results of Experiments: Test LOS 4
3.4 DISCUSSION ON TEST RESULTS

3.4.1 TYPICAL STARTER BAR DETAIL (Tests LOS 1, LOS 3, LOS 4)

The results of tests performed on the typical starter bar detail (see Sections 1.2.3.3 and 1.2.3.4) provide sufficient evidence of detail performance under dilation type loading. The response behaviours of the three test specimens are shown in Figure 3.69.

![Graph showing force-displacement responses of tests LOS 1, LOS 3, and LOS 4](image)

Fig. 3.69 Comparison of force-displacement responses of tests LOS 1, LOS 3 and LOS 4

A comparison of test results indicates that a characteristic mode of failure may occur with starter bar details subjected to axial loading. The relevant observations are as follows:

- In each case, the topping slab fractured a short distance beyond the point of starter bar curtailment. It would appear that the transition in topping stiffness and the reduced ability for crack control directly influences topping fracture.

- Topping fracture is accompanied by a rapid breakdown in composite bond strength between the topping slab and hollow core unit. It is likely that composite bond capacity will have begun to deteriorate prior to the sudden topping fracture.

---

Chapter 3: Loss of Support (LOS) Tests: Discussion on Test Results
• The onset of topping mesh fracture can be expected to occur at around 6 mm horizontal displacement. The tensile resistance of mesh will be fully depleted at about 20 mm displacement. Hence, a useful rule of thumb for the ductility of topping mesh under a dilation loading regime is $1 + 2 = 3$, in terms of quarter inch increments.

• It is likely that collapse of the hollow core unit will occur at a lesser horizontal displacement than the seating length. This is due to spalling of the support ledge, but may also be result from tensile fracture at the end of the hollow core section. In each case, the actual configuration of seating is important, especially with regard to restraint provided by cast-in-place concrete in unit shear keys.

3.4.2 HAIRPIN DETAIL (Test LOS 2)

The results from this test indicate the following:

• Support tie details involving bars embedded into the support and grouted into the hollow core voids can provide a controlled ductile response to dilation forces.

• From the test, it is apparent that efficient bond capacity associated with deformed bars can mitigate ductile capacity. Hence deformed bars are not suited to applications where significant ductility may be required.

• Details involving cutback hollow core flanges and fully grouted voids result in much improved topping bond capacity.

3.4.3 COMPARISON OF WORK DONE

Table 3.13 shows the comparison of work done by the completed tests (test LOS 3 was terminated at 15 mm displacement). The efficiency of the embedded reinforcement is calculated as the ratio of work done $U$ (N-mm) to the product of the effective volume of embedded bar $V_{se} \text{ (mm}^3\text{)}$ and actual yield stress $f_y \text{ (N/mm}^2\text{)}$. Hence, the dimensionless ratio that results may be regarded as resistance mechanism efficiency index. Based on strain gauge readings taken from the tests (see Figs 3.38 and 3.39) the effective length of embedment has been taken as 300 mm for each case.
Table 3.13  Comparison of external work done by the connection detail of tests LOS 1, LOS 2 and LOS 4, and efficiency index of each

<table>
<thead>
<tr>
<th>Test</th>
<th>Resistance Mechanism</th>
<th>$V_{se} = 300$ $A_s$ (mm$^2$)</th>
<th>$f_y$ (N/mm$^2$)</th>
<th>Work Done (kN-mm)</th>
<th>$U/(V_{se} f_y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS 1</td>
<td>665 topping mesh</td>
<td>52800</td>
<td>528</td>
<td>1710</td>
<td>0.061</td>
</tr>
<tr>
<td>LOS 2</td>
<td>6-D10 hairpins</td>
<td>282600</td>
<td>312</td>
<td>18700</td>
<td>0.212</td>
</tr>
<tr>
<td>LOS 4</td>
<td>665 topping mesh</td>
<td>52800</td>
<td>528</td>
<td>2220</td>
<td>0.080</td>
</tr>
</tbody>
</table>

From the last column of Table 3.13, it is evident that the average work efficiency of the 665 topping mesh mechanism was one third that of the embedded D10 hairpins.
4

Loss Of Support Tests Involving Cyclic Loading

4.1 GENERAL

It has been said: “Engineering is an exact science based on assumptions”. In the context of seismic design, perhaps the most important concern is that a given set of details will not compromise fundamental design assumptions when subjected to severe cyclic loading. Hence, to effect good designs, the engineers of (especially) seismic resisting structures must repeatedly and systematically defer “exact science” and concentrate on managing the assumptions.

Considering fundamental design assumptions, a particular case is the behaviour of starter bar and embedded bar details subjected to pronounced cyclic loading. It was observed in the earlier tests (test LOS 2) that efficient bar bond resulted in large strains, and was apparent that cyclic loading would cause bar buckling. This observation may be projected into the topping slab, where dilation and cyclic loading could adversely affect the performance of starter and continuity bars. In view of poor topping bond performance, it was considered that the components of bar buckling might completely dislodge the composite topping slab. Such a result would directly affect assumptions regarding consistent load path networks in seismic resisting floor diaphragms.

Two tests were conducted to observe the relative effects of cyclic dilation loading on both topping bars and embedded bars. In the first test, topping bars were designated as 8-HD12 at 150 mm centres, which may typically appear as continuity reinforcement over the ends of long flooring units. A selection of embedded tie details were then tested in parallel cores to establish if bar buckling tendencies may be induced by tie bar placement and geometry.

4.2 TEST METHODOLOGY

4.2.1 GENERAL PROCEDURE

To remain consistent with earlier experiments, the same basic methodology of previous tests was adopted, but with provision for applying cyclic tension and compression forces. As such, the general emphasis of testing was to apply axial tension force across the support interface so to extract the flooring unit from its seating. At a point when the reinforcing bar strain had exceeded strain at strain hardening $e_{sh}$, compressive force was applied in order to close the
cracks caused by dilation loading and to compress reinforcement that crossed the principal crack.

4.2.2 DESCRIPTION OF TEST EQUIPMENT

4.2.2.1 General

Test equipment, including the three categories of test specimens, actuating system and logging componentry was essentially identical to that described in Chapter 3.

4.2.2.2 Precast Support Beams

A precast concrete support beam was constructed for each test (see Figs 3.1) as described in Section 3.2.2.2. To achieve cyclic loading capacity (tests LOS5 and LOS6), this detail was enhanced by adding two extra bolts to each sole plate and matching anchor plates on the opposite face of the precast support beam.

4.2.2.3 Horizontal Displacement

(a) Actuating System

Horizontal displacement was applied to the test specimens by two parallel acting hydraulic rams, each rated at 43 tonnes (see Figs 3.2). The hollow core flooring unit was secured onto the support carriages by two 310 UC 97 beams placed above and below the precast section, and bolted to it by eight 24 mm diameter threaded rods, as described in Section 3.2.2.3.

(b) Measurements

Principal horizontal displacements were measured by placing two 100 mm linear displacement potentiometers near each support carriage, connecting between target plates mounted on the hollow core unit and rigidly braced uprights bolted to the laboratory strong-floor. Potential uplift of the topping slab was monitored by two sets of potentiometers placed on isolated stands that were epoxied into the top flange hollow core unit (see Fig. 3.4). Further potentiometers were attached to the concrete support block to monitor movement of the test rig in relation to the floor. Force measurements were accorded by 44 tonne load cells positioned between the respective hydraulic rams and support carriages.

4.2.2.4 Reinforcing Bar Strain Gauges

Electrical resistance strain gauges were connected to principal reinforcement to establish bar strain characteristics. Ordinary 3% extension strain gauges were used. These were supplied by Tokyo Sokki Kenkyujo Co. gauge type FLA-5-11, with 120Ω resistance and 5.0 mm gauge length.
Surface preparations and the method of fixing electrical resistance strain gauges was carried out in accordance with the departmental guidelines for these procedures [Hill, 1992].

4.2.2.5 **Data Logger Unit**

The load cells, potentiometers and strain gauges were all connected to the Metabyte logger which converted voltage changes caused by linear displacement into digital values. These values were recorded against respective scan numbers that were manually taken throughout the tests. At the conclusion of a test, the logged information was converted to an ASCII file that was then imported into Excel (spreadsheet program) for subsequent editing and data extraction.

4.2.3 **MATERIALS AND CONSTRUCTION**

4.2.3.1 **General**

The emphasis of this experimental programme was to reflect the general performance of pretensioned floor construction. The construction methods and curing was of a standard that might be expected on a well-supervised construction site.

4.2.3.2 **Precast Pretensioned Hollow Core Units**

Hollow core units were of the same type, dimensions and quality as described in Chapter 3.

4.2.3.3 **Prestressing Strand**

Prestressing strand properties were identical to those described in Section 3.2.3.3.

4.2.3.4 **Cast-In-Place Topping Concrete**

Topping concrete was selected and cured as described in Section 3.2.3.4.

4.2.3.5 **Reinforcing Steel**

(a) **Starter and Tie Bars**

12 mm diameter Grade 430 (HD12) continuity bars and 12 mm and 16 mm diameter Grade 300 (R12 and R16) plain round tie bars complying with the appropriate New Zealand standard [Standards New Zealand, 1989] were supplied cut and bent from recognised reinforcing steel merchants. The actual tensile characteristics of continuity and tie reinforcements are deferred to individual LOS tests.

---

*Chapter 4: Cyclic LOS Tests: Test Methodology*
(b) **Welded Wire Fabric (Mesh)**

665 mesh (5.3 mm diameter) complying with the appropriate New Zealand standard [Standards New Zealand, 1975] was placed in the topping slab, but was not included in the principal crack region.

### 4.3 RESULTS OF EXPERIMENTS

#### 4.3.1 LOS 5

##### 4.3.1.1 General

The initial cyclic LOS test was directed at a typical continuity bar detail (Fig. 4.1). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit that was pretensioned with seven 12.7 mm diameter Supergrade strands. A total of eight 12 mm diameter Grade 430 (HD12) starter bars were placed at 150 mm centres, and extended the full length of the topping slab to avoid topping fracture at the curtailment. 665 mesh was placed over the hollow core unit and terminated before the end of the seating length (assumed to be the critical section).

![Fig. 4.1](image)

**Fig. 4.1** Support configuration of LOS 5 test that involved a typical continuity bar detail

##### 4.3.1.2 Instrumentation

(a) **Forces and Displacements**

Forces and displacements were measured in accordance with the methods described in Section 3.2.2 (also Fig. 3.4).
(b) Reinforcement

Standard 3% extension electrical resistance strain gauges (as described in Section 3.2.2) were employed on two individual items of starter reinforcement. These gauges were configured so that a gauge was situated directly over the expected cracking plane, at the end of the hollow core section. The strain gauges were set at 50 mm centres about the expected cracking zone (Fig. 4.2).

Fig. 4.2 Strain gauge positions on continuity bars used in the LOS 5 test

4.3.1.3 Cast-In-Place Topping Concrete

Concrete was ordered at a specified slump of 120 mm and accepted at a snatch sample slump of 150 mm (Table 4.1). The mix would be described as workable and generally good for topping concrete, although some very early bleed water was observed.

Table 4.1 Characteristics of cast-in-place topping concrete for test LOS 5

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 at 28 days</td>
<td>13</td>
<td>120</td>
<td>150</td>
<td>28 at 21 days</td>
</tr>
</tbody>
</table>

4.3.1.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) HD12 Continuity Bars

The characteristics of HD12 bars were identical to those described in Table 3.4
4.3.1.5 Results of Testing

(a) Initial Response

The initial response to horizontal loading showed a very stiff system (Fig. 4.3) with first cracking over the support region observed at around 150 kN and 0.07 mm displacement. The peak elastic response occurred at 0.27 mm horizontal displacement and a force of just over 300 kN. This was followed by a sudden increase in displacement due to tensile fracture of the concrete, resulting in 0.8 mm displacement at 320 kN of applied force. From this point onward, continuity bars in the topping slab governed the section tensile strength.

![Graph of horizontal force vs. horizontal displacement](image)

**Fig. 4.3** Force-displacement response of continuity bar detail under cyclic loading

(b) First Compression Cycle

The first compression cycle was initiated at 4.5 mm displacement. This displacement was selected due to the relatively uniform distribution of bar strains at gauge stations, apart from those under the critical section (i.e., N4 and S4). The distributions of bar strains at this point are shown in Figure 4.4(a), from where it can be seen that all gauge stations indicate yield strain had been exceeded ($\varepsilon_y = 0.22\%$).

The first compression cycle did not result in noticeable buckling, although it was apparent that a degree of topping bond loss and uplift had occurred during this phase (see Fig. 4.6).
(c) Second Compression Cycle

The second compression cycle was initiated at about twice the displacement of the first cycle (i.e., 9.4 mm). The distributions of bar strains at this point are shown in figure 4.4(b), where it can be seen that several gauges indicate strains well in excess of strain hardening (with $\varepsilon_{sh}$ taken as 1.35%). Similar to the first compression cycle, there was no noticeable tendency for the slab to buckle. However, the debonding and uplift that had been observed during the first cycle became more apparent.

Fig. 4.4(a) Strain in continuity bars at start of first compression cycle (also see Fig. 4.2)

Fig. 4.4(b) Strain in continuity bars at start of second compression cycle
Fig. 4.4(c) Strain in continuity bars at start of third (buckling) compression cycle

(d) Third (Buckling) Compression Cycle

The third compression cycle was commenced at approximately five times the initial compression cycle (i.e., 21.3 mm). The recovery of tension force involved a closing displacement of 7.0 mm, followed by stiffening of the response at the start of the compression run. At 12.7 mm displacement (a further closing of 1.6 mm) and compression force of 103 kN, the slab buckled and completely lifted from the precast unit. The compression run was terminated at 8.7 mm displacement and 167 kN. At this stage, the slab had lifted to a height of 11.5 mm at the position of the linear potentiometer (see Fig. 4.5) and approximately 30 mm at the highest point.

Fig. 4.5 Ratio of vertical uplift of buckled slab to horizontal displacement
It is clear from Figures 4.6 and 4.7 that the topping slab had separated from the precast unit well in advance of buckling, with 0.4 mm uplift recorded at the initiation of the first compression cycle and 0.7 mm at the initiation of the second.

Fig. 4.6  Vertical uplift of topping slab in relation to applied horizontal force

Fig. 4.7  Slab uplift that occurred before buckling, indicated at the initiation of each compression cycle

Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 5
4.3.1.6 Analysis of Test Results

As expected, the sum of topping slab crack openings approximately equalled the total measured test rig displacement at the end of each tension cycle. Consequently, the topping slab buckling that occurred under compression loading was instigated by bar buckling within the crack spaces. In this situation, the mechanism that initiates slab uplift is derived as the shear components $V$ of plastic bending moments that develop in the buckling zones of reinforcing bars (Fig. 4.8).

![Diagram](image)

**Fig. 4.8** Uplift components caused by formation of plastic hinges in a buckling bar

From this perspective, it is apparent that the cyclic amplitude and frequency of displacements, the size and strength of topping slab reinforcement and composite bond capacity will directly influence the potential for slab buckling.

Immediately prior to failure, the principal topping slab cracks were mostly concentrated within a distance of around 300 mm from the line of support. At failure, plastic hinges formed at two primary crack locations about 150 mm apart, allowing rotation and uplift of the topping slab. It was evident that the stiff portions of topping slab between these principal cracks would also influence the eventual buckling displacement pattern (Fig. 4.9).

![Diagram](image)

**Fig. 4.9** Displaced shape of buckled topping slab at end of third compression run

*Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 5*
Dimensionally, Detail F is identical to Detail A. However, Detail F was placed under the top bar of the support block, and hence, was provided with a layer of confining reinforcement. Consequently, this detail did not experience the bar buckling and spalling of adjacent topping concrete that were a feature with Detail A.

The results of testing show similar performance to detail E, with positive response provided near the critical section and a degree of staging in strain gauge response relative to embedded length due to strain penetration (Fig. 4.23).

![Detail F (R16) strain gauge recordings](image)

**Fig. 4.23** Strain gauge recordings for Detail F

### 4.4 DISCUSSION ON TEST RESULTS

#### 4.4.1 STARTER AND CONTINUITY BAR DETAIL (Test LOS 5)

It can be concluded from test LOS 5 that topping slab uplift may occur where flooring units are subjected to the combined effects of frame dilation and cyclic loading. It is certain that the characteristics and quantity of topping slab reinforcement will influence uplift tendencies, as will the tensile bond capacity exhibited between precast and composite concretes. It is evident that bond loss and separation had occurred between the precast and topping concrete surfaces well in advance of slab buckling. This is consistent with composite bond losses observed in the earlier LOS tests that involved similar support details subjected to monotonic loading.

The specimen tested may be considered to represent a lower bound, in that the bond surface of the precast hollow core unit exhibited a minimum of roughening and grouted edge keys were not incorporated. However, the topping slab uplift observed in this experiment was both severe
4.3.2 LOS 6

4.3.2.1 General

The intention of the second cyclic LOS test was to establish the extent to which bar geometry and placement might affect tie bar behaviour. The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit that was pretensioned with seven 12.7 mm diameter Supergrade strands. The top flanges of the six cores of the hollow core unit were removed and a plain bar tie detail was placed and grouted into each (Fig. 4.12). All the tie details involved plain round bars, however the diameters and configurations of each were varied so as to highlight variations in performance under uniformly applied cyclic displacements. 665 mesh was placed over the hollow core unit and terminated before the end of the seating length (assumed to be the critical section).

Fig. 4.12 Layout of embedded support tie details for test LOS 6

Performance variations arising from tie bar configuration was observed in earlier experiments, in which the geometry of a particular tie detail caused serious concrete fracture under tensile loading [Mejia-McMaster and Park, 1994]. From this, it was considered that particular details that performed well in monotonic tensile tests might cause deterioration in the support region when subjected to cyclic loads. Of particular interest were details involving inclined ties that would produce an upward force component when placed in compression.

The geometry, placement and strain gauge positions of the six tie variations are shown in Figure 4.13. Details A, D, E and F were single leg R16 ties. Details B and C were double leg R12 ties. Detail C differed from all other details in that it was anchored directly around the upper HD20
bar of the support beam. This was purposely done to examine the effects of anchoring ties directly to support member reinforcement, as may be required in some construction situations.

Fig 4.13 Tie detail configurations examined in test LOS 6
4.3.2.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 3.2.2 (also Fig. 3.4).

(b) Reinforcement

Standard 3% extension electrical resistance strain gauges (as described in Section 3.2.2) were employed on individual tie details (see Fig. 4.13). These gauges were configured so that a gauge was situated directly over the expected cracking plane, at the end of the hollow core section. The strain gauges were set at 100 mm centres about the expected cracking zone.

4.3.2.3 Cast-In-Place Topping Concrete

Concrete was ordered at a specified slump of 100 mm and accepted at a snatch sample slump of 85 mm (Table 4.2). The mix would be described as having sufficient workability and well suited for topping concrete, showing very little bleeding or segregation.

<table>
<thead>
<tr>
<th>Design Strength</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
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<tbody>
<tr>
<td>25 at 28 days</td>
<td>13</td>
<td>100</td>
<td>85</td>
<td>34 at 34 days</td>
</tr>
</tbody>
</table>

4.3.2.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) R16 Tie Reinforcement

The tensile characteristics of R16 bars used in test LOS 6 are shown in Table 4.3

Table 4.3 Characteristics of R16 reinforcing steel

<table>
<thead>
<tr>
<th>Avg. Yield Stress (MPa)</th>
<th>Avg. UTS (MPa)</th>
<th>Avg. Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>313</td>
<td>488</td>
<td>28.9</td>
</tr>
</tbody>
</table>

Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 6
(e) R12 Tie Reinforcement

The tensile characteristics of R12 bars used in test LOS 6 are shown in Table 4.4

Table 4.4 Characteristics of R12 reinforcing steel

<table>
<thead>
<tr>
<th>Avg. Yield Stress (MPa)</th>
<th>Avg. UTS (MPa)</th>
<th>Avg. Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>343</td>
<td>486</td>
<td>27.7</td>
</tr>
</tbody>
</table>

4.3.2.5 Results of Testing

(a) Initial Response

The initial response to horizontal loading showed a stiff system (Fig. 4.14) with first cracking over the support region observed at around 200 kN and 0.1 mm displacement. The peak elastic response occurred at 0.25 mm horizontal displacement and a force of just over 280 kN. This was followed by a sudden increase in displacement due to tensile fracture of the concrete, resulting in 0.97 mm displacement at 247 kN of applied force. From this point onward, tie bars in the hollow core voids governed the section tensile strength.

![Graph showing combined force-displacement response of tie bar details under cyclic loading](image)

Fig. 4.14 Combined force-displacement response of tie bar details under cyclic loading
(b) Compression Cycles

The first compression cycle was initiated at 3.7 mm displacement. Successive compression cycles were initiated at increments of around 4.0 mm (see Table 4.5), with a total of 12 load reversals being applied during the experiment.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>27.1</td>
<td>31.2</td>
<td>35.2</td>
<td>39.1</td>
<td>43.0</td>
<td>46.5</td>
</tr>
</tbody>
</table>

4.3.2.6 Specific Analysis

(a) General

For each of the six details tested, reinforcing bar strains were recorded from the strain gauge positions illustrated in Figure 4.13. The strains recorded from test LOS 6 show considerable variation, with an interesting general tendency for bar strains to dwell in compression after the application of a number of load reversals. Because of the complexity of loading on individual bars, it is difficult to establish the significance of a predominance of compression strain readings at bar positions away from the critical section. However, for the exercise of comparing the performance of various tie details, it is considered sufficient to draw conclusions based on relative behaviours, such as load response at a particular gauge position of a detail towards the latter stages of the test.

Understandably, a number of strain gauges failed at various stages during the test. Strain gauge failure is indicated by a simple termination of the plot line on the graph of strain versus scan number.

(b) Detail A (Fig. 4.13)

Detail A was the only detail that caused rupture of the topping slab during a compression run (Fig. 4.15). However, this did not occur until the eleventh load cycle, and after a closing displacement of 21 mm. It was fully expected that Detail A could cause such a topping slab rupture due to the diameter and inclination of the tie leg, coupled with a lack of effective confinement.

The topping rupture was accompanied by a localised buckling failure of the tie bar. Bar buckling was predestined to occur at the bend between horizontal and inclined portions, which formed a shape conducive to buckling. Hence, bar buckling was concentrated in the immediate vicinity of this bend.

Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 6
It was apparent from the following load cycle that concrete spalling was unlikely to progress much further, largely due to the facilitated nature of tie bar buckling.

![Image of a load cell with readings](image)

**Fig. 4.15** Spalled topping slab concrete caused by buckling of Detail A

Referring to Figure 4.16, it is evident from gauges #3 and #4 that a degree of strain penetration had occurred. The relative amplitude of gauge #2 suggests that the majority of strain penetration had been experienced at around scan number 139, half way through the test. The loss of cyclic response amplitude recorded by gauge #3 at scan 199 coincides with the onset of bar buckling, which suggests a rapid softening of the detail due to formation of the buckling mechanism.

![Graph of strain gauge recordings](image)

**Fig. 4.16** Strain gauge recordings for Detail A

*Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 6*
(c) Detail B (Fig. 4.13)

Detail B was one of two R12 double leg ties (paperclip ties) tested. With this particular detail, bar anchorage was achieved by providing 200 mm of embedment into the support block.

The results (Fig. 4.17) indicate an active response to loading along the extent of the upper tie leg during the early stages of testing. The increasing response at gauges away from the critical section suggests that an appreciable amount of strain penetration occurred. The gradual fall in response amplitude at gauge stations #2 and #3 over the latter two thirds testing most likely indicates bond degradation between the strain gauge and bar elements. It is considered that the gradual dampening of response at gauge #4 was due to the onset of bar buckling at the critical section, which was clearly observed during the latter stages of the test.

![Graph of Detail B (R12)](image)

**Fig. 4.18** Strain gauge recordings for Detail B

(d) Detail C (Fig. 4.13)

The second R12 paperclip detail was anchored around an upper HD20 bar of the support block, which was nominally confined by R12 stirrups at 150 mm centres (see Figs 3.1). An additional strain gauge was placed on the bottom tie leg (denoted as gauge #1B) to compare response across the section depth.

The results of testing showed considerable deformation of the HD20 anchorage bar (Fig. 4.19), and is reflected in Figure 4.20 by the generally diminished tie response after about scan number 65. As the test progressed, subsequent concrete spalling and deformation of the anchorage bar allowed an increasing amount of free tie movement about the anchorage point. This is indicated by the lag effect shown at the latter stages of testing, in particular, the reducing frequency of cyclic straining from around scan number 161 onwards. Another feature of this detail that

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*Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 6*
became evident toward the latter stages of testing is the sharp increase in response magnitude. This was most likely caused by the positive form of restraint afforded by an anchored connection at a stage when the stiffness of adjacent details had diminished due to the breakdown of bar bond mechanisms.

![Image](image_url)

**Fig. 4.19**  Deformation (end of test) of HD20 bar acting as tie anchorage

![Graph](image_url)

**Fig. 4.20**  Strain gauge recordings for Detail C

(e) **Detail D (Fig. 4.13)**

Although of short duration, the results from Detail D provide clear evidence of bar bond degradation and associated strain penetration effects that most specifically apply to plain round reinforcement (Fig. 4.21). The staged response increments at strain gauge stations occur in direct relation to embedded distances from the critical section, suggesting a systematic breakdown in bar bond under cyclic loading.
Fig. 4.21 Strain gauge recordings for Detail D

(f) Detail E (Fig. 4.13)

Detail E represented a horizontal tie placed centrally in the hollow core void, and is similar to a detail examined in earlier experiments [Mejia-McMaster and Park, 1994]. The prolonged response of gauge #1 (Fig. 4.22) was very beneficial in that it indicates a detail with sustained resistance to cyclic loading at large displacements. The increase in response magnitude towards the latter stages of testing further suggests that the detail retained a degree of axial stiffness in relation to the adjacent details.

Fig. 4.22 Strain gauge recordings for Detail E
(g) **Detail F (Fig. 4.13)**

Dimensionally, Detail F is identical to Detail A. However, Detail F was placed under the top bar of the support block, and hence, was provided with a layer of confining reinforcement. Consequently, this detail did not experience the bar buckling and spalling of adjacent topping concrete that were a feature with Detail A.

The results of testing show similar performance to detail E, with positive response provided near the critical section and a degree of staging in strain gauge response relative to embedded length due to strain penetration (Fig. 4.23).

![Detail F (R16)](image)

**Fig. 4.23** Strain gauge recordings for Detail F

### 4.4 DISCUSSION ON TEST RESULTS

#### 4.4.1 STARTER AND CONTINUITY BAR DETAIL (Test LOS 5)

It can be concluded from test LOS 5 that topping slab uplift may occur where flooring units are subjected to the combined effects of frame dilation and cyclic loading. It is certain that the characteristics and quantity of topping slab reinforcement will influence uplift tendencies, as will the tensile bond capacity exhibited between precast and composite concretes. It is evident that bond loss and separation had occurred between the precast and topping concrete surfaces well in advance of slab buckling. This is consistent with composite bond losses observed in the earlier LOS tests that involved similar support details subjected to monotonic loading.

The specimen tested may be considered to represent a lower bound, in that the bond surface of the precast hollow core unit exhibited a minimum of roughening and grouted edge keys were not incorporated. However, the topping slab uplift observed in this experiment was both severe

*Chapter 4: Cyclic LOS Tests: Results of Experiments: Test LOS 6*
and immediate, suggesting that a positive means of uplift control is more necessary than preferential.

Whether uplift control can be achieved through roughening amplitude or grouted keys is debatable. In the author's opinion, tensile control achieved through the roughening amplitude of extruded concrete surfaces will be very limited. However, uplift restraint provided by edge key details will largely depend on maintaining construction joint integrity. It must be considered that edge keys usually represent only a 40 mm wide strip of plain concrete at 1.2 m centres, and are probably subject to physical degradation under two-way frame actions during a severe earthquake.

It has been observed (Tests LOS 2 and LOS 6) that the provision of cut-back hollow core flanges and grouted voids will greatly increase topping bond capacity in the support region. However, if the recommended detailing is not practicable, it is considered that either vertical ties or the inclusion of steel fibre reinforcement in the topping slab could provide some integrity across the horizontal plane of the edge key joints.

4.4.2 EMBEDDED TIE BAR DETAILS (Test LOS 6)

4.4.2.1 General

The main emphasis of test LOS 6 was to determine the comparative performance of plain bar details embedded in grouted hollow core voids. At the outset, it was considered that any of the details tested would be generally suitable for applications involving potential loss of support.

4.4.2.2 Detail A

The topping concrete spalling associated with Detail A was fully anticipated, although, its occurrence was of less significance than originally presumed. Essentially, the composite bond capacity afforded by removal of hollow core flanges appeared sufficient to control the progress of spalling that results from bar buckling. Also, the ready formation of a plastic hinge at the bar buckling node deferred axial forces by facilitating movement of the flooring unit. This mitigated the adverse effects of axial thrusts that may have acted to strip the bar from the topping slab, directly over the support beam.

From a different perspective, the bar at the buckling node would be subjected to considerable mechanical working under prolonged cyclic loading. Hence, the toughness properties of reinforcement may need to be taken into account if not to eventually affect the performance of this detail.
4.4.2.3 Detail B

Detail B provided no surprises and indicated that R12 paperclip details can afford sufficient ductility for loss of support applications. However, this detail was observed to buckle within the space of the critical crack opening. Therefore as expected, it appears that 12 mm diameter (and smaller) bar details might be prone to buckling at lower cyclic amplitudes.

4.4.2.4 Detail C

In order to gauge the effects of anchorage, Detail C (a shortened paperclip detail) was connected directly to the upper HD20 bar of the support block. Although the tie detail performed well, it is evident that considerable deformation of the anchorage bar is likely to result, especially where effective confinement is not available. Hence, Detail C may be unsuitable for applications that involve cyclic actions of the support member since induced bar deformations could promote the premature buckling of principal reinforcement.

4.4.2.5 Detail D

Detail D provided satisfactory performance, and gave clear indication of the effect of strain penetration that is required to achieve tie bar ductility.

4.4.2.6 Detail E

From the measured response and observations, Detail E provided perhaps the most suitable performance as a means of providing a ductile support tie. The detail, itself, is horizontal and embedded centrally in the hollow core void. Hence, it would seem well configured to sustain the type of loading regime placed on ductile support ties. It is evident that Detail E developed the required strain penetration, yet maintained stiffness toward the latter stages of testing. As such, bar buckling was not observed with this detail.

4.4.2.7 Detail F

Detail F is geometrically identical to Detail A, however, it was strategically placed below the top layer of reinforcement in the support block. It is evident that the top reinforcing layer to control bar buckling and spalling tendencies provided sufficient confinement. It is also considered that Detail F behaved in a similar manner to Detail E, but with a slightly more erratic response.
The Buckling of Topping Slabs

5.1 GENERAL

One of the most compelling observations from experiments involving precast flooring support is the effect of reinforcing details on composite topping behaviour. The tests that were reported in Chapter 3 indicate that diminishing composite bond strengths result when support details are displaced beyond their elastic limits. As reported in Chapter 4, a typical continuity bar detail subjected to inelastic cyclic displacements experienced topping slab uplift via the buckling components of continuity reinforcement.

The primary consideration is that reinforcing bars are frequently placed in composite topping slabs and may play an important part in the overall design strategy for seismic resisting diaphragms. For instance, it is not uncommon in routine designs for appreciable volumes of small to medium sized bars (e.g., 10 mm to 20 mm diameter) to be placed in continuous bands within the topping slab to act as diaphragm chord ties (see section 2.3.2). Furthermore, trimmer bars are required in the vicinity of floor openings and minor drag bar configurations may be treated in the much the same manner as chord tie reinforcement.

Clearly, the greatest risk of topping slab buckling in seismic resisting diaphragms will occur in regions where concentrated forces arise. The in-plane forces that may be derived from plastic hinge development in beams and arch actions in diaphragms have been discussed in Section 2.3. There is a general requirement to examine of the influence of in-plane forces on the incidence of topping slab uplift.

Buckling analysis usually implies the development of effective length criteria based on classical Euler buckling theory. However, it is widely acknowledged [Young, 1989] that plate buckling stresses calculated by this method tend to over-estimate the buckling capacity of actual members. This discrepancy is further complicated with concrete as a base material, since the calculated slab buckling stresses can far exceed practical concrete crushing strengths.

The Euler buckling theory is founded on elastic response to concentric axial load and the premise that an orchestration of sine curves will develop. It is certainly an elegant solution, but is essentially defined by only one parameter: the assumption of a concordant elastic deflection profile. Hence, for applications involving plastically deformed concrete members it is difficult to fully reconcile the Euler buckling criterion with the actual mechanics, and it therefore requires fairly broad modifications for use in practical concrete design [Park and Paulay, 1975].

Chapter 5: Topping Slab Buckling: General
The most difficult aspect to resolve in the Euler buckling model is the profoundly elastic assumption of section flexural rigidity, EI. Although this notation has been strongly adhered to, it really has no rational basis for use in plastic member analyses. However, it is a convenient bench-mark from which to apply mostly empirical modifications to section properties.

In Section 2.2.1.3, an alternative method was presented for the rational estimation of flexural rigidity. Although the author is confident that the equivalent flexural rigidity (EI*) equation (Equation 2.40) has direct applications to concrete section analysis, its simple incorporation into the Euler equation for plastic buckling analysis would only concede to a single parameter model based on deflected shape. Although feasible, such an approach would be still depend on a method (probably empirical) for statistically weighting the often greatly varied effective flexural rigidity over the critical length.

As such, the equivalent flexural rigidity equation would appear more useful as a means for verifying the flexural rigidities used in existing formulae. However, it is considered that EI* could be employed in the Euler buckling approach to estimate the critical buckling stress of nominally elastic slender elements, and especially elements subjected to uniform bending moments (e.g., top-loaded tilt panel walls). A useful aspect of the EI* method is that it will indicate a decrease in flexural rigidity below EIg in both the homogeneous and cracked states when (i) an amount of bending curvature is present and (ii) materials depart from linear stress-strain behaviour.

Alternative approaches to buckling are available, in which rational treatments of the base materials and structural element types are usually incorporated. One important method for the study of concrete wall stability has been developed and tested at the University of Canterbury [Goodsr, 1985]. This is a well-considered approach and it contains many aspects that are equally applicable to the present study, including plastic bending capacity and the effects of reinforcement.

However, it remains problematic in that the presumed curvature distribution of the Goodsr model is not directly applicable to the topping slabs in question. The deterministic requirement of a topping slab buckling analysis is the maximum permissible distance between two points of lateral restraint. A topping slab buckling failure will involve a mechanism of discrete plastic hinges (up to three hinges) of relatively short length, interspersed with comparatively long and unaffected portions of slab. Hence, the uniform radius of curvature over buckling length adopted by Goodsr may adequately define a buckling hinge length, but cannot be used to express the above multiple hinge mechanism. As such, the model best describes a localised buckling mode (generally) as opposed to a member buckling mode. The plastic hinge lengths of ductile structural walls, for which Goodsr's model was derived, are certainly long enough to accommodate the uniform curvature assumption. As a consequence, several design parameters based on this model have been incorporated into Chapter 12 of the New Zealand concrete structures design standard [Standards New Zealand, 1995].
5.2 DEVELOPMENT OF A SLAB BUCKLING MODEL

5.2.1 BUCKLING CRITERIA

5.2.1.1 General

As discussed above, there will be discrepancies between an assumed elastic deflection profile and the true deflection behaviour of any axially loaded concrete slab at buckling. Under the most favourable of conditions, the deflected profiles of reinforced concrete sections are mostly treated as estimates. Conversely, experience has shown that the equilibrium mechanics of concrete sections are very influential and have perhaps the greatest reliability in calculations. This important difference becomes even more distinct when the section is approaching its ultimate limit state.

A deflected shape parameter is fundamental in the development of a buckling model, and must describe a profile that is compatible with the behaviour of plastic failure mechanisms. From this perspective, it is considered that the deformation formats used in established methods of plastic analysis are equally applicable. However, integrated member stability also requires a parameter that can define the incipient point of buckling as a conclusive point of transition. Hence, this important additional parameter should function similarly in manner to the harmonic shape assumptions of the Euler model, yet be rigorously applicable to plasticity models.

5.2.1.2 Conservation of Energy

Energy methods are universal in their application to structures, being applicable in both elastic and plastic analysis. Consequently, these methods prove useful for concrete section analysis because they allow a unified approach when dealing with a material that may respond in both the elastic and plastic states.

It is considered that an element of structure can be regarded as either an isolated system, or at least a system that can be isolated in some manner. Since practical structural mechanics does not involve defined conditions of energy flow (as in the thermodynamic sense), it is reasonable to assume that no quantifiable amounts of energy will diffuse across boundaries of adjoining structural elements. This greatly simplifies the application of energy methods and the underlying conservation of energy principle. This principle may be stated as “the constancy of energy of an isolated system” in which “energy may be converted from one form to another, but is not created or destroyed” [Walker, 1995].

The conservation of energy principle would seem equally applicable to compression members at the incipient point of buckling. In a slender member (especially), the phenomenon of buckling implies a rapid transition from a compression-only state to a bending-only state. In real member collapse mechanisms, the compression-only state may involve linear deformations that are either elastic or elastic-plastic, whereas the bending-only state will primarily involve plasticity.
Hence, it is reasonable logic that for a rapidly buckling mechanism to substantially occur and remain consistent with the laws of mechanics, the existing quantity of stored strain energy "may be converted from one form (of energy) to another, but is not created or destroyed". Thus, the strain energy of force times linear displacement must either be converted into the strain energy of plastic bending moment times rotation, or be substantially transferred to some other energy form. Now, as buckling progresses toward collapse, the associated reduction in axial force potential is directly converted into increased bending rotation, since this is the sole resistance mechanism. Therefore, the assumed mode of conversion will feature strongly throughout the collapse process and must be regarded as the fundamental mode of energy conversion. On this basis, it is proposed that buckling is imminent at a stage when the stored quantity of strain energy due to axial compression $U_c$ becomes equal to the sum of strain energies $U_b$ required to form a collapse mechanism containing a number of plastic hinges. Hence, written in terms of a simple energy balance, this may be expressed as:

$$U_c = \sum U_b$$

(5.1)

Because concrete members will usually exhibit plastic material response prior to buckling, the total strain energy must be related to individual stress-strain curves of concrete and steel reinforcement (see Figure 2.11):

$$U_c = \iint_{\text{vol}} f_c \, ds \, dV + \iint_{\text{vol}} f_s \, ds \, dV$$

(5.2)

The evaluation of bending moment plastic hinges is potentially more difficult than summations of strain energy under simple axial compression. Section geometry, reinforcement and loading configurations will certainly influence the nature and timing of hinge formation. One of the key issues is the point in the moment-curvature relationship at which plasticity accelerates, since this will also define the neutral axis depth applicable to section equilibrium in a buckling model. The differences that may occur in transition rates between elastic and plastic behaviour are illustrated by the moment-curvature relationships of Figures 5.1 and 5.2. In both cases, members are effectively subjected to axial compression via prestress forces.

Limiting the argument to topping slabs, it can be shown (Fig. 5.4) that even under substantial axial force a reasonably rapid transition will occur between elastic and plastic behaviour. Also as discussed, equivalent flexural rigidity ($EI^*$) analysis will indicate a reduction in flexural rigidity under the dual conditions of curvature and departure from linear stress-strain response. Hence, for sections subjected to high axial compressive stress, the introduction of a small amount of bending curvature can sharply reduce the calculated $EI^*$ value.

A final consideration is the enhanced bending moment capacity of sections resisting axial compression. The increase in moment capacity over a member subjected to bending-only can be quite substantial. Figure 5.1 indicates that accurate estimates can be made of moment-curvature response by using established analysis techniques and realistic material properties. Since non-closure of cracks under cyclic actions will influence slab buckling, a conservative first estimate in design would be to adopt the bending-only capacity of the section (i.e., zero axial force).

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Fig. 5.1  Moment–curvature response of prestressed beam with slow transition between elastic and plastic behaviour [Collins and Mitchell, 1987]

Fig. 5.2  Moment–curvature response of prestressed beam with rapid transition between elastic and plastic behaviour [Collins and Mitchell, 1987]
**Fig. 5.3**  Typical topping slab configuration

**Fig. 5.4**  Moment-curvature response of typical topping slab under varying levels of axial stress. The irregularity highlighted in the plot for 0.3f'_c axial compression (and edited from the other plots for clarity) occurs at the transition between uncracked and cracked section response. This is a common result for lightly reinforced members, indicating an accelerated plastic response due to sudden increase in curvature at a given bending moment.

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The basic concept of column buckling is one of a slender prismatic member subjected to increasing axial force, until such a level of force is reached that sudden buckling occurs and collapse follows. With very slender members, where the Euler buckling criteria is most appropriate, a sudden (or snap-through) collapse pattern is generally expected. With members of decreasing slenderness, it is expected that an increasing degree of measurable axial displacement will occur before the onset of buckling. However, the members of broadest concern are usually neither very squat nor very slender, and it is reasonable premise that these members will exhibit an average-to-rapid loss of axial load resistance. Hence, it is considered that the relatively slender and lightly reinforced concrete topping slabs will exhibit such behaviour. The observations of test LOS 5 (reported in Chapter 3) and accelerated curvature response (see Fig. 5.4) are testimonial to this assumption.

5.2.1.3 Plastic Buckling Mechanism

The assumed shape at the onset of buckling is determined by the development of a plastic hinge mechanism. This developed mechanism will also be influenced by the conditions of restraint, whether end conditions are considered as pinned or fixed. The three conditions that may apply to a topping slab, and their associated plastic hinge mechanisms at buckling, are shown in Figure 5.5. In the slab buckling model, it is considered that the effects of cyclic loading will have caused sufficient loss in composite bond to nullify lateral bond restraint. This assumption is also supported by the test observations reported in Chapter 3.

Fig. 5.5  Slab buckling mechanisms assumed in model

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The fixed-fixed \((n_f = 2)\) condition in Figure 5.5 is most relevant to practical situations that involve composite topping slabs.

In terms of plastic hinges and associated rotations, the sum of internal energy in the plastic hinge mechanism may be written as:

\[
\sum U_b = \sum M_p \theta
\]  
(5.3)

For uniform moment capacities and with \(n_f\) equalling the number of fixed end conditions applicable to a length of slab that may buckle, combining into Equation 5.1 gives:

\[
U_e = (n_f + 2) M_p \theta
\]  
(5.4)

With the reasonable assumption that a plastic hinge will form near the mid point of the buckling length, the mid-span displacement at collapse may be taken simply as moment \(M_p\) divided by design axial force \(N^*\) (similarly as assumed in the Euler model). Hence:

\[
\frac{M_p}{N^*} = \frac{\theta L_{cr}}{2} \quad \text{or} \quad \theta = \frac{2M_p}{N^* L_{cr}}
\]  
(5.5)

Substituting this expression for \(\theta\) into Equation 5.4 gives:

\[
U_e = 2 (n_f + 2) \frac{M_p^2}{N^* L_{cr}}
\]  
(5.6)

Reverting to Equation 5.2, the strain energy due to axial deformation when expressed as the summation of concrete and steel components of a prismatic and uniformly reinforced topping slab gives:

\[
U_e = (u_c A_c + u_s A_s) L_{cr}
\]  
(5.7)

where \(u_c\) and \(u_s\) are the individual strain energy densities of concrete and steel.

Assuming the characteristic stress-strain response of unconfined concrete, the strain energy density of concrete may be written as:

\[
u_c = f'_c \int_0^{\varepsilon_c} \frac{2 \varepsilon_s}{\varepsilon_o} - \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^2 d \varepsilon
\]  
(5.8)

which gives:

\[
u_c = \frac{f'_c \varepsilon_c^2}{\varepsilon_o} \left( 1 - \frac{\varepsilon_c}{3\varepsilon_o} \right)
\]  
(5.9)
For prismatic concrete sections under axial compression, the strain corresponding to peak stress is often taken as \( \varepsilon_0 = 0.002 \). Hence, steel reinforcement with yield strength greater than 400 MPa is unlikely to reach the yield condition in compression. The use of higher grade steels such as Grade 430 reinforcement is increasingly common, and the yield strength of wire mesh will be at least 500 MPa.

For bonded reinforcement the compressive strain of steel will be equal to that of concrete, which gives the strain energy density of steel reinforcement as:

\[
U_s = \frac{f_s^2}{2E_s} = \frac{\varepsilon_s^2 E_s}{2}
\]  

(5.10)

Therefore, combining Equations 5.9 and 5.10 into Equation 5.7 gives:

\[
U_c = L_{cr} \varepsilon_c^2 \left[ f'_s A_e \left( 1 - \frac{\varepsilon_c}{3\varepsilon_0} \right) + \frac{E_s A_s}{2} \right]
\]  

(5.11)

and substituting Equation 5.11 into Equation 5.6 gives the general equation for critical buckling length \( L_{cr} \) as:

\[
L_{cr} = \frac{M_p}{N^*} \sqrt{\frac{2(n_f + 2)}{f' \varepsilon_e^2 \left( 1 - \frac{\varepsilon_c}{3\varepsilon_0} \right) + \frac{E_s A_s}{2}}}
\]  

(5.12)

In most topping slabs, the contribution to total strain energy density from steel reinforcement is insignificant in practical terms, and may be ignored. Also, concrete strain \( \varepsilon_c \) can be expressed as a function of concrete stress \( f_c \) as follows:

\[
\varepsilon_c = \varepsilon_0 \left( 1 - \sqrt{1 - \frac{f_c}{f'_c}} \right)
\]  

(5.13)

and Equation 5.12 may be re-written as:

\[
L_{cr} = \frac{M_p}{A_e} \sqrt{\frac{3(n_f + 2)}{\varepsilon_0 f_c f'_c \left[ 1 - \left( 1 + \frac{f_c}{2f'_c} \right) \sqrt{1 - \frac{f_c}{f'_c}} \right]}}
\]  

(5.14)

As discussed, it is customary to adopt a concrete strain at peak stress of \( \varepsilon_0 = 0.002 \), giving:

---

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\[ L_{cr} = \frac{M_p}{A_g} \sqrt{\frac{1500 \left( n_r + 2 \right)}{f_c f'_{c} \left[ 1 - \left( 1 + \frac{f_c}{2f'_{c}} \right) \left( 1 - \frac{f_c}{f'_{c}} \right) \right]}} \]  

(5.15)

### 5.2.1.4 Practical Failure Criteria for Singly Reinforced Slabs

In recognition that reinforcement may contribute only a very small proportion of the total bending moment capacity, a further modification is especially applicable to singly reinforced slab members. Specifically, the reinforcement of axially compressed and singly reinforced slabs subjected to increasing bending curvature is likely to undergo a transition between compressive and tensile stress due to shifting neutral axis depths. Hence, there will be a point during the stage of accelerating bending curvature when reinforcement will provide nil contribution to bending capacity.

The above observation can be exaggerated to the case of an unreinforced (plain) concrete slab. In this situation, moment capacity will fully depend on distance between the centroids of concrete compression and the applied axial force, which is assumed to act through the slab centreline. In a practical sense, it is clear that slab localities would be severely prone to buckling failure from the instant that flexural cracking moment was exceeded. Hence, on the understanding that reinforcement may perform an insignificant and variable role, and that there will be other detrimental effects from non-closure of cracks etc., it is considered appropriate to use the flexural cracking moment \( M_{cr} \) as moment capacity for plain concrete slabs:

\[ M_p = M_{cr} = \left( f_c + f_r \right) \frac{bh^2}{6} \]  

(5.16)

where:

\[ f_c = \frac{N^*}{A_g} \]  

(5.17)

\[ f_r = 0.8\sqrt{f'_{c}} \]  

(5.18)

Furthermore, as the level of axial stress approaches the design crushing strength of concrete, the combined effects of compression and bending should not exceed the peak concrete strength value. The crushing strength of concrete in columns is usually taken as 85% of the test cylinder strength; the deviation being attributed to relative differences in the size and shape of columns, and segregation of fine aggregates at construction (which may be less critical for topping slabs) [Park and Paulay, 1975].

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Therefore, it is considered that the effective crushing strength of concrete for use in the slab buckling model should also be taken as $0.85f'_c$, since this figure reflects the results of crushing tests and provides a practical margin for the added modulus of rupture $f_r$ (see Equation 5.16). Substituting these values into Equation 5.15 yields the critical length equation for a slab member, where flexural actions are determined by the gross concrete section properties and flexural tensile strength:

$$L_{cr} = \frac{h}{6} \left( f_c + 0.8\sqrt{f'_c} \right) \sqrt{\frac{1500 \left( n_f + 2 \right)}{0.85f'_c f_c \left[ 1 - \left( 1 + \frac{f_c}{1.7f'_c} \right) \sqrt{1 - \frac{f_e}{0.85f'_c}} \right]}}$$  \hspace{1cm} (5.19)

Alternatively, when moment capacity is derived from a more detailed section analysis:

$$L_{cr} = \frac{M_p}{A_f} \sqrt{\frac{1500 \left( n_f + 2 \right)}{0.85f'_c f_c \left[ 1 - \left( 1 + \frac{f_c}{1.7f'_c} \right) \sqrt{1 - \frac{f_e}{0.85f'_c}} \right]}}$$  \hspace{1cm} (5.20)

Based on a 65 mm thick slab ($r \approx 19$ mm) of 25 MPa concrete, the critical buckling lengths as described by Equation 5.19 are shown in Figures 5.6, 5.7 and 5.8. The three common cases of braced compression members have been considered, namely pinned ends (free-free: $n_f = 0$, $k_e = 1.0$), fixed one end and pinned the other (fixed-free: $n_f = 1$, $k_e = 0.85$) or fixed both ends (fixed-fixed: $n_f = 2$, $k_e = 0.7$). The stated $k_e$ values are effective length factors for the corresponding Euler buckling cases [Standards New Zealand, 1997]. For the calculation of Euler buckling strengths, column buckling stresses have been used, since plate buckling stresses according to Euler theory would exceed the concrete crushing strength by a factor of 14.

![Graph](image)

**Fig. 5.6** Critical buckling slenderness ratio at proportion of concrete crushing strength (limited to $0.85f'_c$) for a braced strut with pinned ends.

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Fig. 5.7  Critical buckling slenderness ratio at proportion of concrete crushing strength (limited to 0.85f'c) for a braced strut with one pinned end and one fixed end.

Fig. 5.8  Critical buckling slenderness ratio at proportion of concrete crushing strength (limited to 0.85f'c) for a braced strut with two fixed ends.

The effects of end fixity are shown in Figure 5.9. It is evident from Equation 5.19 that critical buckling lengths are proportional to the square root of the number of plastic hinges that must form to allow collapse, with n_h = 4 taken as maximum (i.e., n_f + 2). Furthermore, the general equation (Equation 5.12) may be simply adapted to the case of sway members, in that plastic hinges will only form at member ends. As such, the quantity of strain energy associated with $2M_p\theta$ at the column mid-height will not be required, and by re-defining $\theta$ (see Equation 5.5), the numerator term under the square root sign in Equation 5.12 may be written as:

---

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\[
\frac{2}{2} (n_f + 0) = n_f
\]  
(5.21)

Hence, for a sway member with two pinned ends \((n_f = 0)\), the collapse length is equal to zero (i.e., collapse is independent of buckling). Based on this observation, the effects of differing plastic hinge moment capacities may also be incorporated, since zero moment capacity may be interpreted as the least proportion of maximum moment capacity \(M_p\). Therefore, the end fixity term may be replaced by a general summation, where \(n_0\) is the number of plastic hinge rotations and the \(M_{\theta(i)}\) terms are corresponding hinge moment capacities. Since hinge rotations are relative to their separation lengths at buckling, the length division factors are \(\lambda = 1\) for sway members and \(\lambda = 2\) for braced members, giving:

\[
\lambda \frac{\sum_{n_0} M_{\theta(i)}}{M_p}
\]  
(5.22)

where:

\[
M_{\theta(i)} \leq M_p \quad \text{and} \quad 0 \leq n_0 \leq 4
\]  
(5.23)

Therefore, the general expression for member buckling (Equation 5.12) may be written:

\[
L_{cr} = \sqrt{\frac{\lambda M_p \sum_{n_0} M_{\theta(i)}}{N^2 \varepsilon_c^2 \left[ \frac{f'_c A_c}{\varepsilon_0} \left( 1 - \frac{\varepsilon_c}{3 \varepsilon_0} \right) + \frac{E_s A_s}{2} \right]}}
\]  
(5.24)

![Graph showing critical slenderness ratios at proportion of concrete crushing strength limited to 0.85\(f'_c\) for a 65mm slab with given end fixity and \(\lambda = 2\)]

**Fig. 5.9** Comparisons of critical slenderness ratios at proportion of concrete crushing strength (limited to 0.85\(f'_c\)) for a 65mm slab with given end fixity and \(\lambda = 2\)

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It is worth noting that for plastically deformed sway members, the length division factor used in Equation 5.24 may be taken greater than unity. This is because the characteristics of plastic hinges that develop in sway members are predisposed to the relative positions of bending inflection (see Section 2.2.1.4). Hence, it is conceivable that the effective length division factor may lie between $\lambda = 1$ for a complete column length and $\lambda = 2$ for two equal lengths between critical plastic hinges.

This and other plasticity effects are not considered further here, since the present buckling analysis is primarily directed at optimising the spacing of lateral restraints in topping slabs (i.e., braced member assumptions apply). However, the appropriate $\lambda$ factors are ultimately determined from deflections associated with plastic hinge rotations, which may be estimated for actual sway members by the methods developed in Chapter 2 of this thesis.

To best determine the critical buckling lengths of slabs, an envelope of slenderness ratios should be constructed to allow for the transition between first cracking moment capacity and the plastic moment capacity assumed at failure (i.e., with concrete strain $\varepsilon_c = 0.003$). As mentioned earlier, the plastic moment capacity of a slab will reach a peak value under axial compression which is somewhat less than $0.85f'_c$. This is illustrated in Figure 5.10, where unlike the combined stresses assumption of elastic design, peak moment capacity may fall markedly under increasing axial compression.

*Fig. 5.10* Comparisons of bending moment capacities at first cracking ($M_{cr}$ from elastic analysis) and $M_1$ at a limiting concrete strain of $\varepsilon_c = 0.003$, for a 65 mm thick singly reinforced topping slab under axial compression.

When plastic moment capacities $M_1$ from Figure 5.10 are applied in Equation 5.20, the resulting slenderness ratios also reflect increased buckling length at lower axial loads, and decreased critical length at high axial load in relation to the $M_{cr}$ envelope (Figure 5.11).
Fig. 5.11  Comparisons of critical buckling slenderness ratios at first cracking (\(M_{cr}\) from elastic analysis) and \(M_1\) at a limiting concrete strain of \(\varepsilon_C = 0.003\), for a 65 mm thick singly reinforced topping slab.

By limiting the critical buckling slenderness \(L/r\) to the least value envelope given by Figure 5.11, it is evident that the Euler buckling model would determine the strength of very slender elements (\(L/r \geq 360\)). For elements of intermediate slenderness (100 \(\leq L/r \leq 360\)), the strain energy model (SE method) assuming elastic bending capacity (i.e., \(M_p = M_{cr}\)) would provide the least estimate, and for stocky elements (\(L/r \leq 100\)) the SE method assuming plastic moment capacity would govern.

In practice, the application of slab buckling length equations is appropriate for regions where sizeable in-plane forces might occur in floor diaphragms. These regions and the probable nature and magnitude of forces have been discussed in Section 2.3.

It is generally accepted that in-plane nodal forces arising from seismic actions will be distributed into both the topping and precast concrete sections. However, it is usual in design practice to assume that the topping slab alone will resist in-plane forces. Although this assumption may appear overly conservative, there are instances where edge-to-edge contact cannot be assumed between precast flooring members, thus deferring the transfer of compressive forces \(\propto\) the monolithic topping slab. A primary example of this is in typical flat slab and double tee floor construction, where the edges between precast members are unlikely to provide a reliable contact surface for the transfer of strut forces. However, with flat slab and tee units, adequate surface roughening for composite bond is not difficult: to achieve and ties may be readily incorporated.

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The fundamental concern with topping uplift and buckling lies with extruded hollow core flooring, since the early breakdown of topping bond in laboratory tests was observed for extruded hollow core units without special surface roughening.

With regard to in-plane strutting forces, extruded hollow core flooring has the ready-made advantage of grouted shear keys that can facilitate the transfer of compression forces between adjacent members. Therefore, the situations that may require closer attention occur when (i) forces result from flexure as discussed in Section 2.3.1.2 and (ii) compression forces are generated under reverse seismic actions via chord tie reinforcement placed in the topping slab (see Section 2.3.2.2).

(a) Resistance to Flexure

Reverting to the example given in Section 2.3.1.2, a rational assessment can be made regarding the required spacing of topping ties to control slab buckling. The orthogonal over-strength bending moments $M_{0x}$ and $M_{0y}$ arise from concurrent plastic hinge formation during a severe seismic event. The resultant coupling force $N_{oxy}$ has been derived from the vector sum of moments and the effective section depth $d$. It is assumed that topping bond has degraded (as observed in experiments) and that the contribution of the topping slab must be considered as a stand-alone mechanism. The effective width of slab that resists $N_{oxy}$ is taken as the diagonal distance between the plastic hinge lengths formed adjacent to the column faces (see Fig. 5.12).

![Diagram](image)

**Fig. 5.12** Effective width of slab and critical buckling length for topping slab resistance to concurrent beam flexure at columns

Because the only effective reinforcement in the potential buckling zone is topping mesh, the critical slenderness may be conveniently taken straight from Figure 5.11. The axial stress as a proportion of $f'_c$ is: $1000 \text{ kN} / (25 \text{ MPa} \times (1000 \times 65) \text{mm}^2) = 0.62$. Since this is greater than approximately $0.5f'_c$, the plastic moment capacity is applicable, giving a slenderness ratio of about 74. Hence, the critical buckling length is estimated as $74 \times 19 \text{ mm} = 1.4 \text{ m}$, and a line of ties placed at say $0.85 \times 1.4 = 1.2 \text{ m}$ from the compression face should be considered.
(b) Resistance to Chord Forces

It is common in routine design for appreciable volumes of chord tie reinforcement to be placed in the topping slab near the supports and transverse to the span of precast flooring units. During a severe earthquake this band of reinforcement acts as a chord tie, thus controlling diaphragm actions and preventing the progress (unzipping) of cracks into the diaphragm along the joints between precast units (Fig. 5.13).

![Fig. 5.13 Chord-tie reinforcement in the topping slabs of seismic resisting diaphragms](image)

For chord-tie reinforcement to take effect, it must develop forces that are close to yield strength. Therefore, and depending on selected ductility factors, it is likely that some yielding of this reinforcement will occur during a strong earthquake. Hence, under cyclic response, it is reasonable to assume that the force of yielded bars will need to be resisted in compression by a topping slab with reduced composite bond.

In these situations, the volume of reinforcement should be taken into account. For example, if a unit (one metre) width of 65 mm slab has 12-H16 bars, then the force exerted on the slab face under compression is \((12 \times 201) \text{mm}^2 \times 430 \text{ MPa} = 1040 \text{ kN}\). Therefore, the concrete compression may be taken as approximately \(1040 \text{ kN} / (65 \times 1000) \text{mm}^2 = 16 \text{ MPa}\). From Equation 5.13, this equates to an axial strain of \(\varepsilon_e = 800 \text{ microstrain}\) for 25 MPa concrete. Also, under 16 MPa compression and with reinforcement as above, the ideal moment capacity of the section at the limiting strain of \(\varepsilon_u = 0.003\) is 11.2 kNm. Referring to Equation 5.24, the estimated critical length \(L_{cr}\) based on the common assumption of \(\varepsilon_0 = 0.002\) is:

\[
L_{cr} = \frac{2 \times 11.2 \times 10^6 \times \sum 44.8 \times 10^6}{1.04 \times 10^6 \times (800 \times 10^{-6})^2 \times 25 \times 62590 \left(1 - \frac{800 \times 10^{-6}}{3 \times 0.002}\right) + 200 \times 10^3 \times 2410} = 1280 \text{ mm}
\]

Therefore, in hollow core construction, ductile topping ties should be considered at each shear key (i.e., placed at 1.2 m centres) for the above volume of yielded chord-tie reinforcement.

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5.3 BAR BUCKLING MODEL

5.3.1 BUCKLING CRITERIA

5.3.1.1 General

Based on rational material parameters, the buckling methodology developed in Section 5.2 for reinforced concrete members can likewise be applied to steel members. In particular, the response of reinforcing bars may be examined in a similar manner to slabs members in the preceding Section. The non-composite nature of typical steel members also allows for some simplifications in applying the developed principles.

5.3.1.2 Conservation of Energy

Proceeding as in Section 5.2.1.2, the sum of internal strain energy due to linear deformation is equated to the required strain energy in bending to allow a plastic collapse mechanism:

$$U_c = \sum_n U_p$$

(5.25)

Considering $n_0$ plastic hinge rotations (see Fig. 5.14), Equation 5.25 may be expanded to:

$$A_b L_{cr} \int_{\epsilon_o} \epsilon \, d\epsilon = n_0 M_p \theta$$

(5.26)

where the rotation $\theta$ is likewise defined in accordance with Equation 5.5.

The integral of the LHS of Equation 5.26 gives the strain energy density of steel $u_o$, and is taken as the area under the characteristic stress-strain curve of the material. For the plastic buckling of steel, the material stress-strain response must be considered beyond yield strength.

![Buckling model of steel bar](image)

Fig. 5.14 Buckling model of steel bar

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Chapter 5: Topping Slab Buckling: Bar Buckling Model
5.3.1.3 Equilibrium

The critical (equilibrium) displacement at buckling is taken as \( \delta_{cr} = \frac{M_p}{N^*} \), which gives \( \theta = \lambda \cdot M_p / N^* \cdot L_{cr} \). Therefore, the critical length may be written as:

\[
L_{cr} = \sqrt{\frac{M_p^2 \lambda n_0}{N^* A_b u_s}} \quad (5.27)
\]

For steels exhibiting a distinct yield plateau, strain energy density up to the point of strain hardening may be written:

\[
u_s = f_s \left( e_s - \frac{f_s}{2E_s} \right) \quad (5.28)
\]

Observing that \( N^* \) is the product of \( A_b \cdot f_y \), Equation 5.27 can therefore be simplified to:

\[
L_{cr} = \frac{M_p}{N^*} \sqrt{\frac{\lambda n_0}{e_s - \frac{f_s}{2E_s}}} \quad (5.29)
\]

A further simplification may be made when \( f_s \) is less than the yield strength \( f_y \) (i.e., elastic buckling):

\[
L_{cr} = \frac{M_s}{N^*} \sqrt{\frac{2E_s \lambda n_0}{f_s}} \quad (5.30)
\]

As mentioned, the ratio of \( M_p / N^* \) in Equation 5.30 is the critical displacement. This value may be derived for the particular situation in accordance with axial load levels, initial member displacement and section geometry. The general equation may therefore be written:

\[
L_{cr} = \delta_{cr} \sqrt{\frac{\lambda n_0}{e_s - \frac{f_s}{2E_s}}} \quad (5.31)
\]

Column buckling is imminent when the distance between the centroids of axial compression force and compression bending resistance has been exceeded at potential plastic hinge regions. Consequently, if both \( M_p \) and \( N^* \) in Equation 5.29 involve the material at yield strength (i.e., \( f_s = f_y \)) then the ratio will be that of plastic section modulus to cross section area, and therefore equal to the centroidal distance of the first moment of area about the neutral axis. Thus, for stocky members subject to plasticity, the critical displacement is:

\[
\delta_p = \bar{y} = \frac{\sum A \bar{y}}{A} \quad (5.32)
\]

Chapter 5: Topping Slab Buckling: Bar Buckling Model
For members subjected to elastic buckling, the critical displacement must also increase to reflect the (initially) elastic state of stress at the incipient point of buckling. Hence, the critical displacement value will vary between the respective centroids of plastic and elastic bending stress. As such, the radius of gyration may be taken as an intermediate value. Because the radius of gyration represents the area A pin-pointed at a distance r so that $A \cdot r^2 = I$, then the section modulus $I\gamma$ must equal $I/r$, since this is also the fibre distance. Therefore, at equal stress, $M_e / N^* = A \cdot r^2 / A \cdot r = r$, and for elastic members of intermediate slenderness:

$$\delta_{le} = r = \sqrt{\frac{I}{A}}$$

(5.33)

For slender members, the centroid of elastic bending stress is considered most appropriate. This is especially applicable to thin-walled hollow sections where the centre of bending resistance is concentrated near the section extreme fibre. Therefore for slender elastic members, the critical displacement may be taken as:

$$\delta_{le} = \bar{y} = \frac{M}{\sum \frac{c}{A}}$$

(5.34)

Verification of the above equations are made by direct comparison with buckling slenderness ratios calculated in accordance with Chapter 6 of the Steel Structures Standard [Standards New Zealand, 1997]. In each case, the member is “compact” and has a from factor $k_f$ of 1.0. For simplicity, the “compression member section constant” $\alpha_b$ has been taken as zero (0) in Table 6.3.3(2) of the standard. All members are assumed to be braced with pin-ends ($k_e = 1.0$). For the SE method and Euler buckling calculations, it is assumed that steel exhibits linear stress-strain response up to yield strength. The modulus of elasticity has been taken as 200 GPa.

For the SE method calculations, Equation 5.30 has been used with ratio $M_e / N^*$ taken as the relevant $\delta_e$ value given in Table 5.1 or 5.2. For braced pin-ended members, both $n_0$ and $\lambda$ are equal to 2.0 (i.e, two plastic hinge rotations at mid-height, giving two corresponding length divisions).

<table>
<thead>
<tr>
<th>Table 5.1</th>
<th>100 x 100 x 6 Square Hollow Section (Grade 350) (see Fig 5.15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (mm$^2$)</td>
<td>I (mm$^4$)</td>
</tr>
<tr>
<td>2132</td>
<td>3.03 e$^6$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.2</th>
<th>86.6 x 86.6 Solid Square Section (Grade 250) (see Fig. 5.16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (mm$^2$)</td>
<td>I (mm$^4$)</td>
</tr>
<tr>
<td>7500</td>
<td>4.688 e$^6$</td>
</tr>
</tbody>
</table>

Chapter 5: Topping Slab Buckling: Bar Buckling Model
Fig. 5.15  Buckling slenderness ratios of a 100 x 6 Box Section (Grade 350) calculated in accordance with (i) NZS3404: 1997 (ii) the SE method for plastic, intermediate and elastic centroids, and (iii) classic Euler buckling theory

Fig. 5.16  Buckling slenderness ratios of a 86.6 x 86.6 Solid Square Section (Grade 250) calculated in accordance with (i) NZS3404: 1997 (ii) the SE method for plastic, intermediate and elastic centroids, and (iii) classic Euler buckling theory
It should be noted that the curve described by NZS3404: 1997 is based on Euler buckling theory, but with typical modifications for member class effects, increasing stockiness at reduced slenderness and initial deflections, etc [see Trahair and Bradford, 1988].

Assuming slender elastic behaviour, the SE method and classic Euler theory may be related by a simple expression. The critical buckling length according to Euler theory for braced pin-end members is:

$$ L_{Ec} = r \sqrt{\frac{\pi^2 E_s}{f_s}} $$  \hspace{1cm} (5.35)

The corresponding critical length for braced pin-end slender elastic members by the SE method is:

$$ L_{cr} = \delta_{sc} \sqrt{\frac{8 E_s}{f_s}} $$  \hspace{1cm} (5.36)

Hence, the ratio between Euler buckling length and the SE method buckling length may be defined as a section constant:

$$ \frac{L_{Ec}}{L_{cr}} = \frac{r}{\delta_{sc}} \sqrt{\frac{\pi^2}{8}} $$  \hspace{1cm} (5.37)

In the preceding examples, the ratio described by Equation 5.37 is 1.02 for the 100 x 6 SHS and 0.97 for the 86.6 x 86.6 Solid Square Section.

However, despite the fact these theories may produce identical results, there are important conceptual differences between the two methods. Referring to the Equation 5.37, the radius of gyration $r$ commonly employed in the Euler buckling theory is for convenience, being derived from the ratio of I/A that underlies the Euler theory. The radius of gyration applied in statics actually has limited physical meaning because it violates the parallel axis theorem.

Conversely, the critical displacement $\delta_{cr}$ in the SE method has important physical meaning, being the distance to the centroid of bending resistance applicable under the given state of stress. Hence, the critical displacement is an equilibrium requirement that will vary between elastic and plastic bending, and may be derived accordingly.

### 5.3.2 ELASTIC-PLASTIC TRANSITION

#### 5.3.2.1 General

As demonstrated earlier, the underlying theory of the SE method does not limit the solution to tangent modulus theory or reduced modulus theory approximations for inelastic buckling. The underlying parameter is strain energy density, which may be rigorously derived for any stress-strain response. Hence, for typical mild steels that exhibit a distinct yield plateau (Lüders strain
region), the disappearance of an elastic modulus at yield ($E_s \to \text{zero}$) has no impact on the validity of the theory.

5.3.2.2 The Bauschinger Effect

In seismic resisting reinforced concrete research, the cyclic stress-strain response of reinforcing steel has been the subject of much study. In particular, cyclic response and the induced Bauschinger effect has been examined in some detail, notably by Kent and Park, Thompson and Park, Spurr and Paulay, Mander et al, Tjokrodimumjo and Fenwick and Restrepo-Posada et al [see Park and Paulay, 1975: Restrepo-Posada, 1993].

The Bauschinger effect is important in that a reduced stiffness is experienced, and non-linear response develops at a strain much lower than the yield strain (Fig. 5.17). It is recognised that the Bauschinger effect likewise influences the behaviour of reinforced concrete members subjected to load reversals beyond the elastic range.

![Stress-strain behaviour in the yield plateau region, showing Bauschinger effect under load reversal [Restrepo-Posada, 1993]](image)
Ultimately, the Bauschinger effect will reduce the buckling length of reinforcing bars where cover concrete has spalled under cyclic actions and effective confinement has not been provided. This may apply to regions of seismic resisting floor diaphragms where chord ties and drag bars constitute part of the force transfer system. In particular, these reinforcing details may coincide with (i.e., be connected to or lie adjacent to) the ductile structural elements where plasticity is expected to occur.

Taking Figure 5.16 as a typical example of softening response in the yield plateau range, a theoretical buckling envelope can be established for reinforcing bars in the transition between elastic and plastic response. The strain described by line c-g in Figure 5.16 corresponds with the point of zero stress and the rejoin point where yield stress is achieved. The magnitude of c-g is shown as approximately three times the monotonic loading yield strain, as indicated by lines o-a and o-e.

A simple but sufficiently accurate approximation is to treat the curves b-g and c-e as parabolic. As such, the parabolic stress equation normally assigned to unconfined concrete may be adapted to describe this curve and the associated strain energy density (see Equations 5.8, 5.9 and Figure 5.18). As discussed, the peak strain increment described by line c-g in Figure 5.16 is taken as three times the monotonic yield strain (i.e., $3\cdot f_y/E_o$). Therefore $\varepsilon_O = 3\cdot \varepsilon_y$.

\[
f_s = f_y \left[ \frac{2 \varepsilon_s}{\varepsilon_o} - \left( \frac{\varepsilon_s}{\varepsilon_o} \right)^2 \right]
\]

(5.38)

and:

\[
u_s = \frac{f_y \varepsilon_s^3}{6 \varepsilon_o} \left( 1 - \frac{\varepsilon_s}{3 \varepsilon_o} \right)
\]

(5.39)

Fig. 5.18  Bauschinger effect approximated by parabolic relationship for yield plateau range (Equation 5.38), in comparison to monotonic stress-strain response

**Chapter 5: Topping Slab Buckling: Bar Buckling Model**
Table 5.3  Properties of reinforcing H16 (Grade 500) reinforcing bar

<table>
<thead>
<tr>
<th>$A_b$</th>
<th>$f_y$</th>
<th>$\delta_p$</th>
<th>$E_s$</th>
<th>$\varepsilon_y$</th>
<th>$\varepsilon_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mm$^2$</td>
<td>MPa</td>
<td>(mm)</td>
<td>GPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>201</td>
<td>500</td>
<td>3.4</td>
<td>200</td>
<td>0.0025</td>
<td>0.0075</td>
</tr>
</tbody>
</table>

For calculation of critical buckling length, Equation 5.27 may be written:

$$L_{cr} = \frac{M_p}{A_b} \sqrt{\frac{\lambda n_0}{f_1 u_s}}$$  \hspace{1cm} (5.40)

where $M_p$ is equal to the product $S f_s$. Hence, for a round bar element, the ratio of $M_p/A_b$ in Equation 5.40 may be written:

$$M_p \over A_b = \delta_p f_s = \frac{2d_0 f_s}{3\pi}$$  \hspace{1cm} (5.41)

For the buckling model, the bar element is assumed to be braced and have fixed ends:

$$\lambda = 2 \quad \text{and} \quad n_0 = 4$$  \hspace{1cm} (5.42)

The buckling slenderness values (Fig. 5.19) are calculated from the figures in Table 5.4:

Table 5.4  Strain energy density under monotonic and cyclic loading between zero strain and $\varepsilon_0 = 3 \cdot \varepsilon_y$ for H16 (Grade 500) bar

<table>
<thead>
<tr>
<th>$\varepsilon_s$ (monotonic)</th>
<th>$f_s$ (monotonic)</th>
<th>$u_s$ (monotonic)</th>
<th>$\delta_p f_s$ (monotonic)</th>
<th>$f_s$ (cyclic)</th>
<th>$u_s$ (cyclic)</th>
<th>$\delta_p f_s$ (cyclic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MPa)</td>
<td>(N/mm$^2$)</td>
<td>(N/mm)</td>
<td>(MPa)</td>
<td>(N/mm$^2$)</td>
<td>(MPa)</td>
<td>(N/mm$^2$)</td>
</tr>
<tr>
<td>0.0005</td>
<td>100</td>
<td>0.025</td>
<td>340</td>
<td>64</td>
<td>0.016</td>
<td>218</td>
</tr>
<tr>
<td>0.0010</td>
<td>200</td>
<td>0.100</td>
<td>680</td>
<td>124</td>
<td>0.064</td>
<td>422</td>
</tr>
<tr>
<td>0.0015</td>
<td>300</td>
<td>0.225</td>
<td>1020</td>
<td>180</td>
<td>0.140</td>
<td>612</td>
</tr>
<tr>
<td>0.0020</td>
<td>400</td>
<td>0.400</td>
<td>1360</td>
<td>231</td>
<td>0.243</td>
<td>785</td>
</tr>
<tr>
<td>0.0025</td>
<td>500</td>
<td>0.625</td>
<td>1700</td>
<td>278</td>
<td>0.370</td>
<td>945</td>
</tr>
<tr>
<td>0.0035</td>
<td>500</td>
<td>1.125</td>
<td>1700</td>
<td>358</td>
<td>0.690</td>
<td>1217</td>
</tr>
<tr>
<td>0.0040</td>
<td>500</td>
<td>1.375</td>
<td>1700</td>
<td>391</td>
<td>0.877</td>
<td>1329</td>
</tr>
<tr>
<td>0.0045</td>
<td>500</td>
<td>1.625</td>
<td>1700</td>
<td>420</td>
<td>1.080</td>
<td>1428</td>
</tr>
<tr>
<td>0.0050</td>
<td>500</td>
<td>1.875</td>
<td>1700</td>
<td>444</td>
<td>1.296</td>
<td>1510</td>
</tr>
<tr>
<td>0.0055</td>
<td>500</td>
<td>2.125</td>
<td>1700</td>
<td>464</td>
<td>1.524</td>
<td>1578</td>
</tr>
<tr>
<td>0.0060</td>
<td>500</td>
<td>2.375</td>
<td>1700</td>
<td>480</td>
<td>1.760</td>
<td>1632</td>
</tr>
<tr>
<td>0.0065</td>
<td>500</td>
<td>2.625</td>
<td>1700</td>
<td>491</td>
<td>2.003</td>
<td>1669</td>
</tr>
<tr>
<td>0.0070</td>
<td>500</td>
<td>2.875</td>
<td>1700</td>
<td>498</td>
<td>2.250</td>
<td>1693</td>
</tr>
<tr>
<td>0.0075</td>
<td>500</td>
<td>3.125</td>
<td>1700</td>
<td>500</td>
<td>2.500</td>
<td>1700</td>
</tr>
</tbody>
</table>

Chapter 5: Topping Slab Buckling: Bar Buckling Model
Fig. 5.19  Critical buckling slenderness ratios $L/r$ for Grade 500 reinforcing bar under monotonic compression and cyclic compression incorporating the Bauschinger effect in the yield plateau range (see Fig. 5.18)

Although Table 5.4 specifically involves 16 mm bar, it will be noted that Figure 5.19 applies to all round bars exhibiting the stress-strain behaviour described by Figure 5.18, as the slenderness ratio is in linear proportion to the centroid distance $\delta_p$. Also, the use of traditional terminology $L/r$ is for comparison and convenience (since $r = d_b/4$). The plastic slenderness ratio $L/\delta_p$ (involving definable lengths) holds physical meaning within the model (Fig. 5.20).

Fig. 5.20  Critical length displacement ratios $L/\delta$ for Grade 500 reinforcing bar under monotonic compression and cyclic compression incorporating the Bauschinger effect in the yield plateau range (see Figs 5.18 and 5.19)

Chapter 5: Topping Slab Buckling: Bar Buckling Model
5.3.2.3 Practical Buckling

It is widely recognised that the theoretical effective length factor of $k_e = 0.5$ for braced fixed-end members is unrealistic. This is because a degree of rotation is inevitable, thus weakening the sinusoidal deflected profile assumptions that are essential to the Euler theory. In structural steel design, where comparatively slender members are common, the minimum effective length factor for idealised end conditions is generally taken as not less than $k_e = 0.7$ (Fig. 5.21).

<table>
<thead>
<tr>
<th>Buckled shape</th>
<th>Braced member</th>
<th>Sway member</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective length factor ($k_e$)</td>
<td>0.7</td>
<td>0.85</td>
</tr>
<tr>
<td>Symbols for end restraint conditions</td>
<td>Rotation fixed, translation fixed</td>
<td>Rotation free, translation fixed</td>
</tr>
</tbody>
</table>

Fig. 5.21 Effective length factors for members with given conditions of end restraint [Standards New Zealand, 1997]

A final observation is to compare the slenderness ratio for monotonic loading calculated by the SE method, with that calculated from Euler theory modified by a suitable end condition constant. The Euler buckling stress was given by Equation 5.35 for the case of braced pin-end members. Since the braced pin-end member is the most fundamental case of Euler buckling, the end condition constant is simply taken as $C = 1$ (and is therefore ignored in Equation 5.35).

Considering the end condition constant $C$ from a Mechanical Engineering perspective for the practical design of so-called Euler columns. It has been stated: “if liberal factors of safety are employed, and if the column load is accurately known, then a value of C not exceeding 1.2 for both ends fixed, or for one end rounded and one end fixed, is not unreasonable” [Shigley, 1986]. The theoretical and recommended $C$ values for practical design are shown in Table 5.5:

Table 5.5 End Condition Constants for Euler Columns [Shigley, 1986]

<table>
<thead>
<tr>
<th>Column end conditions</th>
<th>Theoretical value</th>
<th>Conservative value</th>
<th>Recommended value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-free</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>Rounded-rounded</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Fixed-rounded</td>
<td>2</td>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>Fixed-fixed</td>
<td>4</td>
<td>1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

* To be used only with liberal factors of safety when the column load is accurately known.

Chapter 5: Topping Slab Buckling: Bar Buckling Model
The constant C is applied to Equation 5.43:

$$\frac{L_{cr}}{r} = \sqrt{\frac{C \pi^2 E_s}{f_s}}$$  \hspace{1cm} (5.43)

The comparative slenderness ratios for Grade 500 steel bars calculated by the two methods (Equations 5.40 and 5.43 with C = 1.2) are shown in Figure 5.22 for bar stresses of 100, 200, 300, 400 and 500 MPa as shown in Table 5.4 for monotonic loading.

![Graph showing slenderness ratios for Grade 500 steel bars](image)

**Fig. 5.22** Elastic critical slenderness ratios for Grade 500 round steel bars, calculated as Euler columns (with C = 1.2), and as Herlihy columns (by the SE method)

Referring to Figure 5.19, it is evident that a critical slenderness of about 30 (7.5d₀) is estimated for Grade 500 steel bars subjected to a compression strain of 3\(\varepsilon_y\) (0.75%) under cyclic loading. At 0.95\(f_y\) (475 MPa) the critical slenderness is estimated as 40 (10d₀) for bars subjected to the Baushinger effect. The comparative buckling slenderness for bars under monotonic loading to 0.95\(f_y\) is estimated as 70 (17.5d₀).

Therefore, for bars incorporated into regions of seismic resisting diaphragms where plasticity effects are expected during a severe earthquake, effective ties should be provided at 10d₀ centres or less, depending on the anticipated level of plastic strain. It is apparent that the onset of strain hardening will increase both the quantities of \(f_s\) and \(u_s\) in Equation 5.40. This may effectively accelerate shortening of the critical buckling length, which is proportional to the dimensionless ratio:

$$\sqrt{\frac{f_s}{u_s}}$$  \hspace{1cm} (5.44)

Hence, in simplest form, Equation 5.40 may be written:

$$L_{cr} = \delta_p \sqrt{\frac{f_s \lambda n_0}{u_s}}$$  \hspace{1cm} (5.45)

*Chapter 5: Topping Slab Buckling: Bar Buckling Model*
A perfect-elastic perfect-plastic stress-strain behaviour has been used to calculate the buckling response of steel members as shown by Figures 5.15 and 5.16. Hence, the curves described by the SE method do not show a transition between the tangents of parabolic and horizontal lines. However, with higher-grade steels especially, an appreciable transition may occur between elastic and plastic behaviour under monotonic compression loading, similar in manner but less pronounced than described by the Bauschinger effect. Therefore, a transition of the type shown by Figure 5.19 for cyclically loaded bars would result for those steels, but initiating from a point on the monotonic loading curve.

As a consequence to the SE method buckling model, the following applies with regard to constructing a generalised design curve:

(i) Considering movement of the stress centroid between elastic and plastic bending resistance, the curve will vary between the critical displacements of slender elastic response ($\delta_{se}$) and plastic response ($\delta_p$) in direct proportion to the axial load ratio $N*/N_0$. Thus, the design critical displacement $\delta^*$ may be written:

$$\delta^* = \delta_{se} - \frac{f_x}{f_y} (\delta_{se} - \delta_p)$$  \hspace{1cm} (5.46)

and the design critical buckling length is defined as:

$$L_{cr} = \left( \delta_{se} - \frac{f_x}{f_y} (\delta_{se} - \delta_p) \right) \sqrt{\frac{f_y \lambda n_0}{u_s}}$$  \hspace{1cm} (5.47)

(ii) The transition between parabolic and horizontal linear curves will initiate at the point of departure from initial elastic modulus, and become collinear with the horizontal line when yield stress had been achieved.

The curve that results from Equation 5.47 (yet unmodified for member effects) describes a buckling envelope very similar in appearance to the curve described by NZS 3404:1997, but with less conservative estimates of squat member buckling strengths. This is illustrated by Figure 5.23, where critical buckling lengths have been calculated for a 100 x 6 SHS (Grade 450) in accordance with Equation 5.47 and with strain energy densities $u_s$ derived from Figure 5.24. The member is assumed to be braced with pin-ends ($k_e = 1$; both $n_0$ and $\lambda = 2$), and the section constant has been taken as $\alpha_b = 0$ in Table 6.6.3(2) of NZS 3404:1997.

Although hypothetical, the stress-strain relationship shown in Figure 5.24 is based on observed compression testing of steel bars. The relationship assumes a 0.15% proof stress, with departure from the initial elastic modulus commencing at $0.75f_y$ (337.5 MPa) and reaching yield stress at 0.375% strain. The strains corresponding to steel stress labels in figure 5.24 are 0.00169 (337.5), 0.0023 (414), 0.003 (443) and 0.00375 (450). From the above information, a sufficiently accurate estimate of strain energy density $u_s$ can be made by area measurement.

*Chapter 5: Topping Slab Buckling: Bar Buckling Model*
Fig. 5.23 Buckling slenderness ratios of a 100 x 6 Box Section (Grade 450) calculated in accordance with (i) NZS3404: 1997 and (ii) the SE method with allowance made for reducing critical displacement $\delta^*$ under increasing axial load and a reducing elastic modulus prior to yielding (see Fig. 5.24).

Fig. 5.24 Hypothetical stress-strain relationship, showing a reducing elastic modulus prior to yielding.
5.4 CONCLUSIONS

5.4.1 GENERAL

In this Chapter, a general buckling theory has been proposed to facilitate the plasticity and composite section effects of reinforced concrete compression members. It is considered that the theory provides a more rational approach to buckling analysis than the traditional Euler theory, since equilibrium requirements and true material response may be incorporated into the solution. It is not possible to provide this level of amenity via the Euler solution, since mostly empirical notions of section stiffness response must be adopted.

Perhaps most importantly, the SE method theory principally requires knowledge of moment-curvature relationships in potential plastic hinges regions, which can now be ascertained with some accuracy. The regions away from plastic hinge locations do not exert a major influence within the theory. As such, the SE method is most adept for the analysis of ductile reinforced concrete members, since detailed modifications to the flexural rigidity EI in Euler theory must involve the entire member length in a mixture of guesswork and effort.

Members under combined actions of bending moment and compression have not been examined much beyond the effects of bending capacity on buckling length. However, it is considered that the underlying principle of energy balance must still apply, where the sum of strain energies due to compression $U_c$ and bending $U_b$ must equal the strain energy sum of a plastic collapse mechanism. Hence, Equation 5.1 may be more generally written as:

$$U_c + U_b = \sum U_p$$

(5.48)

It is worth noting that there has been no empirical assumptions or adjustments used in the development of the SE method theory to this stage. Hence, Equation 5.47 (illustrated in Figure 5.23) is a primitive form of the critical buckling solution. Sufficient examples have been provided to validate the theory for use in both concrete and steel, elastic and plastic. Although an almost perfect correlation can be achieved with both pure and modified Euler theory (see Figures 5.15, 5.16, 5.22 and 5.23) it is important to note that the basis of the SE method theory is the conservation of energy principle, and is therefore quite separate from the Euler theory. Consequently, the SE method theory shares only one common assumption with the Euler theory: that bending moment is the product of force and displacement (i.e., the P-delta effect).

Some necessary assumptions have been made in the given examples. For instance, members have been considered as straight and free from residual stresses, etc. In the further development of a buckling theory, these effects would need to be closely examined. However, referring to Section 5.3.2.3, it is considered that modification by end condition constants (C in Equation 5.43) should not be applicable. This is because the development of plastic hinges essential to the SE method theory will be less sensitive to misalignments than the Euler theory. As such, the SE method theory simply requires that no admissible plastic hinge rotations will form in order to satisfy equal energy principles, and allow member yield in accordance with statics.
5.4.2 CONCRETE SLABS

The critical buckling length of typical lightly reinforced slabs, calculated by the SE method, indicates that the buckling strength of very slender elements may be greater than predicted by the Euler theory (Fig. 5.11). This is the result of increased bending strength exhibited by concrete members resisting low-to-medium levels of axial compression. However, with increasing axial stress, the critical slenderness may decrease more significantly than estimated by elastic buckling assumptions.

The influence of bending response in the SE method is illustrated by Figure 5.11. Since the method incorporates the anticipated bending strength (see Fig. 5.10), the point of interception between elastic and ideal strengths at approximately $0.52f'_c$ in Figure 5.10 is reflected by the respective slenderness intercepts in Figure 5.11.

Likewise, the decreasing ideal moment capacity of highly compressed concrete members is reflected by the respective slenderness ratios in Figure 5.11. Hence, the SE method argues that a direct relationship exists between compression-bending interactions and the buckling strengths of reinforced concrete members.

From the analysis of typical slab configurations, it is concluded that designers must give some consideration to potential regions of buckling. This is especially the case when fully ductile behaviour may occur. From the developed buckling models, it is apparent that intense axial stress and cyclic actions will reduce the buckling capacity of these elements.

5.4.3 STEEL BARS

(a) Structural Steel Members

A précis analysis of structural steel members has been included, since the strong correlation with highly developed buckling methods given in modern steel design standards provides considerable validation of the SE method theory (Figures 5.15, 5.16 and 5.23). However, since modifications have not been made for specific member effects such as local buckling, the SE method estimates higher buckling strengths for squat members than the Standards method.

(b) Reinforcing Bars

As observed in test LOS 5, the buckling of topping slabs may be influenced by the behaviour of reinforcing steel. Under cyclic response, the associated softening response of steel bars (the Bauschinger effect) can result in non-closure of principle cracks. This effectively reduces the local bending capacity and facilitates slab buckling under lower levels of axial compression.

Examination of the Bauschinger effect indicates that a significant decrease in bar buckling lengths can be expected to occur under cyclic actions (see Fig. 5.19). Based on the parabolic

Chapter 5: Topping Slab Buckling: Conclusions
softening envelope of Equation 5.38 (see Fig. 5.18), Grade 500 bars reaching yield strength under cyclic strains up to $3 \cdot \varepsilon_y$, should be provided with lateral ties spaced at not more than $10d_b$. 

Chapter 5: Topping Slab Buckling: Conclusions
Composite Topping Bond

6.1 GENERAL

At the outset of testing, an initial estimate was made of the bond stress that could be resisted in shear between a composite topping and the clean surface of an as-extruded hollow core unit (Section 3.3.1.1). Although it was recognized that the hollow core surface was not intentionally roughened, experiments have indicated that the horizontal shear provisions for composite toppings may be conservative for units subjected to flexure and shear [Scott, 1973].

The 0.55 MPa provision of the design standard [Standards New Zealand, 1995] may be traced back to at least as far as ACI 318-63, where an ultimate strength design factor of 1.9 was permitted on a basic allowable stress of 40 psi (0.275 MPa) for clean and roughened surfaces. Hence, a basic ultimate strength value of 0.55 MPa (80 psi) was eventually adopted for the first New Zealand code of practice (NZS 5101: 1982).

However, the tests reported in Chapter 3 have highlighted that the horizontal shear strengths accorded by the design standards and associated literature are significantly less than the shear strengths observed under dilation type loading. The composite bond values reported in these documents are not disputed, but it is clear that they apply to the horizontal shear stresses arising from flexural actions (i.e., shear flow stresses) rather than direct horizontal shear.

It is considered that the deficit of shear strength results from the limited scope for stress redistribution to occur under a direct shear loading regime. By virtue of its name, direct shear involves the most fundamental concept of shearing; that of a definitive force acting tangentially to a planar surface of known area, so that strength is afforded by a near uniform distribution of shear stress. This concept certainly applies at the interface between topping slabs and precast flooring units in the support region.

To establish the direct shear capacity of topping-precast bond surfaces, a short series of tests were performed on precast surfaces, constructed with varying forms and amplitudes of surface roughening. These specimens were subjected to a direct shear force until failure of the bond surface (or specimen) occurred.
6.2 TEST METHODOLOGY

6.2.1 GENERAL PROCEDURE

Each test set-up was comprised of two precast components plus a cast-in-place topping. The precast components consisted of an anchor slab, which acted as a supporting element of structure, and a shear slab that acted a precast flooring unit. The cast-in-place topping effectively acted as a restraint between the two precast items, and with regard to the translating (shear) slab, the restraint force was resisted by composite bond.

6.2.2 DESCRIPTION OF TEST EQUIPMENT

6.2.2.1 General

The test equipment involved an actuating system in the form of two 43 tonne rams, linear strain potentiometers plus a pen plotter device. The potentiometers were configured so that force-displacement relationships could be traced throughout the test.

6.2.2.2 Test Set-up

The test rig set-up and components of the test are shown in Figures 6.1 and 6.2

![Diagram](image)

Fig. 6.1 Test set-up for establishment of composite bond strengths

Chapter 6: Composite Topping Bond: Test Methodology
Fig. 6.2 View of test composite bond set-up

6.2.2.3 Horizontal Displacements

(a) Actuating System

Horizontal displacement was applied to the test specimens by two parallel acting hydraulic rams, as used for the LOS test series (see Figs 3.2). The rams were connected to the shear slabs via steel channel sections and four 20 mm threaded bars cast into the slab. The shear slabs were mounted on rollers on smooth levelled shim stock, to reduce the restrictive effects of an uneven surface.
(b) Measurements

Principal horizontal displacements were measured by placing two 20 mm linear displacement potentiometers against the end of the shear slab. The associated force measurements were taken directly from the load cells mounted between the rams and connection eyelets. These readings were bridged to a Watanabe X-Y pen plotter, and readings were also scanned to the Metrabyte logger unit.

6.2.3 MATERIALS AND CONSTRUCTION

6.2.3.1 General

The shear slabs were constructed at a precasting yard and the surfaces were roughed (where applicable) in accordance with standard construction practice.

6.2.3.2 Anchor Slabs

The anchor slabs were cast-in-place in the laboratory from 30 MPa concrete and moist cured for three days.

6.2.3.3 Shear Slabs

Table 6.1 Characteristics of shear slab concrete at testing

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Crushing Strength (MPa)</th>
<th>Splitting Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 at 28 days</td>
<td>47.7</td>
<td>3.5</td>
</tr>
</tbody>
</table>

At testing, the shear slab concrete achieved a cylinder splitting strength of $0.51\sqrt{f'_c}$.

The shear slabs were precast in a factory from concrete used in typical prestressed concrete manufacture. The bond surfaces were screeded level, followed by the relevant surface treatment.

The following bond interactions (times number tested) were considered (see Figs 6.3 to 6.7)

- Steel Trowel Finish (2) representing a smooth clean surface
- Wooden Float Finish (1) representing a textured clean surface
- Light Broom Finish (3) representing low-to-medium roughening with laitance
- Retarder + Water Blast (1) representing medium roughening amplitude free of laitance
- Medium Broom + Lightweight Topping (1) representing medium roughening with laitance, bonding with lightweight concrete topping

Chapter 6: Composite Topping Bond: Test Methodology
Fig. 6.3  Steel Trowel test surface

Fig. 6.4  Wooden Float test surface

Chapter 6: Composite Topping Bond: Test Methodology
Fig. 6.5  Light Broom test surface

Fig. 6.6  Retarder + Water Blast test surface
Fig. 6.7  Medium Broom + Lightweight Topping test surface

6.2.3.4 Cast-In-Place Topping Concrete

The precast surfaces were lightly dampened prior to placing of topping concrete. Moist curing was applied to the toppings for five days.

Table 6.2  Characteristics of cast-in-place topping concrete at testing

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Crushing Strength (MPa)</th>
<th>Splitting Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 at 28 days</td>
<td>24.3</td>
<td>2.8</td>
</tr>
</tbody>
</table>

At testing, the topping concrete achieved a cylinder splitting strength of $0.57 \sqrt{f'_c}$.

The structural lightweight concrete was comprised of expanded clay shale (Liapor) lightweight aggregate and normal weight sands, with a target dry density of around 18 kN/m³.

Table 6.3  Characteristics of cast-in-place lightweight topping concrete at time of testing

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Crushing Strength (MPa)</th>
<th>Splitting Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 at 28 days</td>
<td>31.6</td>
<td>2.7</td>
</tr>
</tbody>
</table>

At testing, the lightweight topping concrete achieved a cylinder splitting strength of $0.49 \sqrt{f'_c}$.

Chapter 6: Composite Topping Bond: Test Methodology
6.3 RESULTS OF EXPERIMENTS

6.3.1 BOND STRENGTH

6.3.1.1 General

The bond tests returned results that were generally consistent with expectations based on observation of earlier experiments and experience. The bond stress $v_h$ was calculated as the average of horizontal force $V_h$ divided by bond area $A$:

$$v_h = \frac{V_h}{A} \quad (6.1)$$

The shear slab extensions were continued beyond peak stress response for an approximate minimum of 10 mm. The graphed results are principally from logger data, with peak stress corrections taken from the X-Y plotter diagrams.

6.3.1.2 Steel Trowel Finish

Two specimens were examined, representing smooth clean surfaces in which adhesion between topping and precast concrete is influential. It is considered that these surfaces provide a reasonable comparison with those of as-extruded hollow core units used in the LOS testing series. The results of these tests are shown in Figures 6.8 and 6.9.

![Steel Trowel Finish (ST1)](image)

**Fig. 6.8** Bond-displacement response of test ST1

*Chapter 6: Composite Topping Bond: Results of Experiments*
Fig. 6.9    Bond-displacement response of test ST2

6.3.1.3 Wooden Float Finish

One specimen was tested, representing a textured clean surface without deliberate roughening. The results of this test are shown in Figure 6.10.

Fig. 6.10    Bond-displacement response of Wooden Float Finish test
6.3.1.4 **Light Broom Finish**

Three Light Broom specimens were tested, since the roughening amplitude is close to that achieved with broomed extruded hollow core units. The results of these tests are shown in Figures 6.11, 6.12 and 6.13.

![Light Broom Finish (LB1)](image)

**Fig. 6.11** Bond-displacement response of test LB1

![Light Broom Finish (LB2)](image)

**Fig. 6.12** Bond-displacement response of test LB2

*Chapter 6: Composite Topping Bond: Results of Experiments*
6.3.1.5 Retarder + Water Blast Finish

This standard of finish is commonly used at construction joints in precast concrete. It is achieved by applying a retarding agent when the concrete surface is green, and subsequent water blasting when stripping strength has been achieved. The results of this bond test are shown in Figure 6.14:

Fig. 6.13 Bond-displacement response of test LB3

Fig. 6.14 Bond-displacement response of Retarder + Water Blast test

Chapter 6: Composite Topping Bond: Results of Experiments
6.3.1.6 Medium Broom + Lightweight Topping

A structural lightweight concrete topping was placed over a medium broom surface. The results of this test are shown in Figure 6.15:

Fig.6.15 Bond-displacement response of Medium Broom + Lightweight Topping test

6.4 DISCUSSION ON TEST RESULTS

6.4.1 GENERAL

The direct shear test results give reasonable indications of composite topping bond strengths under dilation type loading. Furthermore, the tests verify that composite bond failure is very brittle and conclusive, with insignificant residual bond strengths being developed.

The observed test strengths would suggest that the surfaces listed in Section 6.2.3.3 could be generalised into three categories of direct shear bond strength (Table 6.4 and Fig. 6.16).

Table 6.4 Bond strength categories for surfaces under direct shear stress

<table>
<thead>
<tr>
<th>Low Bond</th>
<th>Medium Bond</th>
<th>High Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Trowel Finish</td>
<td>Light Broom Finish Wooden Float Finish</td>
<td>Medium Broom Finish Retarder + Water Blast Finish</td>
</tr>
</tbody>
</table>

Chapter 6: Composite Topping Bond: Results of Experiments
6.4.2 STEEL TROWEL FINISH

It is clear from the tests that direct shear bond strengths of as-extruded surfaces are significantly lower than indicated by shear flow tests. This observation is entirely consistent with the early composite bond loss of LOS tests reported in Chapter 3. Hence, it is concluded that a degree of surface roughening must be provided for any precast flooring unit that may be subjected to dilation type loading.

6.4.3 LIGHT BROOM FINISH

This surface type is considered to represent the amplitude of roughening commonly achieved with broomed extruded hollow core units. It is difficult to achieve heavier amplitudes due to the tendency for upper surfaces of extruded sections to collapse under broom pressure. However, it is also possible that hollow core surfaces with light broom roughening will have superior bond strength than the test specimens due to lesser laitance associated with dry mix concrete. The test average is 0.53 MPa, and it is therefore feasible that with an appropriate capacity reduction factor, lightly broomed hollow core units will provide bond strengths that are comparative to the 0.55 MPa specified in design standards.

6.4.4 WOODEN FLOAT FINISH

The textured surface of the wooden float surface produced a good level of bond strength. It is probable that this surface developed greater bond strength than the light broom surfaces due to comparable roughness but less prominent laitance. This is because brooming creates a ridged roughing pattern with surface laitance appearing in the ridges, thus reducing shear strength.

Chapter 6: Composite Topping Bond: Discussion on Test Results
6.4.5 **RETARDER + WATER BLAST FINISH**

In practice, this class of bond surface has shown to perform very well. In the test, full bond capacity was not achieved due to fracture of the shear slab, as is indicated in the residual shear region of Figure 6.14. Hence, it is almost certain that somewhat greater reliable bond strengths than shown can be achieved by this surface.

6.4.6 **MEDIUM BROOM + LIGHTWEIGHT TOPPING**

Due to considerably deeper striations resulting from a medium broom, much greater bond strength was achieved with this surface than with the light broom surfaces. It is important to note that the structural lightweight concrete topping did not appear to reduce bond strength. This is because the matrix contained a mixture of hard crusher and river sands, and since bond strength typically depends on the integrity of the cement/sand mortar fraction, lightweight coarse aggregates had no appreciable effect on composite bond strength.

6.5 **CONCLUSIONS**

The most important conclusion is that the bond surfaces of smooth "as-extruded" hollow core units, which have performed well in shear flow situations, are almost certain to perform badly in situations involving direct shear. Hence, where the effects of frame dilation are possible and support tie details depend on local composite bond strength, a degree of surface roughening must be applied to these surfaces.

From the results of testing, it appears that surface laitance has a significant mitigating effect on composite bond strength, and may be more influential than the amplitude of surface roughening. Hence, surfaces with appreciable roughening may not bond effectively if the brooming striations include an excess of laitance material. In production, this situation can arise when surface brooming is applied too early to wet or fatty mixes.

Bond surfaces that involve a hard interface (e.g., retarder and water blast surfaces) exhibit superior bond characteristics. These surfaces are almost totally impractical for precast floor unit production, however, the principle should be observed. It appears that a medium depth of brooming at the initial set stage would provide a satisfactory level of composite bond strength.
7

Cantilever Tests

7.1 GENERAL

Whether intentional or not, most seating connections between composite topped precast flooring and the support structure will develop an amount of bending moment resistance. In general, this has been a beneficial feature since it aids the serviceability of the composite floor and provides the type of structural continuum that is unique to concrete as an engineering material. There are very few known cases (although some do exist) where accidental carry-over of bending moments from precast floors to support elements has caused support member distress.

The proposed use of alternative support tie details as direct substitutions for traditional starter and continuity bar details may suggest that restraint associated with moment continuity will be affected. Hence, it was considered that the cantilever performance of hollow core slabs with composite topping and typical tie details (both traditional and proposed) should be tested for strength-rotation behaviour.

7.2 TEST METHODOLOGY

7.2.1 GENERAL PROCEDURE

The test set-up involved a 1.2 metre long section of 200 mm deep extruded hollow core unit with 65 mm composite topping, supported on a solid support (see Figs 7.1, 7.2 and 7.3). The support block was restrained against rotation and a vertical point load was applied near the end of floor cantilever. From this perspective the moment-rotation relationship was established, with concurrent bar strains data-logged from electrical resistance strain gauges attached to reinforcement.

Three specimens were constructed, representing one traditional deformed bar starter/continuity detail, and two embedded plain bar details. Topping mesh was provided, but terminated before the critical section so as not to influence bending strengths. To ensure that the critical section coincided with starting positions of strain gauges, a 10 mm deep crack inducing groove was formed in the topping at casting.

Chapter 7: Cantilever Tests: General
Fig. 7.1  Cantilever test set-up

Fig 7.2  View of cantilever set-up prior to testing

Chapter 7: Cantilever Tests: Test Methodology
7.2.2 DESCRIPTION OF TEST EQUIPMENT

7.2.2.1 General

Items of test equipment were either identical or similar to those used in earlier experiments. A small steel crosshead was constructed specifically for the tests, which was bolted to the top of the finished floor section. A pin connection was detailed to allow necessary rotation between the floor section and the hydraulic ram during testing.

7.2.2.2 Precast Support Beams

A precast concrete support beam was constructed for each test (see Fig. 7.1), with sufficient anchorage provided to resist the torsion induced by cantilever loading.

7.2.2.3 Vertical Force

(a) Actuating System

Vertical force was applied to the test specimens by a 20 tonne double-acting hydraulic ram swivel mounted to a steel crosshead (Fig. 7.1). The ram was operated via a hand pump.

Chapter 7: Cantilever Tests: Test Methodology
(b) Measurements

Principal vertical displacements were measured by placing two 100 mm linear displacement potentiometers to the underside of the cantilever and coincident with the line of vertical force (see Fig. 7.1). Further potentiometers were placed against the support block to measure support rotations that might have transpired during testing.

Corresponding force measurements were taken via a calibrated load cell mounted between the stem of the ram and the connection eyelet.

7.2.2.4 Reinforcing Bar Strain Gauges

Electrical resistance strain gauges were connected to principal reinforcement to establish bar strain characteristics. Ordinary 3% extension strain gauges were used. These were supplied by Tokyo Sokki Kenkyujo Co. gauge type FLA-5-11, with 120Ω resistance and 5 mm gauge length.

Surface preparations and the method of fixing electrical resistance strain gauges was carried out in accordance with the departmental guidelines for these procedures [Hill, 1992].

7.2.2.5 Data Logger Unit

The load cell, potentiometers and strain gauges were all connected to the Metrabyte logger, which converted voltage changes caused by linear displacement into digital values. These values were recorded against respective scan numbers that were manually taken throughout the tests. At the conclusion of a test, the logged information was converted to an ASCII file that was then imported into Excel (spreadsheet program) for subsequent editing and data extraction.

7.2.3 MATERIALS AND CONSTRUCTION

7.2.3.1 General

The emphasis of this experimental programme was to reflect the general performance of pretensioned floor construction. The construction methods and curing was of a standard that might be expected on a well-supervised construction site.

7.2.3.2 Precast Pretensioned Hollow Core Units

Hollow core units were of the same type, dimensions and quality as described in Chapter 3.

7.2.3.3 Prestressing Strand

Prestressing strand properties were identical to those described in Section 3.2.3.3.
7.2.3.4 Cast-In-Place Topping Concrete

Topping concrete was selected and cured as described in Section 3.2.3.4.

7.2.3.5 Reinforcing Steel

(a) Starter/Continuity and Tie Bars

12 mm diameter Grade 430 (HD12) starter/continuity bars and 12 mm and 16 mm diameter Grade 300 (R12 and R16) plain round tie bars complying with the appropriate New Zealand standard [Standards New Zealand, 1989] were supplied cut and bent from recognised reinforcing steel merchants. The actual tensile characteristics of continuity and tie reinforcements are deferred to individual cantilever tests.

(b) Welded Wire Fabric (Mesh)

665 mesh (5.3 mm diameter) complying with the appropriate New Zealand standard [Standards New Zealand, 1975] was placed in the topping slab, but was not included in the principal crack region.

7.3 RESULTS OF EXPERIMENTS

7.3.1 CANTI-1

7.3.1.1 General

The initial cantilever test (Canti-1) was directed at a typical starter/continuity bar detail (Fig. 7.4). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit. Four 12 mm diameter Grade 430 (HD12) starter/continuity bars were placed at 300 mm centres, and extended the full length of the cantilever slab. 665 mesh was placed over the hollow core unit and terminated before the end of the seating length.

7.3.1.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 7.2.2.

(b) Reinforcement

Standard 3% extension electrical resistance strain gauges (as described in Section 7.2.2) were employed on two items of starter reinforcement. These were configured so that a gauge was
situated directly over the induced cracking plane. The strain gauges were set at 50 mm centres away from the induced critical section (Fig. 7.5).

![Diagram](image)

Fig. 7.4 Configuration of CANTI-1 test involving a typical starter/continuity bar detail

![Diagram](image)

Fig. 7.5 Strain gauge positions on starter/continuity bars used in the CANTI-1 test

7.3.1.3 **Cast-In-Place Topping Concrete**

Concrete was ordered at a specified slump of 90 mm and accepted at a snatch sample slump of 110 mm (Table 7.1)
Table 7.1 Characteristics of cast-in-place topping concrete for CANTI-1 test

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 at 28 days</td>
<td>19</td>
<td>90</td>
<td>110</td>
<td>34 at 28 days</td>
</tr>
</tbody>
</table>

7.3.1.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) HD12 Continuity Bars

The tensile characteristics of HD12 bars were as described in Table 7.2

Table 7.2 Characteristics of HD12 starters used in CANTI-1 test

<table>
<thead>
<tr>
<th>Avg. Yield Stress (MPa)</th>
<th>Avg. Strength at Fracture (MPa)</th>
<th>Avg. Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>443</td>
<td>617</td>
<td>19.4</td>
</tr>
</tbody>
</table>

7.3.1.5 Results of Testing

The moment-rotation relationship (Fig. 7.6) shows an initially stiff elastic reaction with first cracking at 12.4 kNm. Moment capacity then increased to a distinct yield point at 39.7 kNm and 0.004 radians rotation. The moment capacity fell briefly to around 38 kNm, then gradually increased to a maximum of 51.9 kNm at 0.05 radians as the effects of strain hardening became apparent.

Based on the characteristic yield strength of bar, the calculated ideal bending strength with all bars yielded is around 43 kNm (d = 226 mm). Because significant plasticity occurred below this strength, it is evident that the yield strength of bars was not achieved in exact unison. The plots of the bar strain versus cantilever rotation (Figs 7.7 and 7.8) further support this assertion. These plots indicate that bar strains in the respective N and S gauge series developed in very different fashion. In the early stages, the S gauge series showed the greater strain response, but were dramatically superseded by the N gauge series as cantilever rotation increased.

On the strain-rotation plots, data points have been plotted to the termination point, where gauges had either failed or exceeded the gauge limit.

Chapter 7: Cantilever Tests: Results of Experiments: CANTI-1
Fig. 7.6  Moment-rotation response of CANTI-1 test

Fig. 7.7  Bar strain response of the N gauge series, showing dramatic strain increases at gauges N1 and N2 at approximately 0.015 radians, and a significant late response from gauge N3
Fig. 7.8 Bar strain response of the S gauge series, showing greater early strain response than the N series gauges, but less in the latter stages of testing and limited contribution from gauge S3

Notable in both Figures 7.7 and 7.8 is the generally insignificant response of gauges number 3 and the almost negligible response of gauges number 4, 5 and 6. It must be appreciated that the number 3 gauges were located only 100 mm (8.3 bar diameters) away from the critical section.

Also notable in Figure 7.8 is the diminished response of gauge N1 with increasing rotation.

7.3.2 CANTI-2

7.3.2.1 General

The second cantilever test (CANTI-2) was directed at an embedded R16 tie detail that had performed satisfactorily in dilation tests (Fig. 7.9). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit. Two 16 mm diameter Grade 300 (R16) tie bars were placed into cut out cores and grouted integrally with the topping concrete. 655 mesh was placed over the hollow core unit and terminated before the end of the seating length.

7.3.2.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 7.2.2.
Reinforcement

Standard 3% extension electrical resistance strain gauges (as described in Section 7.2.2) were employed on the two items of tie reinforcement. These were configured so that a gauge was situated 25 mm either side of the induced cracking plane. The strain gauges were set at 50 mm centres away from the induced critical section (Fig. 7.10).

Fig. 7.9 Configuration of CANTI-2 test involving an embedded R16 tie detail

Fig. 7.10 Strain gauge positions on embedded R16 ties used in the CANTI-2 test
7.3.2.3 Cast-In-Place Topping Concrete

Concrete was ordered at a specified slump of 90 mm and accepted at a snatch sample slump of 125 mm (Table 7.3).

Table 7.3 Characteristics of cast-in-place topping concrete for CANTI-2 test

<table>
<thead>
<tr>
<th>Design Strength</th>
<th>Max. Aggregate Size</th>
<th>Ordered Slump</th>
<th>Received Slump</th>
<th>Test Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MPa) 30 at 28 days</td>
<td>13</td>
<td>90</td>
<td>125</td>
<td>28 at 17 days</td>
</tr>
</tbody>
</table>

7.3.2.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) R16 Tie Bars

The tensile characteristics of R16 bars were as described in Table 7.4

Table 7.4 Characteristics of R16 tie bars used in CANTI-2 test

<table>
<thead>
<tr>
<th>Avg. Yield Stress (MPa)</th>
<th>Avg. Strength at Fracture (MPa)</th>
<th>Avg. Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>326</td>
<td>467</td>
<td>30.7</td>
</tr>
</tbody>
</table>

7.3.2.5 Results of Testing

The moment-rotation relationship (Fig. 7.11) shows an initially stiff elastic reaction with first cracking at 15.0 kNm, resulting in a softened response. Plastic behaviour commenced at around 22.5 kNm and rotation of 0.01 radians, followed by gradually increasing moment capacity as the effects of strain hardening became apparent.

Based on the characteristic yield strength of bar, the calculated ideal bending strength with both bars yielded is approximately 27 kNm (d = 217 mm). Similar to the CANTI-1 test, significant plasticity occurred below the calculated yield moment value. However, unlike CANTI-1 it is most likely that this resulted from a progressive decrease in effective depth due to bond loss over the inclined tie portion. Because the plain bar ties were inclined at 16 degrees to the horizontal, 50 mm of bond loss corresponds with a 14 mm reduction in effective depth. The plots of bar strain versus cantilever rotation (Figs 7.12 and 7.13) indicate that considerable
strain penetration occurred during the test, which supports the assertion of a reducing effective depth.

On the strain-rotation plots, data points have been plotted to the termination point, where gauges had either failed or exceeded the gauge limit.

Fig. 7.11  Moment-rotation response of CANTI-2 test

Fig. 7.12  Bar strain response of the N gauge series, showing the progression of strain penetration at gauges N3, N4 and N5

Chapter 7: Cantilever Tests: Results of Experiments: CANTI-2
Fig. 7.13 Bar strain response of the S gauge series, showing the progression of strain penetration at gauges N3, N4 and N5.

Notable in both Figures 7.12 and 7.13 is the delayed strain response at gauges 1 and 2, compared with rapid strain increases at gauges 3, 4 and 5.

7.3.3 CANTI-3

7.3.3.1 General

The third cantilever test (CANTI-3) was directed at an embedded R12 tie detail that were based on a proposed (paperclip) detail (Fig. 7.14). The floor configuration involved 65 mm of cast-in-place topping over a 200 mm hollow core extruded flooring unit. Four 12 mm diameter Grade 300 (R12) tie bars were placed into two cut-out cores (i.e., two bars per core) and grouted integrally with the topping concrete. 665 mesh was placed over the hollow core unit and terminated before the end of the seating length.

7.3.3.2 Instrumentation

(a) Forces and Displacements

Forces and displacements were measured in accordance with the methods described in Section 7.2.2.

Chapter 7: Cantilever Tests: Results of Experiments: CANTI-2
(b) **Reinforcement**

Standard 3% extension electrical resistance strain gauges (as described in Section 7.2.2) were employed on two items of tie reinforcement. The configuration was such that two gauges were situated directly over the induced cracking plane (one on the top leg, one on the bottom). The strain gauges were set at 50 mm centres away from the induced critical section (Fig. 7.15)

**Fig. 7.14** Configuration of CANTI-3 test involving an embedded R12 tie detail

**Fig. 7.15** Strain gauge positions on embedded R12 ties used in the CANTI-3 test
7.3.3.3 Cast-In-Place Topping Concrete

Concrete was ordered at a specified slump of 90 mm and accepted at a snatch sample slump of 125 mm (Table 7.5).

Table 7.5 Characteristics of cast-in-place topping concrete for CANTI-3 test

<table>
<thead>
<tr>
<th>Design Strength (MPa)</th>
<th>Max. Aggregate Size (mm)</th>
<th>Ordered Slump (mm)</th>
<th>Received Slump (mm)</th>
<th>Test Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 at 28 days</td>
<td>13</td>
<td>90</td>
<td>135</td>
<td>24 at 20 days</td>
</tr>
</tbody>
</table>

7.3.3.4 Reinforcement

(a) 665 Mesh

The characteristics of hard drawn wire mesh were identical to those described in Table 3.2

(b) R12 Tie Bars

The tensile characteristics of R12 bars were as described in Table 7.6

Table 7.6 Characteristics of R12 tie bars used in CANTI-3 test

<table>
<thead>
<tr>
<th>Avg. Yield Stress (MPa)</th>
<th>Avg. Strength at Fracture (MPa)</th>
<th>Avg. Strain at Fracture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>342</td>
<td>486</td>
<td>27.2</td>
</tr>
</tbody>
</table>

7.3.3.5 Results of Testing

The moment-rotation relationship (Fig. 7.16) shows an initially stiff elastic reaction with first cracking at 25.1 kNm, resulting in a softened response. Plastic behaviour commenced at around 48 kNm and rotation of 0.007 radians. The subsequent response involved almost perfect-plastic behaviour at an average 53.5 kNm over the remaining 80% of the test.

Based on the characteristic yield strength of bar, the calculated ideal first yield moment is approximately 46 kNm (d = 228 mm). Unlike the two prior tests, the calculated first yield moment was slightly lower than the actual test value. The plots of bar strain versus cantilever rotation (Figs 7.17 and 7.18) indicate that considerable strain penetration occurred during the test, which supports the assertion of a reducing effective depth.
Fig. 7.16  Moment-rotation response of CANTI-3 test

Fig. 7.17  Bar strain response of the N gauge series, showing an apparent strain transition at gauges N2 and N4

Chapter 7: Cantilever Tests: Results of Experiments: CANTI-3
Fig. 7.18  Bar strain response of the S gauge series, showing an apparent strain transition at gauges S2, S3 and S4

Notable in both Figures 7.17 and 7.18 is the unanticipated behaviour at gauges 3, 4 and 5, showing apparent transitions from tensile to compressive strain under increasing rotation. These results are quite contrary to expectations; however, it is difficult to uniformly dismiss the data as a fundamental measurement error. For example, the strain transitions are gradual and seemingly independent of local gauge failure, and therefore cannot be simply linked to the instantaneous "zero shift" effect. Also, closer inspection suggests that distinct correlations exist between the slopes of plotted strain profiles at respective gauge positions such as N3 and N5 in Figure 7.17 and S3, S4 and S5 in Figure 7.18.

The phenomenon of a progressive strain transition in plain bar details is not unprecedented. An interesting feature of test LOS-6 (see Section 4.3.2) was the tendency towards an entirely compressive strain response at gauge locations remote from the critical section. From within the compressive strain domain, these gauges continued to reflect the externally applied cyclic forces, including the eventual diminishing response caused by the Bauschinger effect and bar buckling at the critical section. A characteristic evident in both the LOS-6 and CANTI-3 data is that the apparent compressive strain transition is most pronounced at gauges located furthest from the predominantly tensile critical section.
7.4 **CALCULATED RESPONSE**

7.4.1 **GENERAL**

The three details tested comprise a small representative sample of the possible combinations of support tie and starter/continuity details.

In general, typical continuity details involving deformed bars may vary slightly from the detail tested (CANTI-1) in that the quantity of reinforcement could differ. Conversely, support tie details involving plain bars may show significant variations, as influenced by support member configurations and strength requirements in the advent of support loss.

In order to quantify the performance of such details, comparisons are made between the theoretical and observed moment-rotation relationships of the three test specimens.

7.4.2 **STRENGTH AND STIFFNESS**

For the three test details, it is considered that moment-curvature relationships calculated up the point of reinforcement strain hardening is sufficient for comparison between theoretical and observed moment-rotation response. For the calculation of characteristic bending moments, the strain-equilibrium approach is used (as throughout this thesis), with converging neutral axis depths at a given value of concrete compressive strain.

In the prediction of rotational behaviour, the main difficulty lies in estimating effective plastic hinge lengths. In this instance, the method developed in Section 2.2.1.4 for estimation of effective plastic hinge lengths is utilised. In accordance with the calculated moment-curvature relationships, the method of Section 2.2.1.4 assumes the common parabolic relationship between unconfined concrete stress and compressive strain. Equation 2.52 from Section 2.2.1.4 is reproduced here, with the ratio of plastic hinge length \(l_p\) to shear-span \(z\) derived as:

\[
\frac{l_p}{z} = \sqrt{\left(\frac{\Psi_p}{\Psi_{\Lambda}} - 1\right)^2 + \frac{M_p}{3M_y\left(\frac{\Psi_p}{\Psi_{\Lambda}} - 1\right)}} - \left(\frac{\Psi_p}{\Psi_{\Lambda}} - 1\right)
\]  

(7.1)

and:

\[
\Psi_{\Lambda} = M_p \left(\phi_p - \phi_y\right)
\]

(7.2)

In the above expression, the strain energy per unit length \(\Psi_p\) corresponds with the plastic bending moment \(M_p\) at bending curvature \(\phi_p\). All other terms relate to the condition at first yield. Referring to Section 2.2.1.3, \(\Psi_p\) may be calculated as:

---

*Chapter 7: Cantilever Tests: Calculated Response*
\[ \psi_p = \frac{b f_c' e_c}{\varphi e_o} \left( \frac{1}{3} - \frac{e_c}{2e_o} \right) + \sum_i A_s f_y \left( \varphi d - e_c - \frac{e_y}{2} \right) \]  

(7.3)

As with the measured rotations, the theoretical rotations are simply calculated as deflection \( \delta \) divided by shear span \( z \), giving:

\[ \theta_p = \frac{\varphi_y z}{3} + \left( \varphi_p - \varphi_y \right) f_y \left( 1 - \frac{\varsigma_p}{2z} \right) \]  

(7.4)

The theoretical bending moment capacities and matching rotations at selected values of concrete compressive strain are shown in Table 7.7.

**Table 7.7**  Calculated moment capacities, effective plastic hinge lengths and rotations of test specimens

<table>
<thead>
<tr>
<th>( \varepsilon_y/\varepsilon_v )</th>
<th>( \varepsilon_c )</th>
<th>( M_p )</th>
<th>( M_y )</th>
<th>( \varphi_p )</th>
<th>( \varphi_y )</th>
<th>( \psi_p )</th>
<th>( I_p )</th>
<th>( \theta_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(kNm)</td>
<td>(kNm)</td>
<td>(0/m)</td>
<td>(0/m)</td>
<td>(kNm/m)</td>
<td>(mm)</td>
<td>(mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CANTI-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>0.00039</td>
<td>43.18</td>
<td>43.18</td>
<td>0.01158</td>
<td>0.01157</td>
<td>0.2504</td>
<td>333</td>
<td>0.0039</td>
</tr>
<tr>
<td>2.46</td>
<td>0.0006</td>
<td>43.72</td>
<td>43.18</td>
<td>0.02679</td>
<td>0.01157</td>
<td>0.9111</td>
<td>258</td>
<td>0.0073</td>
</tr>
<tr>
<td>4.36</td>
<td>0.0008</td>
<td>44.06</td>
<td>43.18</td>
<td>0.04624</td>
<td>0.01157</td>
<td>1.766</td>
<td>215</td>
<td>0.0105</td>
</tr>
<tr>
<td>6.68</td>
<td>0.001</td>
<td>44.26</td>
<td>43.18</td>
<td>0.06993</td>
<td>0.01157</td>
<td>2.812</td>
<td>186</td>
<td>0.0137</td>
</tr>
<tr>
<td>CANTI-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>0.00029</td>
<td>27.27</td>
<td>27.27</td>
<td>0.00893</td>
<td>0.00892</td>
<td>0.1219</td>
<td>333</td>
<td>0.0039</td>
</tr>
<tr>
<td>4.53</td>
<td>0.0006</td>
<td>27.71</td>
<td>27.27</td>
<td>0.03681</td>
<td>0.00892</td>
<td>0.8882</td>
<td>212</td>
<td>0.0083</td>
</tr>
<tr>
<td>7.96</td>
<td>0.0008</td>
<td>27.86</td>
<td>27.27</td>
<td>0.06349</td>
<td>0.00892</td>
<td>1.630</td>
<td>175</td>
<td>0.0117</td>
</tr>
<tr>
<td>12.06</td>
<td>0.001</td>
<td>27.96</td>
<td>27.27</td>
<td>0.09524</td>
<td>0.00892</td>
<td>2.516</td>
<td>150</td>
<td>0.0150</td>
</tr>
<tr>
<td>41.03</td>
<td>0.002</td>
<td>28.13</td>
<td>27.27</td>
<td>0.31750</td>
<td>0.00892</td>
<td>8.755</td>
<td>92</td>
<td>0.0299</td>
</tr>
</tbody>
</table>

*Chapter 7: Cantilever Tests: Calculated Response*
The calculated values from Table 7.7 are plotted against observed moment-rotation diagrams for tests CANTI-1, CANTI-2 and CANTI-3 respectively (Figures 7.19, 7.20 and 7.21):

Fig. 7.19 Comparison between observed and theoretical rotational stiffness $K_\theta$ for test CANTI-1 (from Table 7.7)
Fig. 7.20  Comparison between observed and theoretical rotational stiffness $K_0$ for test CANTI-2 (from Table 7.7)

Fig. 7.21  Comparison between observed and theoretical rotational stiffness $K_0$ for test CANTI-3 (from Table 7.7)

Chapter 7: Cantilever Tests: Calculated Response
7.5 APPLICATION OF TEST RESULTS

7.5.1 GENERAL

The cantilever tests have provided useful information on the characteristic strength, stiffness and bond behaviour of both the traditional starter/continuity bar and special plain bar details that have been advocated for hollow core floors in structures prone to dilation effects.

7.5.2 EFFECTIVE CONTINUITY

The comparative strength and stiffness of details is shown in Figure 7.22. From there, it is evident that the R12 “paperclip” detail of test CANTI-3 can provide very similar initial stiffness characteristics to the traditional starter/continuity detail of test CANTI-1, and significantly more post-yield strength. The secant stiffness values up to 40 kNm bending strength for the CANTI-1 and CANTI-3 tests were 10 000 kNm/rad and 9000 kNm/rad respectively.

The R16 detail of test CANTI-2 exhibited good ductility, but clearly lacked the desired levels of initial stiffness and strength for efficient performance as a continuity detail. The secant stiffness value up to 20 kNm bending strength was 2500 kNm/rad.

![Graph of moment-rotation response of cantilever test details](attachment:image.png)

**Fig. 7.22** Moment-rotation response of cantilever test details involving 200mm hollow core flooring with 65mm composite topping

*Chapter 7: Cantilever Tests: Application of Test Results*
The derived stiffness values may be incorporated into the analysis of flooring members for the determination of effective continuity moments under gravity loads. Referring to Figure 7.23, flexibility equations may be developed to include the rotational stiffness at supports i and j, as provided by the relative reinforcing detail. It is assumed that under gravity loads, a net clockwise rotation will result at support i (i.e., \( -\theta_i \)) and an anti-clockwise rotation at support j (i.e., \( +\theta_j \)), therefore:

\[
\begin{align*}
\frac{M_j L}{3EI} - \frac{M_j L}{6EI} &= \rho_i + \theta_i = 0 \\
-\frac{M_i L}{6EI} + \frac{M_j L}{3EI} &= \rho_j - \theta_j = 0
\end{align*}
\]  

(7.5)

Fig. 7.23  Flexibility actions assumed in continuity analysis

In Equation 7.5, the terms \( \rho_i \) and \( \rho_j \) relate to the rotations caused at respective simple supports by the configuration of applied loads. For the common case of a uniformly distributed weight \( w \) per unit length (udl) acting over the entire span \( L \):

\[
\rho_i = \rho_j = \frac{WL^3}{24EI}
\]  

(7.6)

Also, the moment terms in Equation 7.5 can be written as the product of rotational stiffness and rotation, so that \( M_i = K_{0i} \theta_i \), etc. Hence, Equation 7.5 becomes:

\[
\begin{align*}
&+ \left( \frac{K_{0i} L}{3EI} + 1 \right) \theta_i - \left( \frac{K_{0j} L}{6EI} \right) \theta_j = \rho_i \\
&- \left( \frac{K_{0i} L}{6EI} \right) \theta_i + \left( \frac{K_{0j} L}{3EI} + 1 \right) \theta_j = -\rho_j
\end{align*}
\]  

(7.7)

or in the form of a matrix equation:

\[
\begin{bmatrix}
\frac{K_{0i} L}{3EI} + 1 & -\frac{K_{0j} L}{6EI} \\
-\frac{K_{0i} L}{6EI} & \frac{K_{0j} L}{3EI} + 1
\end{bmatrix}
\begin{bmatrix}
\theta_i \\
\theta_j
\end{bmatrix}
= 
\begin{bmatrix}
\rho_i \\
-\rho_j
\end{bmatrix}
\]  

(7.8)

Chapter 7: Cantilever Tests: Application of Test Results
Of course, modern structural analysis programs implicitly solve these equations with direct assignment of rotational stiffness values. However, if there happens to be a power-up failure:

\[
M_i = \frac{K_{oi}wL^2}{12} \cdot \frac{K_{oj}L + 6EI}{K_{oi}K_{oj}L^2 + 4EI[(K_{oi} + K_{oj})L + 3EI]}
\]  

(7.9)

\[
M_j = \frac{K_{oj}wL^3}{12} \cdot \frac{K_{oi}L + 6EI}{K_{oi}K_{oj}L^2 + 4EI[(K_{oi} + K_{oj})L + 3EI]}
\]  

(7.10)

In Equations 7.9 and 7.10, \(M_i\) and \(M_j\) are considered as the negative bending reactions of a flooring unit of length \(L\) that supports a uniformly distributed load \(w\).

Considering the practical design of a 200 mm deep hollow core unit with 65 mm topping in a car park building. The weight of unit plus topping is \(G = 4.0\) kPa and the design floor load for car parking is \(Q = 2.5\) kPa. The flooring units are not propped at construction, so that the continuity effect is mainly introduced under the superimposed live load. Normal volume changes such as creep and shrinkage occur in the flooring units, and promote the formation of topping cracks over supports. For the Ultimate Limit State under factored loads of \(1.2G + 1.6Q\), the applied load in addition to the self-weight of unit plus topping concrete (unrestrained at construction) is \(w = [(1.2 - 1.0) \times 4.0 + 1.6 \times 2.5] = 4.8\) kPa \(\approx 5.8\) kN/m.

The properties of a typical hollow core flooring unit are given in Table 7.8, with the developed end moments relating to combinations of support tie details given in Table 7.9.

**Table 7.8** Properties of 200 mm hollow core with 65 mm topping used in analysis (see Equations 7.9 and 7.10).

<table>
<thead>
<tr>
<th>(L) (m)</th>
<th>(W) (kN/m)</th>
<th>(EI) (Nmm²)</th>
<th>(M^*) (mid-span) (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.0</td>
<td>5.8</td>
<td>40e¹²</td>
<td>159.7</td>
</tr>
</tbody>
</table>

**Table 7.9** End continuity moments developed by support details, with the associated reduction in mid-span bending moments at the Ultimate Limit State.

<table>
<thead>
<tr>
<th>(K_{ei}) (kNm/rad)</th>
<th>(K_{oj}) (kNm/rad)</th>
<th>(M_i) (kNm)</th>
<th>(M_j) (kNm)</th>
<th>Reduction in M* (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9000 (CANTI-3)</td>
<td>9000 (CANTI-3)</td>
<td>32.3</td>
<td>32.3</td>
<td>20.2</td>
</tr>
<tr>
<td>9000 (CANTI-3)</td>
<td>2500 (CANTI-2)</td>
<td>36.7</td>
<td>12.9</td>
<td>15.6</td>
</tr>
<tr>
<td>9000 (CANTI-3)</td>
<td>0 (NIL)</td>
<td>39.7</td>
<td>0</td>
<td>12.4</td>
</tr>
<tr>
<td>2500 (CANTI-2)</td>
<td>2500 (CANTI-2)</td>
<td>15.0</td>
<td>15.0</td>
<td>9.4</td>
</tr>
<tr>
<td>2500 (CANTI-2)</td>
<td>0 (NIL)</td>
<td>16.4</td>
<td>0</td>
<td>5.1</td>
</tr>
</tbody>
</table>

*Chapter 7: Cantilever Tests: Application of Test Results*
From Table 7.9, it is evident that reductions in mid-span moments of between 12% and 20% are reasonable for the given hollow core configuration incorporating CANTI-3 support details.

It is important that design for continuity in precast prestressed flooring is used with discretion, since successful continuity depends on physical contact between unit ends and the supports. Hence, volume change effects of creep and shrinkage will influence the development of continuity moments, and rotational stiffness in particular.

In this regard, it is generally considered that hollow core flooring is well suited to continuity development because extruded dry-mix concrete is a relatively stable material. However, with precast units manufactured from normal slump concrete, dimensional changes are more important. For example, single tee, double tee and prestressed joist units are all prone to creep effects under concentrated axial prestress.

A mixture of prestressing technique and selective use of continuity can mitigate the potentially adverse effects of creep shortening. For example, partial prestressing has proven to be advantageous in tee construction, and a judicious balance between axial prestress and continuity (where available) has proved satisfactory with shored joist construction.

In both tee and joist construction, it is apparent that excessive axial prestress is neither economical nor beneficial. For instance, there is a general tendency for the initial precamber of heat cured tee units to exceed deflection values calculated by purely elastic theory. Hence, even with partial prestressing, care needs to be taken with eventual cambers, and especially if the casting schedule is somewhat ahead of the delivery schedule (rare, but it still does happen).

In joist construction, limiting axial prestress will help control creep effects and therefore improve the efficiency of continuity development. Also, the composite section properties of shored joist flooring are considerably greater than the bare section properties of prestressed joists. Thus, topping concrete that incorporates creep restraint over the supports via continuity reinforcement should be considered as an initial control mechanism that gives long-term benefits.
7.6 DISCUSSION ON RESULTS

7.6.1 CHARACTERISTICS OF BAR BOND

The cantilever test details were comprised of three fairly distinct support tie configurations. In particular, the starter/continuity bar detail (CANTI-1) differed markedly from the other details because it involved typical use of deformed bars. The CANTI-2 and CANTI-3 tests both concerned plain bar details with proven ability as support ties under dilatation actions. However, these details were also appreciably different, with the “paperclip” detail (CANTI-3) provided two legs of reinforcement at each tie location.

Hence, it is interesting to note that three substantially different details exhibited a general tendency for the strains associated with strain hardening to concentrate near the critical section. Thus, under pronounced rotations, the required bond lengths for each detail were generally quite short. This is clearly illustrated by the deformed bars of the CANTI-1 test, where it can be seen (Figures 7.7 and 7.8) that strain response beyond gauges N3 and S3 remained practically unchanged under increasing rotation. As such, the critical bar bond region for the deformed bars mostly occurred within 100 mm from the critical section.

The R16 bars of the CANTI-2 test detail naturally exhibited greater bond loss than the deformed bars of CANTI-1. The procession of bond loss is illustrated in Figures 7.12 and 7.13, where strain gauge response increased sharply at distinct rotation intervals. However, the effective bond development length was again quite short for such a detail. For example, gauges N4 and S4 were located only 125 mm from the critical section, yet exhibited an increased response at around 0.025 radians. This is more than four times the rotation associated with continuity moments at the ultimate limit state (see Table 7.8).

An interesting aspect of the CANTI-3 test was the apparent drift to from tension to compression strain at gauges situated away from the critical section. The exact cause of this phenomenon has not been verified, and could be the material of further research. However, it would, at the very least, seem to discount tensile bond demand at these locations. If the apparent sign shift of gauges is ignored, Figure 7.18 in particular indicates a concentrated strain profile, with gauges located only 50 mm and 100 mm from the critical section showing very limited response under pronounced rotation.

Based on the consistency of results, it is concluded that components of bending curvature may significantly influence the characteristic bond lengths of both plain and deformed reinforcing bars. This observation is supported by contemporary research at the University of Canterbury [Oliver, 1998]. In the testing of the “paperclip” support tie details, it was found that the friction component associated with the kinking of bars acted to restrict the bond lengths in plain bars. This is an important observation, since the success of support tie details in dilation loading situations depends on the ability of plain bars to slip within concrete and provide sufficient gauge length for extension compatibility.
It is now evident that the geometry of tie details may significantly influence the relationship between support rotation, strain penetration and rotational stiffness. As demonstrated in Chapter 4, the debonding of plain bars can be expected to occur under cyclic axial loading. With regard to bending, this result was also reflected in the CANTI-2 test by the conspicuous series of strain increments that occurred under increasing rotation (see Figures 7.12 and 7.13).

Tie details of the CANTI-2 and CANTI-3 varieties both exhibited strain penetration under cyclic axial loading. Therefore, the appreciably greater strain penetration exhibited by the CANTI-2 detail over the CANTI-3 detail under bending action is most likely attributable to inclined ties. Simple mechanics dictate that a completely vertical tie bar anchored between the points of support and load application would develop axial tensile force that is unaffected by support rotation. Hence, an inclined tie that is anchored within the shear span may also develop a proportion of axial force that is independent of bending interactions.

7.6.2 PREDICTED BEHAVIOUR

Comparisons between observed and theoretical moment-rotation behaviour are shown in Figures 7.19, 7.20 and 7.21. From there, it is evident that Equations 7.1 to 7.4 may allow reasonable predictions of effective plastic hinge lengths and resulting rotation response.

The most accurately predicted response is that of CANTI-1, which involved fully bonded deformed bars. The least accurately predicted response is that of CANTI-2, which involved a plain bar detail and considerable bond loss. The CANTI-3 detail involved plain bars with some bond loss, which is reflected in the general accuracy of the predicted response.

In both specimens that involved plain bars, it is clear that eventual bond loss and strain penetration under increasing rotations weakened the assumption of bonded reinforcement in the moment-curvature analysis. Hence, actual curvatures were greater than calculated curvatures, which is reflected in the "flattening" of the observed moment-rotation behaviours. This effect most pronounced with the CANTI-2 detail, which exhibited considerably more bond loss than the CANTI-3 detail.

It is evident that neither of the plain bar details exhibited enhanced bending strengths due to strain hardening of reinforcement, as is immediately apparent in the CANTI-1 test. This is most likely due to the described "flattening" effect of bond loss, since bar strains in excess of strain hardening were recorded at strain gauge locations.

Consequently, the moment-rotation behaviours predicted by Equation 7.1 would relate more closely to fully bonded (deformed bar) reinforcement under the same configurations. The predicted response of CANTI-1 (see Figure 7.19) is certainly sufficient for design purposes involving rotational stiffness with respect to continuity bars. It is considered that with small empirical modifications, the CANTI-3 detail may be similarly predicted.

Chapter 7: Cantilever Tests: Discussion on Results
7.7 CONCLUSIONS

From the observed and theoretical behaviour of the Cantilever tests, the following general conclusions are made:

The “paperclip” detail (CANTI-3) is able to achieve a level of rotational stiffness that is comparable to traditional deformed bar continuity details.

The rotational stiffness provided by CANTI-1 and CANTI-3 details are sufficient to provide useful levels of redistribution to mid-span bending moments.

Rotational stiffness can be predicted with reasonable accuracy by the analytical methods set out in Sections 2.2.1.3 and 2.2.1.4. However, the stiffness response of plain bar details will be affected by bond loss, which may limit the accuracy of the analytical method for predicting rotational stiffness.

Plain bar details with inclined legs may be particularly susceptible to bond loss, as indicated by progressions of strain development and relative rotational stiffness. This behaviour would limit their use as continuity details, but may enhance their performance as a support tie under dilation effects.

Strain gauge readings indicate that details may rely on relatively short bond lengths to achieve bending moment capacity at given stages. It is evident that the deformed bar detail (CANTI-1) required very short bond lengths from the critical section to achieve moment capacity over the entire test duration. It is considered that local friction effects may enhance bond and therefore reduce required bond lengths.

The strain gauge readings in the CANTI-3 test showed an apparent shift from tension to compression under increasing rotation. This phenomenon appears to be unrelated to the peculiarities of electronic equipment, since the transition is gradual, occurs at more than one gauge station and maintains a general empathy with the test procedure. Hence, the phenomenon should probably warrant further investigation. Nevertheless, it is unlikely that this observation exerted much influence on the particular test outcome, and it is not considered further here.
8

Reinforcing Bar Bond

8.1 GENERAL

An interesting and important observation from both this research and a contemporary study into flooring unit support [Oliver, 1998], is the apparent effect of tie bar kinking on reinforcing bar bond performance. In Section 7.4.3, generally diminished bond lengths were associated with continuity bending moments at flooring unit supports. In Oliver’s work, the apparent bond due to friction in kinking bars was shown to be the most likely source of reduced extension capacity of plain round support tie bars. Oliver concluded thus: “Test results indicate that much of the inelastic strains developed by the bars may have been concentrated within the width of the cracked zone by friction forces that could have developed between the bar and the surrounding concrete at the face of the principle transverse crack”. This conclusion is consistent with the findings from experimental research reported in this thesis.

It would be of little surprise to many that the minimum bond lengths of bars, as specified in design standards or codes of practice, are conservative for certain (albeit limited) applications. For instance, a simple comparison with concrete anchor technology will verify that a variety of successful anchorage systems rely on comparatively short bond lengths. In this regard, the very reliable mechanical anchors that employ “undercutting” mechanisms are certainly analogous in their application to plain round bars with hook returns. In both cases, the premature fracture of surface concrete is circumvented via bond loss over the shank length that terminates in a rigid embedded anchorage.

Hence, in terms of anchorage reliability, the concept of providing a debonded length is a defensible one. However, the present concern must lie with estimating the localised effects of friction on the viability of the debonded length. At this point, it is important to note that the described plain bars with hook ends are a prime example of friction development in lieu of pure bond, where the strength attributed to the anchorage is due to the bearing friction.

The following chapter examines a bond model that can allow for the components of friction forces, consistent with the combined actions that often exist in structural concrete members. Since the nature of bar bond is often quite variable and specific to member types and material conditions, emphasis is placed on the comparative effects of friction sources, within a hypothetical bar bond model.
8.2 DEVELOPMENT OF A BOND AND FRICTION MODEL

8.2.1 BOND AND FRICTION CRITERIA

8.2.1.1 General

The concept of reinforcing bar anchorage within a concrete mass has traditionally been related by strength development through pure bond stresses. As such, the concurrent transverse components of force acting on bars at critical (cracked) sections are not considered within the common bond model.

The friction effects from transverse forces may be particularly evident with support tie details, since these typically involve floor support derived from the significant kinking of small diameter reinforcement. From the first principles of bond analysis, it is immediately evident that smaller diameter bars require the least effective bond lengths. Based on an average bond strength $\bar{u}$ over length, the tensile force $T$ of reinforcement is related to development length $l_d$:

$$ T = A_s f_s = \bar{u} \Sigma \omega l_d $$  \hspace{1cm} (8.1)

Therefore, the characteristic development length is related to the diameter $d_b$ of reinforcement as:

$$ l_d = \frac{d_b f_s}{4 \bar{u}} $$  \hspace{1cm} (8.2)

It is envisaged that the effects of friction will act to reduce the effective development length by enhancing the average effective bond stress.

8.2.1.2 Bar Bond

(a) Bond Strength

Straight lengths of plain round reinforcement invoke the most fundamental form of bond action, in which the developed strength depends on adhesion between the bar material and surrounding concrete. For plain bars with surface corrosion, bond strength is often significantly improved through friction that develops at the pitted bar surface. The forces of friction are, of course, most pronounced with deformed bars, where the bearing and shear forces associated with projecting ribs greatly increases the local “bond strength” [see Park and Paulay, 1975].

The bond strength between reinforcement and concrete is usually evaluated by bond stress-slip relationships. Associated forms of this test appear widely in research literature and design standards [Standards New Zealand, 1989]. The test bond-slip fundamentally involves measurement of bar slippage at an average bond stress, which is based on a relatively short bond length (see Fig. 8.1). The derived bond stress-slip relationship will typically show bond stress increasing at a decreasing rate under increments of applied force (Fig. 8.2).
Fig. 8.1 Details of bond tests apparatus [Standards New Zealand, 1989]

Fig. 8.2 Typical nominal shear (bond) versus slip diagram
[Standards New Zealand, 1989]

Chapter 8: Reinforcing Bar Bond: Bond & Friction Model
The bond-slip diagram (Fig. 8.2) is often described by a power function, in which the maximum (or peak) bond stress \( \tau_{\text{max}} \) is given at a corresponding nominal slip displacement \( s_1 \). Hence, the bond stress for a given slip \( s \) (for \( s \leq s_1 \)) is calculated as:

\[
\tau = \tau_{\text{max}} \left( \frac{s}{s_1} \right)^\alpha
\]  

(8.3)

where the coefficient \( \alpha \) defines the curve shape, and may typically lie in the range \( 0 \leq \alpha \leq 1 \). Typical parameters for deformed bars involving both confined and unconfined concrete and plain bars are shown in Table 8.1.

### Table 8.1  
Bond-slip parameters applicable to design in good bond conditions [CEB-FIP, 1990].

<table>
<thead>
<tr>
<th>Bar profile</th>
<th>Concrete</th>
<th>( \tau_{\text{max}} ) (MPa)</th>
<th>( s_1 ) (mm)</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>deformed</td>
<td>confined</td>
<td>( 2.5 f_{\text{ck}} )</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>deformed</td>
<td>unconfined</td>
<td>( 2.0 f_{\text{ck}} )</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>plain</td>
<td>all</td>
<td>( 0.3 f_{\text{ck}} )</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(b) Bond Capacity Function

In the proposed bond model, a bond capacity function \( q(\kappa) \) is introduced. This function does not describe the required bond for strength development, but provides a shape function of bond capacity over the embedded length. Hence, under very low levels bar tension, it is assumed that a near uniform capacity will exist over the embedment length, and that progressive loading will reduce this capacity to zero at the loaded end. However, the capacity will be subject to less deterioration with increasing distance from the loaded end.

As such, the bond capacity function may be described by a power function, similar in appearance to equation 8.3, where the bond capacity \( q \) over embedment length \( L \) is given as:

\[
q(\kappa) = q_\text{u} \left( \frac{x}{L} \right)^n
\]  

(8.4)

where \( q_\text{u} \) is the ultimate bond capacity, and the coefficient \( n \) is normally in the range \( 0 \leq n \leq 1 \). The bond capacity envelopes described by equation 8.4 are shown in Figure 8.3.

It is considered that a number of factors, such as concrete condition, bar surface condition, degree of compaction, confinement and embedment length may all influence the value \( q_\text{u} \). In this study, the bond capacity will be taken as an upper limit of bond strength for 10 mm plain round bars in ideal bond conditions.
Fig. 8.3 Envelopes of bond capacity over embedment length, as described by Equation 8.4 with given values of the coefficient n

8.2.1.3 Friction from Shear

An embedded bar subjected to large shear displacements will resist shear forces by dowel action due to dowel kinking. Under the combined actions of shear and elongation, it is considered that friction associated with dowel action will affect the bond conditions in the vicinity of the critical section (see Fig. 8.4). As discussed, this is especially relevant to the small diameter support ties used in details to control seating failure.

In order to establish the extent of friction forces over a nominal bond length, the normal component of shear must be determined. In this regard, it is possible to estimate bearing stresses along an embedded length of dowel bar by using a beam on elastic foundation analogy [Elliott et al, 1992]. In Elliot’s paper, it is considered that an embedded length of $8d_b$ is adequate to develop dowel action, and that compressive stress will be zero at around $2.5d_b$ from the critical section (i.e., the point of shear application).

Hence, by adopting a reasonable estimate of subgrade reaction (25 N/mm$^3$), the shear force distribution according to the Winkler theory [Young, 1989] for a 10 mm diameter bar is shown in Figure 8.5. It is evident from the analysis that the distance $2.5d_b$ is close to the point of zero stress and that $8d_b$ embedment is sufficient to develop shear strength. Also, and perhaps most importantly, this result appears to be fairly insensitive to the actual choice of subgrade reaction.
Fig. 8.4  Effect of bearing stresses on bar bond development

Fig. 8.5  Distribution of shear force, in accordance with Winkler beam theory, along a 10 mm diameter bar embedded 80 mm (8d₀) into concrete with a unit shear force applied at the critical section

Chapter 8: Reinforcing Bar Bond: Bond & Friction Model
The analytical solution to the curve in figure 8.5 involves a veritable cocktail of circular and hyperbolic functions. However, it is evident that a parabola will allow close approximation of the curve, and may be expressed in polynomial form:

\[ V(t) = V_0 \left[ 5 \left( \frac{x}{h} \right) - 5 \left( \frac{x}{h} \right)^2 - 1 \right] \]  

(8.5)

where \( V_0 \) is the shear force applied at the critical section and \( h \) is the effective embedment length, taken as 100 mm (10d_b).

8.2.1.4 Friction from Curvature

The concept of curvature bond is analogous to the frictional losses due to curvature in post tensioning operations. The bending curvature within a member will also result in angular changes to reinforcement. Therefore, the normal force component due to angular change will develop a friction force with the surrounding concrete. In this way, it is considered that curvature friction may provide bond in regions of constant moment, where the traditional expression for flexural bond (i.e., \( V/E_{cd} \)) breaks down due to an absence of shear force.

8.2.1.5 Friction Coefficient

For smooth steel on concrete, the friction coefficient \( \mu \) is usually taken as 0.7, and this figure is adopted for the proposed friction model. In tests conducted by Oliver [Oliver, 1998] involving 10 mm diameter plain bar, a friction coefficient of \( \mu = 0.86 \) was calculated from experimental results, and therefore compares well with the recommended value. It is considered that the friction coefficient of deformed bars would be at least twice the appropriate value for plain bars.

8.2.2 EQUILIBRIUM MODEL

8.2.2.1 General

In a bond model that includes the effects of friction forces, the actual interactions between bond and friction and the progressions of bond loss under load applications are likely to be extremely complex. In order to preserve simplicity in the proposed model, the combined effects of bond and friction resistance within a force equilibrium model are described by a single envelope of "effective" bond stress over the embedded bar length.
8.2.2.1 Equilibrium

An elemental length of reinforcing bar subjected to concurrent axial tension, shear force and curvature is shown in Figure 8.6.

![Diagram showing bond and friction forces acting on an element of reinforcing bar](image)

**Fig. 8.6** Bond and friction forces acting on an element of reinforcing bar

Assuming that the tensile force $T$ is resisted by bond and friction mechanisms, the differential equation of force equilibrium may be written:

$$ T - \left( T + \frac{dT}{dx} \right) = \Sigma_0 q(x) dx + \mu \left( dV + dN \right) (x) $$

(8.6)

where $\Sigma_0$ is the bar perimeter, $q(x)$ is the bond capacity function according to Equation 8.4, $\mu$ is the friction coefficient, and $V(x)$ and $N(x)$ are the normal forces due to shear and curvature respectively. Furthermore, in terms of flexural bending, the normal differential force $dN$ due to curvature may be written:

$$ dN = T d\theta = T \varphi dx $$

(3.7)

and Equation 8.6 becomes:

$$ - \frac{dT}{dx} = \Sigma_0 q(x) dx + \mu dV(x) + \mu \varphi(x) dx $$

(8.8)

In order to solve the differential equation, Equation 8.8 is written in dimensionless form:

$$ \frac{dT}{T} = \frac{\Sigma_0 q(x)}{T} dx + \frac{\mu}{T} dV(x) + \frac{\mu \varphi(x)}{T} dx $$

(8.9)

*Chapter 8: Reinforcing Bar Bond: Bond & Friction Model*
Equation 8.9 may be solved by separation of variables provided that the tension value $T$ on the right-hand side may be described as a constant that corresponds with the initial condition. This condition exists at $x = 0$, where $T_o$ is the bar tension at the critical section.

$$\frac{dT}{T(x=0)} = \left( \frac{\Sigma_o q(x)}{T_o} - \frac{\mu}{T_o} \frac{dV_s(x)}{dx} + \mu \frac{\varphi(x)}{dx} \right)$$ (8.10)

Integration of Equation 8.10 gives:

$$\ln T = - \left( \frac{\Sigma_o}{T_o} \int q(x) \, dx + \frac{\mu}{T_o} \int V_s(x) \, dx + \mu \int \varphi(x) \, dx \right) + C$$ (8.11)

where $C$ is the arbitrary constant of integration.

8.2.2.3 Bond-Only Model

The common assumption of bar bond in response to direct axial tension is considered first. Thus ignoring friction effects, the first term of the RHS of Equation 8.11 is integrated and the relevant initial condition of $T = T_o$ at $x = 0$ is applied, so that:

$$\ln T(o) = - \frac{\Sigma_o}{T_o(n+1)} \left( \frac{x}{L} \right)^{n+1} + C$$ (8.12)

$$C = \ln T(o)$$ (8.13)

which yields the solution for bar tension $T$ as a function of embedded length $x$:

$$T(x) = T_o \exp \left[ \frac{\Sigma_o}{T_o(n+1)} \left( \frac{x}{L} \right)^{n+1} \right]$$ (8.14)

As discussed in Section 8.2.1.2, the bond stress $q(x)$ described by Equation 8.4 is an envelope of bond capacity that may be influenced by such factors as reinforcing bar and concrete properties, confinement and loading regime. It is not the actual (effective) bond stress required to resist bar tension developed under a total embedment length $L$ and selected envelope shape coefficient $n$, which will approach zero as length $x$ approaches the total embedment length $L$. The effective bond stress envelope may be determined from the fundamental bar bond equation.

Referring to Equation 8.1, the variation of bar tension $T$ with distance $x$ may be written:

$$dT = \Sigma_o u(x) \, dx$$ (8.15)
Therefore, differentiating Equation 8.14 with respect to $x$ and dividing by $\Sigma_0$ gives the effective bond stress:

$$ u(x) = q_u \left(\frac{x}{L}\right)^n \exp\left[-\frac{\Sigma_0 L q_u}{T_0 (n+1)} \left(\frac{x}{L}\right)^{n+1}\right] $$

(8.16)

For a single round bar, the exponential function of Equations 8.14 and 8.16 may be simplified by reducing the terms $\Sigma_0$ and $T_0$ so that:

$$ \exp\left[-\frac{4L q_u}{f_s d_u (n+1)} \left(\frac{x}{L}\right)^{n+1}\right] $$

(8.17)

Based on Equations 8.4, 8.14 and 8.16, relationships between the assumed bond capacity envelope, derived tension forces and corresponding effective bond stresses are shown in Figures 8.8 to 8.11 for plain and deformed bars. In each case, the model has assumed a 10 mm diameter bar with good initial bond conditions (see Table 8.2) and varying degrees of bond capacity loss as described by Equation 8.4 (see Fig. 8.7). As such, a procession of bond loss may be modelled by increments of the coefficient $n$. Thus, the $n = 0$ condition represents undisturbed (uniform) bond capacity over the embedment length, characteristic of pre-yield bar tension. At the other extreme, the $n = 2$ condition represents considerable bond loss over the embedment length, characteristic of strain penetration effects under cyclic loading.

![Graph showing the relationship between the proportion of ultimate bond strength and the proportion of embedded length](image)

**Fig. 8.7** Range of bond capacity shape coefficients $n$ in Equation 8.4, for a 10 mm diameter bar embedded 600 mm into concrete.

**Table 8.2** Characteristics of embedded bars corresponding with Figures 8.8 to 8.11

<table>
<thead>
<tr>
<th>Bar profile</th>
<th>Size</th>
<th>$T_0$ (kN)</th>
<th>$q_u$ (MPa)</th>
<th>$L$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>plain</td>
<td>R10</td>
<td>23.6</td>
<td>4.0</td>
<td>600</td>
</tr>
<tr>
<td>deformed</td>
<td>D10</td>
<td>23.6</td>
<td>12.0</td>
<td>600</td>
</tr>
</tbody>
</table>

*Chapter 8: Reinforcing Bar Bond: Bond & Friction Model*
**Fig. 8.8** Proportion of yield force $T_0$ according to Equation 8.14, developed by a 10 mm plain round bar (Grade 300) over a 600 mm embedded length at bond capacity envelopes given in Figure 8.7 (increasing $n$ corresponds with bond loss)

**Fig. 8.9** Effective bond stress envelopes $u(x)$ according to Equation 8.16, developed by a 10 mm plain round bar (Grade 300) over a 600mm embedded length at bond capacity envelopes given in Figure 8.7 (increasing $n$ corresponds with bond loss)

*Chapter 8: Reinforcing Bar Bond: Bond & Friction Model*
Fig. 8.10  Proportion of yield force $T_0$ according to Equation 8.14, developed by a 10 mm deformed bar (Grade 300) over a 600 mm embedded length at bond capacity envelopes given in Figure 8.7 (increasing $n$ corresponds with bond loss)

Fig. 8.11  Effective bond stress envelopes $u(\lambda)$ according to Equation 8.16, developed by a 10 mm deformed bar (Grade 300) over a 600 mm embedded length at bond capacity envelopes given in Figure 8.7 (increasing $n$ corresponds with bond loss)
The bond capacity function \( q_u \) is not limited to pull-out models, and may be adapted to the general cases of flexural bond and member axial tension. A simple example is to assume that the bond capacity between flexural or axial tension cracks can be described by a sine curve relationship (Fig. 8.12):

![Diagram](image)

**Fig. 8.12** Assumed distribution of bond capacity envelope between flexural cracks, relevant to direction of bar tension

Hence, the bond capacity is written:

\[
q_u = q_u \sin \left( \frac{2\pi x}{L} \right) \tag{8.18}
\]

Substituting Equation 8.18 into Equation 8.11, integrating and applying the initial condition of \( T = T_o \) at \( x = 0 \) gives the solution for a round bar as:

\[
T(u) = T_o \exp \left[ \frac{2Lq_u}{\pi d_u f_s} \left( \cos \left( \frac{2\pi x}{L} \right) - 1 \right) \right] \tag{8.19}
\]

and the corresponding effective bond stress as:

\[
u_u = q_u \sin \left( \frac{2\pi x}{L} \right) \exp \left[ \frac{2Lq_u}{\pi d_u f_s} \left( \cos \left( \frac{2\pi x}{L} \right) - 1 \right) \right] \tag{8.20}
\]

Tension forces and effective bond stresses as described by Equations 8.19 and 8.20 and at varying values of ultimate bond capacity (see Table 8.3) are shown in Figures 8.13 and 8.14.

**Table 8.3** Characteristics of longitudinal bar corresponding with Figures 8.13 and 8.14

<table>
<thead>
<tr>
<th>Bar profile</th>
<th>Size</th>
<th>( f_u ) (MPa)</th>
<th>( q_u ) (MPa)</th>
<th>( L ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>deformed</td>
<td>D10</td>
<td>300</td>
<td>0.2, 4, 8, 16</td>
<td>300</td>
</tr>
</tbody>
</table>

*Chapter 8: Reinforcing Bar Bond: Bond & Friction Model*
Fig. 8.13 Proportion of yield force $T_o$ according to Equation 8.19, developed by a 10 mm deformed bar (Grade 300) over 300 mm bond length at the stated ultimate bond capacities.

Fig. 8.14 Effective bond stress envelopes $u(x)$ according to Equation 8.20, developed by a 10 mm deformed bar (Grade 300) over 300 mm of bond length at the stated ultimate bond capacities.
8.2.3 **FRICTION-BOND MODEL**

8.2.3.1 **General**

At this point, the physical model involving the kinking of bars is revisited. For the particular case of support tie bars, it is considered that directly applied shear force will dominate local friction effects. Since the influence of curvature may feature less strongly in general, it will be ignored for the particular case of tie bar kinking.

Referring to Equations 8.4 and 8.5, and considering the initial condition of \( T = T_0 \) at \( x = 0 \) gives:

\[
\ln T(\theta) = -\frac{1}{T_0} \left\{ \frac{\Sigma_0 L q_u}{(n+1)} \left( \frac{x}{L} \right)^{n+1} + \mu V_o \left[ 5 \left( \frac{x}{h} \right) - 5 \left( \frac{x}{h} \right)^2 - 1 \right] \right\} + C \quad (8.21)
\]

\[
C = \ln T(\theta) - \frac{\mu}{T_0} V_o \quad (8.22)
\]

Combining Equations 8.21 and 8.22 gives the solution for reinforcing bar tension as a function of bond capacity, embedment length and shear force. In Equation 8.21, the absolute magnitude of shear force becomes applicable since friction developed between the bar and concrete is independent of the direction in which the shear force acts.

\[
T_s = T_o \exp - \left[ \frac{\Sigma_0 L q_u}{(n+1)} \left( \frac{x}{L} \right)^{n+1} + \frac{5\mu V_o}{h} \left( \frac{x}{h} \right) \left( 1 - \frac{2\langle x \rangle}{h} \right) \right] \quad (8.23)
\]

and:

\[
u_s = \left[ q_u \left( \frac{x}{L} \right)^n + \frac{5\mu V_o}{\Sigma_0 h} \left( 1 - \frac{2\langle x \rangle}{h} \right) \right] \exp - \left[ \frac{\Sigma_0 L q_u}{(n+1)} \left( \frac{x}{L} \right)^{n+1} + \frac{5\mu V_o}{h} \left( \frac{x}{h} \right) \left( 1 - \frac{2\langle x \rangle}{h} \right) \right] \quad (8.24)
\]

In the above expressions, angle brackets indicate that the contained value may not exceed \( 8d_0 \) (80 mm), as prescribed by the shear force envelope of Equation 8.4 (see Fig. 8.5).

Tension forces and effective bond stresses according to Equations 8.23 and 8.24 at varying bar kinking angles \( \theta = 0^\circ, 10^\circ, 20^\circ \) and \( 30^\circ \) are shown in Figures 8.15 and 8.16, where shear force at the critical section \( V_o = A_b f_y \sin \theta \) (see Fig. 8.6) and \( T_o = A_b f_y \).

**Table 8.4** Characteristics of embedded bars corresponding with Figures 8.8 to 8.11

<table>
<thead>
<tr>
<th>Profile</th>
<th>Size</th>
<th>( q_u ) (MPa)</th>
<th>( \mu )</th>
<th>( n )</th>
<th>( L ) (mm)</th>
<th>( h ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>plain</td>
<td>R10</td>
<td>4.0</td>
<td>0.7</td>
<td>1.0</td>
<td>600</td>
<td>100</td>
</tr>
</tbody>
</table>

*Chapter 8: Reinforcing Bar Bond: Bond & Friction Model*
Fig. 8.15  Proportion of yield force $T_0$ according to Equation 8.23, developed by a 10 mm plain bar (Grade 300) over a 600 mm embedded length, showing influence of friction due to shear from bar kinking at given angles (see Fig. 8.6)

Fig. 8.16  Effective bond stress envelopes $u_x$ according to Equation 8.24, developed in the shear zone of a 10 mm plain bar (Grade 300), showing influence of friction due to shear from bar kinking at given angles (see Fig. 8.6)

Chapter 8: Reinforcing Bar Bond: Bond & Friction Model
Reverting to Equation 8.1, it is evident that for a bar kinking angle of 30 degrees in Figure 8.16, the average bond stress over the first 50 mm of embedment (i.e., approximately 6 MPa) is sufficient to develop 9.4 kN of tension, or 40% of the bar yield strength. Also, the summation of bond stress over embedded bar surface area (see Equation 8.15) will be marginally less than the total bar yield force $T_0$. Thus, the net tension demand over embedment length will equal the total tension at any point less the proportion of force developed through friction bond over the first 50 mm of embedment. As such, the net profile of bar tension demand over embedment length to resist $T_0$ is shown in figure 8.17.

![Plain Bar (R10)](image)

**Fig. 8.17** Proportion of yield force $T_0$ developed by a 10 mm plain bar (Grade 300) over a 600 mm embedded length after net deduction of force developed through friction from bar kinking at given angles

Figure 8.17 indicates that the effects of friction due to shear may have a significant impact on development lengths. For the case of the plain bar shown, an 80% reduction to yield force at 600 mm embedment is augmented to a 100% reduction to yield force at 480 mm embedment. This implies an effective bar anchorage that will directly limit the amount of extension available at lower levels of steel strain. For deformed bars, the above effect will be much more acute, resulting in significantly shorter development lengths and relatively large steel strains under dilation type loading (see Table 8.5, Figures 8.18 to 8.20).

**Table 8.5** Characteristics of embedded bars corresponding with Figures 8.18 to 8.20

<table>
<thead>
<tr>
<th>Profile</th>
<th>Size</th>
<th>$q_0$ (MPa)</th>
<th>$\mu$</th>
<th>$n$</th>
<th>$L$ (mm)</th>
<th>$h$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>deformed</td>
<td>D10</td>
<td>12.0</td>
<td>1.4</td>
<td>1.0</td>
<td>600</td>
<td>100</td>
</tr>
</tbody>
</table>

*Chapter 8: Reinforcing Bar Bond: Bond & Friction Model*
Fig. 8.18 Proportion of yield force $T_o$ according to Equation 8.23, developed by a 10 mm deformed bar (Grade 300) over a 600 mm embedded length, showing influence of friction due to shear from bar kinking at given angles (see Fig. 8.6)

Fig. 8.19 Effective bond stress envelopes $u_{(x)}$ according to Equation 8.24, developed in the shear zone of a 10 mm deformed bar (Grade 300), showing influence of friction due to shear from bar kinking at given angles (see Fig. 8.6)
Fig. 8.20 Proportion of yield force $T_0$ developed by a 10 mm deformed bar (Grade 300) over a 600 mm embedded length after net deduction of force developed through friction from bar kinking at given angles.

8.3 DISCUSSION ON RESULTS

8.3.1 COMPARISON WITH TEST RESULTS

The values given in Table 8.6 are extracted from recent research work on embedded support tie bars [Oliver, 1998]. As discussed earlier, the effects of friction were considered to have caused measurable reductions in tension force resisted by the embedded bar portion. These reductions are subsequently compared with estimated reductions, which are calculated in accordance with Equation 8.23.

Table 8.6 Horizontal force misclose observed during the testing of Unit 2 [Oliver, 1998: (Table 5.2)]

<table>
<thead>
<tr>
<th>Horizontal Displacement (mm)</th>
<th>Applied Horizontal Force (kN)</th>
<th>Measured Paperclip Force (kN)</th>
<th>Continuity Bar Force (kN)</th>
<th>Misclose (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(1) – (2) – (3)</td>
</tr>
<tr>
<td>20</td>
<td>333</td>
<td>106</td>
<td>204</td>
<td>23</td>
</tr>
<tr>
<td>30</td>
<td>135</td>
<td>110</td>
<td>Nil</td>
<td>25</td>
</tr>
<tr>
<td>45</td>
<td>141</td>
<td>110</td>
<td>Nil</td>
<td>28</td>
</tr>
<tr>
<td>47</td>
<td>139</td>
<td>113</td>
<td>Nil</td>
<td>26</td>
</tr>
</tbody>
</table>
From Table 8.6, the average force misclose may be taken as 25 kN. The associated shear force acting at the critical section was 29 kN. These two values were used by Oliver to calculate an effective friction coefficient between the plain bar reinforcement and surrounding concrete of $\mu = 25/29 = 0.86$.

Since the tested paperclip detail of Unit 2 involved four legs of R10 reinforcement, it is evident that an average initial tension force of $T_o = 34$ kN and an average corresponding initial shear force of $V_o = 7.25$ kN acted on each tie leg.

Inspection of Equation 8.24 and Figures 8.15 to 8.20 indicates that the critical bond length affected by shear force is 0.5$b$, (i.e., $5d_b$). Hence, the maximum reduced tension resulting from an introduction of shear force is equal to the value corresponding to $5d_b$ of embedment. If the critical shear bond length of $5d_b$ is adopted and cohesive bond is ignored (i.e., $q_u \rightarrow 0$) the expression for maximum reduced tension due to shear may be written as:

$$T_{(5d_b)} = T_o \exp \left(-\frac{5\mu V_o}{4T_o}\right)$$  \hspace{1cm} (8.25)

Substituting appropriate values derived from the experiment, the estimated reduced tension due to shear for Unit 2 is:

$$\frac{T_{(5d_b)}}{T_o} = \exp \left(-\frac{5 \times 0.86 \times 7.25}{4 \times 34}\right) = 0.8$$

Referring to Table 8.6 and the difference between applied and measured horizontal forces, it is evident that the estimated 20% reduction to initial tension due to shear is comparable to the average observed reduction of 19.3%.

The estimated variation in reduced bar tension, with the initial bar tension ranging between nominal yield force and the ultimate tensile strength, is given in Table 8.7. For each case, the initial shear force and friction coefficient have been taken as constants.

**Table 8.7** Relationship between initial bar tension $T_o$ and the maximum reduced tension due to shear force, $T_{5d_b}$.

<table>
<thead>
<tr>
<th>Initial Bar Tension $T_o$ (kN)</th>
<th>Initial Shear Force $V_o$ (kN)</th>
<th>Friction Coefficient $\mu$</th>
<th>Reduced Tension $T_{5d_b}/T_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.5</td>
<td>7.25</td>
<td>0.86</td>
<td>0.72</td>
</tr>
<tr>
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<td>0.86</td>
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</tr>
<tr>
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<td>0.78</td>
</tr>
<tr>
<td>34.8</td>
<td>7.25</td>
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<td>0.80</td>
</tr>
</tbody>
</table>

*Chapter 8: Reinforcing Bar Bond: Discussion on Results*
8.4 CONCLUSIONS

8.4.1 GENERAL

The following general conclusions are made with regard to the bond theory presented in this chapter:

The friction-bond model assumes that friction effects, resulting from the normal components of direct shear and curvature, may significantly influence the nature of resistance to pullout forces.

In regions of constant bending moment, the friction-bond theory reasons that curvature friction (analogous to post-tension friction) may account for a proportion of the apparent bar bond.

Of the three bond modes considered (i.e., cohesive, direct shear and curvature friction), direct shear may be the singularly most influential source of effective bond in regions that involve dowel bar actions.

Based on the commonly assumed Winkler Beam analogy, the effective length of shear bond zone for a bar subjected to dowel action is five bar diameters ($5d_b$) from the critical section. At this distance, the maximum force reduction due to shear bond will have been achieved. In situations involving deformed bars, this distance may need to be increased somewhat to allow for concrete cone pullout (i.e., the so-called Goto effect).

When compared to test results involving plain tie bar details, the force misclose (i.e., the force reduction over embedment length) that results from friction effects may be estimated with reasonable accuracy.
9

Summary, Conclusions and Recommendations

9.1 GENERAL

A series of experiments were conducted with the intention of isolating various critical aspects of the support and continuity performance of hollow core flooring under (especially) seismic related forces. The main emphasis of testing has been to ascertain the capabilities of support details, specifically starter and tie bars, when subjected to the dilation type loadings associated with plastic hinge formation in support members.

The primary experimental programme involved six assemblages of hollow core flooring units with composite topping. These specimens exhibited variations in support tie details, and were loaded either monotonically and cyclically in order to test the hypotheses of inherent detail weaknesses under particular loading regimes.

Further experiments were conducted to determine more specific behaviour, such as composite topping bond capacity.

A sizeable introductory chapter is featured, in which the historical background of pretensioned flooring use is discussed. This is considered an important inclusion, since the current approach toward support detail design would appear more strongly influenced by a perceived history of performance than by way of substantiated test data.

A reasonable body of theory has been presented that examines the subjects of dynamics with stiffness degradation, elastic-plastic stiffness transition, effective plastic hinge lengths, frame sway compatibility, elastic-plastic buckling, rotational behaviour and bar bond. Although apparently diverse, it is considered that each aspect of theory presented may assist in one or more ways with the general analysis of flooring members and/or the interactions between flooring and ductile support members.
9.2 MONOTONIC DILATION

9.2.1 TYPICAL STARTER BAR DETAIL (Tests LOS 1, LOS 3, LOS 4)

Based on the evidence of testing full-scale specimens, the following is concluded for flooring members subjected to dilation type loading:

- Starter bar details are grossly inadequate for the purposes of maintaining a ductile tie connection between pretensioned flooring units and support members. Hence, starter details provide no control over the potential for loss of support and floor member collapse.

- For flooring members with relatively smooth bond surfaces (i.e., extruded hollow core), complete tensile fracture of the topping at the point of starter curtailment is almost certain. The topping fracture is accompanied by a rapid breakdown in composite bond strength between the topping slab and hollow core unit. It is likely that composite bond capacity will have begun to deteriorate prior to the sudden topping fracture.

- In fractured topping slabs that exhibit debonding, the onset of topping mesh fracture can be expected to occur at around 6 mm horizontal displacement. The tensile resistance of mesh will be fully depleted at about 20 mm displacement.

- When supported onto concrete members, it is likely that collapse of the hollow core unit will occur at a lesser horizontal displacement than the seating length due to spalling of support ledge concrete.

- In practice, the typical occurrence of chases and rebates in floor support regions are likely to exert considerable influence on the nature of topping and unit fracture.

9.2.2 HAIRPIN DETAIL (Test LOS 2)

Based on the evidence of a testing full-scale specimen, the following is concluded for flooring members subjected to dilation type loading:

- Support tie details involving bars embedded into the support and grouted into the hollow core voids can provide a controlled ductile response to dilation forces.

- From the test, it is apparent that efficient bond capacity associated with deformed bars can mitigate ductile capacity. Hence deformed bars are not well conditioned for situations where significant ductility may be required.

- Details involving removed hollow core flanges and fully grouted voids exhibit much improved topping bond capacity.
9.2.3 COMPOSITE TOPPING BOND

Based on the evidence of testing several direct shear specimens, the following is concluded for the bond surface condition of precast flooring members under monotonic dilation type loading:

- Smooth concrete surfaces subjected to direct shear developed (on average) only 37% of the minimum shear strength (0.55 MPa) required by the design standards. Hence, the results of shear flow experiments, on which composite bond capacity has been based, are not applicable to situations that involve direct shear.

9.2.4 CANTILEVER TESTS

Based on the evidence of testing three cantilever specimens, the following is concluded on the continuity performance of precast flooring members under rotational loading:

- Embedded tie bar details may be configured to develop end continuity moments of the same order of magnitude and rotational stiffness as conventional details.

- Due to reduced rotational stiffness that results from strain penetration, inclined plain bar details would appear the least favourable tie configuration for purposes of continuity development.

9.3 CYCLIC DILATION

9.3.1 TYPICAL CONTINUITY BAR DETAIL (Test LOS 5)

Based on the evidence of testing a full-scale specimen, the following is concluded for flooring members subjected to cyclic dilation type loading:

- For flooring members with relatively smooth bond surfaces (i.e., extruded hollow core), the mechanisms that facilitate topping slab uplift (buckling) may be expected to occur when typical quantities of continuity reinforcement are subjected to cyclic dilation type loading.

- Similar such mechanisms could develop in other parts of floor diaphragms where cyclic actions impose compression forces on yielded bars within topping slabs (e.g., chord-tie reinforcement).

- In the test specimen, buckling was observed to be both sudden and conclusive. It was evident that a considerable amount of topping bond degradation had occurred well in advance of slab buckling.
9.3.2 EMBEDDED TIE BAR DETAILS (Test LOS 6)

Based on the evidence of testing a full-scale specimen, the following is concluded for flooring members subjected to cyclic dilation type loading:

- Generally, the cyclic loading response of embedded plain bar details is excellent. For certain details (inclined ties), buckling may eventuate so that topping concrete is spalled and bar popping occurs. However, this effect is considered to be of little importance provided that some mechanical working of the bar is permissible.

- Similarly, smaller diameter tie bars (i.e., 12 mm and less) may be susceptible to buckling at low displacement cycles. Hence, the mechanical toughness of tie steel should be considered for these details.

- Removal of the top flange and grouting of cores would appear to effectively mitigate the problem of topping slab fracture and delamination.

- Considerable strain penetration may be observed when embedded plain bar details are subjected to cyclic loading.

- Directly connecting tie details to reinforcing elements of the support member should be avoided, since the impending lateral bar deformation may counteract the efficiency of confinement.

9.4 ANALYSIS

9.4.1 COMPOSITE BEAM-FLOOR ACTION

- Design information indicates that sufficient bond strength can exist between precast hollow core flooring and perimeter beams to develop the strength capacity of prestressing strands in tension flanges.

- Conventional section analysis indicates that the potential contribution to beam bending moment capacity from prestressing steel in tension flanges could be very significant.

- It is likely that the modular nature of precast flooring units will restrict the migration of tension flange cracks to the width of one precast unit.

- It is very likely that prestressed flooring units having thin flanges and open webs (e.g., tee units) would contribute considerably less to beam bending moment capacity.

- Compatibility analysis gives an estimate of the minimum beam lengths required to achieve prescribed structural rotations at limiting levels on concrete compression strain.
9.4.2 DIAPHRAGM FORCES AND SLAB BUCKLING

- The bending moment components of ductile beams in the vicinity of columns are almost certain to introduce large reaction (coupling) forces into adjacent floor planes.

- The conventional notion of uniformly distributed shear stresses acting along the peripheral elements of diaphragms is inconsequential when compared to nodal forces arising from the bending components of ductile support members.

- Buckling analysis indicates that topping slab regions subjected to large in-plane forces may be prone to buckling over relatively short distances if composite topping ties are not present and correct.

- Buckling analysis concludes that large diameter bars and/or concentrations of bars within topping slabs will reduce the critical buckling length, and therefore increase the required density of composite topping ties.

9.5 RECOMMENDATIONS FOR FUTURE RESEARCH

More information is required on the nature of frame dilation, and this must be determined largely by the physical testing of frame assemblages. However, it is warned that such testing will have limited meaning if the assemblages do not contain at least one module of pretensioned flooring unit. Clearly, there are very important and indivisible matters of composite beam-flooring interaction that need to be figured into the existing frame models.

With regard to the practical testing of diaphragm elements, it is considered that the logistically difficult task of testing full-size elements might not be necessary. It must be appreciated that strut and tie fields are also a partially notional concept, the same being subject to considerable spreading and redistribution within the floor slab. Rather, it is considered that the components of orthogonal bending, characteristic of ductile beams intersecting at corner columns, will induce perhaps the most definitive and plausible of all nodal (localised) reactions.

It is perceived that the following points should be closely examined in any such experiments:

- Contributions to beam strength and stiffness from adjacent flooring units, and the influence of adjacent flooring on the nature of beam plastic hinge development.

- The direct observation of dilation effects, with emphasis on the determination of practical dilation magnitudes in real structures and related design criteria.

For the general understanding of structural mechanics, various items of theory presented in this thesis should be examined, correlated with test data and developed further if appropriate. This would especially include the topics of elastic-plastic transition, rotational stiffness prediction and the plastic buckling theory.

*Chapter 9: Summary, Conclusions & Recommendations.*
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