Systematic ground motion observations in the Canterbury earthquakes and region-specific non-ergodic empirical ground motion modeling

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This paper presents an examination of ground motion observations from 20 near-source strong motion stations during the most significant 10 events in the 2010-2011 Canterbury earthquake sequence to examine region-specific systematic effects based on relaxing the conventional ergodic assumption. The non-ergodic ground motion prediction methodology is applied to the dataset to determine the systematic features of the between- and within-event residuals based on the New Zealand-specific Bradley (2010) ground motion prediction equation. On the basis of similar site-to-site residuals 15 of the 20 stations are grouped into four sub-regions: the Central Business District; and Western, Eastern, and Northern suburbs. Mean site-to-site residuals for these sub-regions then allows for the possibility of non-ergodic ground motion prediction over sub-regions of Canterbury, rather than only at strong motion station locations. The ratio of the total non-ergodic vs. ergodic standard deviation is found to be, on average, consistent with previous studies, however it is emphasized that on a site-by-site basis the non-ergodic standard deviation can easily vary by ±20%.

INTRODUCTION

The 2010-2011 Canterbury earthquake sequence includes the 4 September 2010 $M_w$7.1 Darfield earthquake (e.g. Gledhill et al. 2011, NZSEE 2010) and three subsequent earthquakes of $M_w \geq 5.9$, most notably the 22 February 2011 $M_w$6.2 Christchurch earthquake that resulted in 185 fatalities (NZSEE 2011, Seismological Research Letters 2011). Ground shaking in the Darfield earthquake resulted in widespread liquefaction in eastern Christchurch and in isolated areas throughout the region (Cubrinovski et al. 2010), and substantial damage to unreinforced masonry structures (Dizhur et al. 2010). The $M_w$6.2 Christchurch earthquake caused significant damage to commercial and residential buildings of various eras (Buchanan et al. 2011, Clifton et al. 2011, Kam et al. 2011). The severity and

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spatial extent of liquefaction observed in native soils was profound, and was the dominant cause of damage to residential houses, bridges and underground lifelines (Cubrinovski et al. 2011). The 13 June 2011 $M_w$6.0 earthquake caused further damage to previously damaged structures and severe liquefaction and rockfalls, and similarly for the $M_w$5.8 and $M_w$5.9 earthquakes on 23 December 2011.

As a result of a high-density of strong motion instruments in the Canterbury region (Berrill et al. 2011) operated by GeoNet (www.geonet.org.nz) a significant number of high amplitude near-source ground motion have been recorded during this sequence (e.g. Bradley 2012c, Bradley and Cubrinovski 2011). Of particular note are the large number of strong motions which have been recorded at the same location over these multiple events. Such a relatively unique ground motion dataset allows for the opportunity to directly examine systematic and repeatable ground motion phenomena. Such systematic effects have been qualitatively noted in the Canterbury ground motions (Bradley 2012b), but significant additional insight can be gained by quantitative analysis.

One approach to examine systematic ground motion phenomena is based on the use of empirical ground motion prediction equations (GMPEs) without the so-called non-ergodic assumption (Anderson and Brune 1999). Lin et al. (2011), Walling (2009), and Rodriguez-Marek et al. (2011), among others, represent examples where the non-ergodic ground motion prediction framework has been utilized to examine the reduction in the standard deviation of pseudo-acceleration response spectral ordinates due to relaxation of the ergodic assumption.

In this paper systematic effects in the Canterbury earthquakes are examined within the non-ergodic empirical ground motion prediction framework. The earthquake events and strong motion stations considered are first presented, with particular emphasis placed on the significant ground motion amplitudes considered. The theoretical details of the non-ergodic ground motion methodology utilized are elaborated upon, and the observed results from applying this methodology to the near-source strong motions in the Canterbury earthquake sequence are presented, and compared with previous studies. Finally, the application of the developed non-ergodic modification factors are compared with ergodic predictions and observed ground motions.

**EARTHQUAKE EVENTS AND STRONG MOTION STATIONS CONSIDERED**

In selecting the events to consider in the Canterbury earthquake sequence, a trade-off naturally occurs between the number of events and the representative range of the ground
motion amplitudes produced. Considering a larger number of events provides statistically more robust estimates (i.e. larger sample sizes). However, in order to consider more events the minimum ground motion intensity obviously has to be reduced. As a result, the overall dataset becomes increasingly dominated by smaller amplitude ground motions which are arguably not of primary concern when GMPEs are utilized in PSHA for developing design strong motions. Furthermore, there are an abundance of studies illustrating the lack of correlation between GMPE performance for small and large magnitude events. As a result of the above considerations, only events above magnitude $M_w 4.5$, which produced ground motions of engineering significance in the urban Christchurch area, were considered. This resulted in a set of 10 events, the basic details of which are given in Table 1. It is to be noted in Table 1 that a maximum source-to-site distance for recorded ground motions was also utilized to remove those ground motions which are of low amplitude as discussed further subsequently.

<table>
<thead>
<tr>
<th>ID</th>
<th>Event date</th>
<th>Magnitude, $M_w$</th>
<th>Maximum source-to-site distance, $R_{rup}^{max}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4 September 2010</td>
<td>7.1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>19 October 2010</td>
<td>4.8</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>26 December 2010</td>
<td>4.7</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>22 February 2011</td>
<td>6.2</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>16 April 2011</td>
<td>5.0</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>13 June 2011 (1:01pm)</td>
<td>5.3</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>13 June 2011 (2:20pm)</td>
<td>6.0</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>21 June 2011</td>
<td>5.2</td>
<td>30</td>
</tr>
<tr>
<td>9</td>
<td>23 December 2011 (12:58pm)</td>
<td>5.8</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>23 December 2011 (2:18pm)</td>
<td>5.9</td>
<td>50</td>
</tr>
</tbody>
</table>

$^{1}$Moment magnitudes obtained from GeoNet (www.geonet.org.nz) regional Centroid Moment Tensor (CMT) solutions (Ristau 2008).

For the 4 September 2010, 22 February 2011, 13 June 2011, and 23 December 2011 (i.e. the four largest events) the finite fault inversion studies by Beavan et al. (2011, 2012) were adopted for defining fault source geometry while moment magnitude, $M_w$, was obtained from regional centroid moment tensor solutions (www.geonet.org.nz; last accessed June 2012), based on Ristau (2008). For the remaining six smaller events, regional centroid moment tensor solutions were used to provide the event $M_w$, and a finite fault model was developed assuming the centroid location to be at the center of the finite fault plane along-strike and down-dip, and using the appropriate magnitude scaling relationship for New Zealand (Stirling et al. 2012) for defining along-strike length and down-dip width. Figure 1 illustrates the
study region and the location of the finite fault models of the 10 different earthquake events considered.

A total of 20 strong motion stations were considered in the Christchurch region for the purposes of examining systematic site effects. Figure 1 illustrates the locations of the 20 stations relative to the seismic sources considered, while Table 2 provides a list of the stations and the values of geometric mean PGA which were recorded for each of the 10 events. Figure 2 provides a histogram of the geometric mean PGA values in Table 2. It can be clearly seen that the dataset is comprised of a significant number of high intensity ground motions with values in the range of 0.01-1.41g, and a mean value of 0.183g. For each strong motion record PGA, and elastic pseudo-acceleration response spectral ordinates (SA) for vibration periods from 0.01-10s are computed and considered in the analyses to follow.

For each event, all strong motion stations within the event-specific maximum distance, $R_{lep}^{max}$ (Table 1) were used to compute the between-event residuals with respect to empirical ground motion prediction equations (GMPEs). In some events, for example the 4 September 2010 event, this included several other strong motion stations in addition to those in Table 2, however in the majority of the smaller $M_w$ events, only these 20 strong motion stations were considered.

Figure 1: Location of the finite fault planes of the 10 considered earthquake events, and the location of the 20 strong motions at which systematic site effects are examined. Color coding of the finite fault models is for clarity only.
<table>
<thead>
<tr>
<th>Station</th>
<th>Site class</th>
<th>Num events, ( N_{E} )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</thead>
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<td>CACS</td>
<td>D</td>
<td>10</td>
<td>0.197</td>
<td>0.027</td>
<td>0.020</td>
<td>0.21</td>
<td>0.034</td>
<td>0.081</td>
<td>0.136</td>
<td>0.104</td>
<td>0.073</td>
<td>0.083</td>
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<td>CBGS</td>
<td>D</td>
<td>10</td>
<td>0.158</td>
<td>0.069</td>
<td>0.270</td>
<td>0.50</td>
<td>0.070</td>
<td>0.183</td>
<td>0.163</td>
<td>0.077</td>
<td>0.157</td>
<td>0.210</td>
</tr>
<tr>
<td>CCCC</td>
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<td>6</td>
<td>0.224</td>
<td>0.119</td>
<td>0.227</td>
<td>0.43</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>CHHC</td>
<td>D</td>
<td>10</td>
<td>0.175</td>
<td>0.089</td>
<td>0.162</td>
<td>0.37</td>
<td>0.146</td>
<td>0.199</td>
<td>0.215</td>
<td>0.113</td>
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<td>D</td>
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<td>0.237</td>
<td>0.191</td>
<td>0.132</td>
<td>0.37</td>
<td>0.137</td>
<td>0.159</td>
<td>0.178</td>
<td>-</td>
<td>0.152</td>
<td>0.174</td>
</tr>
<tr>
<td>HPSC</td>
<td>E</td>
<td>10</td>
<td>0.147</td>
<td>0.041</td>
<td>0.049</td>
<td>0.22</td>
<td>0.148</td>
<td>0.183</td>
<td>0.256</td>
<td>0.068</td>
<td>0.199</td>
<td>0.264</td>
</tr>
<tr>
<td>HVSC</td>
<td>C</td>
<td>10</td>
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<td>0.091</td>
<td>0.111</td>
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<td>0.676</td>
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<td>0.013</td>
<td>0.012</td>
<td>0.20</td>
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<td>0.186</td>
<td>0.099</td>
<td>0.067</td>
<td>-</td>
<td>-</td>
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<td>LINC</td>
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<td>0.034</td>
<td>0.020</td>
<td>0.12</td>
<td>0.028</td>
<td>0.026</td>
<td>0.065</td>
<td>0.114</td>
<td>0.062</td>
<td>0.073</td>
</tr>
<tr>
<td>LPCC</td>
<td>D</td>
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<td>0.290</td>
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<td>0.92</td>
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<td>0.146</td>
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<td>0.068</td>
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<td>0.437</td>
</tr>
<tr>
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<td>0.289</td>
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<td>0.070</td>
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<td>PPHS</td>
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<td>10</td>
<td>0.221</td>
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<tr>
<td>PRPC</td>
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<td>0.054</td>
<td>0.087</td>
<td>0.63</td>
<td>0.223</td>
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<td>0.341</td>
<td>0.089</td>
<td>0.296</td>
<td>-</td>
</tr>
<tr>
<td>REHS</td>
<td>D</td>
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<td>0.081</td>
<td>0.245</td>
<td>0.52</td>
<td>0.101</td>
<td>0.188</td>
<td>0.264</td>
<td>0.086</td>
<td>0.204</td>
<td>0.254</td>
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<td>0.202</td>
<td>0.159</td>
</tr>
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<td>ROLC</td>
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<td>0.022</td>
<td>0.18</td>
<td>0.013</td>
<td>0.036</td>
<td>0.045</td>
<td>0.111</td>
<td>0.102</td>
<td>0.062</td>
</tr>
<tr>
<td>SHLC</td>
<td>D</td>
<td>10</td>
<td>0.175</td>
<td>0.072</td>
<td>0.156</td>
<td>0.33</td>
<td>0.116</td>
<td>0.245</td>
<td>0.184</td>
<td>0.076</td>
<td>0.262</td>
<td>0.275</td>
</tr>
<tr>
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<td>0.020</td>
<td>0.034</td>
<td>0.16</td>
<td>0.034</td>
<td>0.132</td>
<td>0.085</td>
<td>0.078</td>
<td>0.066</td>
<td>0.148</td>
</tr>
<tr>
<td>TPLC</td>
<td>D</td>
<td>10</td>
<td>0.266</td>
<td>0.058</td>
<td>0.032</td>
<td>0.11</td>
<td>0.024</td>
<td>0.037</td>
<td>0.065</td>
<td>0.250</td>
<td>0.068</td>
<td>0.081</td>
</tr>
</tbody>
</table>

As defined by the New Zealand Loadings Standard, NZS1170.5 (2004), i.e. B=rock; C=shallow soil; D=deep or soft soil; E=very soft soil.
Figure 2: Histogram of the geometric mean PGA values observed at the 20 stations during the 10 earthquake events.

COMPARISON OF NZ-SPECIFIC GMPE WITH THE CANTERBURY EARTHQUAKES

The representation of (pseudo) spectral acceleration (SA), from event \( e \), at a single location \( s \), for the purposes of ground motion prediction, is generally given by:

\[
\ln SA_{es} = f_{es}(Site, Rup) + \delta B_e + \delta W_{es}
\]

where \( \ln SA_{es} \) is the (natural) logarithm of the observed SA; \( f_{es}(Site, Rup) \) is the median of the predicted logarithm of SA as given by an empirical GMPE, which is a function of the site and earthquake rupture considered; \( \delta B_e \) is the between-event (or inter-event) residual with zero mean and variance \( \tau^2 \); and \( \delta W_{es} \) is the within-event (or intra-event) residual with zero mean and variance \( \sigma^2 \). Based on equation (1), empirical ground motion prediction equations can provide the distribution of SA as:

\[
\ln SA_{es} \sim N(f_{es}, \tau^2 + \sigma^2)
\]

where \( X \sim N(\mu_X, \sigma^2_X) \) is short-hand notation for \( X \) having a normal distribution with mean \( \mu_X \) and variance \( \sigma^2_X \).

Figure 3 illustrates the PGA observations for the 10 considered events as compared to the New Zealand (NZ)-specific site class D prediction of Bradley (2010, 2013a) (herein “Bradley (2010)”). Comparisons for between prediction and observation of SA’s at vibration periods of 0.2, 0.5, 1.0, 1.5, and 3.0 seconds can be found in Bradley (2013b, Appendix 1). On the basis of Figure 3 it can be seen that the Bradley (2010) model: (i) exhibits good scaling of PGA amplitudes with source-to-site distance, \( R_{rup} \), even for very near-source distances; and
(ii) provides a consistent prediction of ground motions from events of different magnitudes. The same trends for PGA are apparent for SA at various vibration periods (Bradley 2013b, Appendix 1). In addition, although not shown explicitly in Figure 3, the standard deviation of the within-event residuals for each event is consistent with the model standard deviation (see Figure 18 of Bradley (2013a) for further details).

It is also worth noting that several large events occurred in quick succession in the Canterbury earthquake sequence. For example, on 13 June 2011 and 23 December 2011 two large events ($M_w 5.3/6.0$ and $M_w 5.8/5.9$, respectively) occurred approximately 80mins apart (see times in Table 1). Both of these events occurred near the east of Christchurch, which has soft surficial soil deposits. As a result, it is speculated that the ground motions recorded in the latter event of these two sequences were affected by the surficial soils having elevated pore water pressures from the strong shaking in the earlier event. Such speculation would be expected to result in a general over prediction of short period SA amplitudes and an under prediction of long period SA amplitudes because of the reduced stiffness of the soft surficial soils due to elevated pore pressures, which is somewhat evident in the PGA observations in Figure 3, as discussed in greater detail by Bradley (2013b). Despite this postulated phenomena, no explicit account of such effects is considered in the subsequent analyses.

The above discussion implies that the Bradley (2010) model provides, generally-speaking, an unbiased prediction of the 10 considered events as a function of various model parameters. However, Figure 3 also alludes to the significant variability in the observations for a given set of model parameters. A large portion of this variability arises due to systematic features of the specific sites at which the ground motions were recorded; features which are not adequately captured in the simple parameterization used in GMPEs. The subsequent sections therefore examine the between- and within-event residuals obtained for the Bradley (2010) GMPE in order to ascertain these site specific systematic effects.

**NON-ERGODIC GROUND MOTION PREDICTION**

The Bradley (2010) GMPE, and GMPEs in general, are often based on the ergodic assumption, i.e. the time-averaged behavior of a random process at a given location is the same as the space-averaged behavior at given instants in time. Practically speaking, the ergodic assumption is invoked when ground motion records from different locations around the globe are combined without discrimination for the purposes of ground motion prediction at a single location. Several studies have illustrated that the ergodic assumption generally
Figure 3: Comparison of the Bradley (2010) GMPE (for site class D) with PGA observations from the 10 considered events. Denoted values of the between-event residual, \( \delta B_e \), are normalized by the between-event standard deviation so that \( \delta B_e = +1.0 \) implies observations with an between-event residual which is one standard deviation above zero.
leads to an over-prediction of ground motion uncertainties because it combines variability in source, path and site effects from different tectonic regions and sites, and some of this variability may in fact be systematic in a site-specific context (Anderson and Brune 1999). The ergodic assumption can be relaxed by considering that the between- and within-event residuals, given in Equation (1), are no longer purely random variables with zero-mean, but systematically depart from this for a given earthquake source (or source region) and given site of interest.

The contemporary non-ergodic methodology for considering such effects employed here has also been adopted by Lin et al. (2011), Walling (2009), and Rodriguez-Marek et al. (2011), among others, and notational convention herein generally follows Al Atik et al. (2010). Firstly, the between-event residual, $\delta B_e$, is separated into a systematic event location-to-location (herein simply ‘location-to-location’) residual (for location $l$), $\delta L_2 L_l$, and a ‘remaining’ between-event residual, $\delta B_{et}^0$ (i.e. $\delta B_e = \delta L_2 L_l + \delta B_{et}^0$). Secondly, the within-event residual, $\delta W_{es}$, is separated into a systematic ‘site-to-site’ (sometimes also referred to as ‘station-to-station’ (Rodriguez-Marek et al. 2011)) residual (for site $s$), $\delta S_2 S_s$, and a ‘remaining’ within-event residual, $\delta W_{es}^0$ (i.e. $\delta W_{es} = \delta S_2 S_s + \delta W_{es}^0$). As a result, Equation (1) can be re-written as:

$$\ln S_{A_{es}} = f_{es}(Site, Rup) + (\delta L_2 L_l + \delta B_{et}^0) + (\delta S_2 S_s + \delta W_{es}^0)$$

(3)

It should be noted that no consideration here is given to a path-specific effect in the within-event residual, as done so by Lin et al. (2011), for example. The principal reason for this omission is that only near-source recordings from moderate-to-large magnitude earthquakes are considered. As a result, ray paths from different sub-faults in the idealized rupture plane can be quite different. This makes determination of the specific ray path to be used in consideration of spatial correlation and path-specific effects non-unique. Furthermore, since only events in the relatively small geographic region of Canterbury are considered, any systematic path effects are likely to be simply represented in the systematic location-to-location residual. Finally, it is noted that all 10 earthquake events considered here are taken to be from the same ‘Canterbury’ region so that all events are considered to have the same location-to-location residual, $\delta L_2 L_l$. Subsequent analyses illustrate that this residual is not a function of event magnitude.
BETWEEN-EVENT RESIDUAL AND ITS COMPONENTS

As noted in Equation (3), the between-event residual, δB_e, is considered to be comprised of two parts. The systematic location-to-location residual can be computed as the mean value of δB_e from all the events considered:

\[
\delta L2L_l = \frac{1}{NE} \sum_{e=1}^{NE} \delta B_e
\]

(4)

where \(NE\) is the number of events (i.e. \(NE = 10\) in this study). For each event the ‘remaining’ portion of the between-event residual, \(\delta B^0_{el}\), can then be computed from:

\[
\delta B^0_{el} = \delta B_e - \delta L2L_l
\]

(5)

By definition, since \(\delta L2L_l\) is the mean of \(\delta B_e\), \(\delta B^0_{el}\) has zero mean. In addition to their mean values, both \(\delta L2L_l\) and \(\delta B^0_{el}\) are uncertain. The uncertainty in \(\delta L2L_l\) results from the fact that it is computed from a finite number of events, and hence its variance can be computed from:

\[
Var[\delta L2L_l] = \tau^2_{L2L} = \frac{\tau^2}{NE}
\]

(6)

where \(\tau^2\) is the variance in the between-event residual, \(\delta B_e\). The variance in the ‘remaining’ between-event residual can be computed simply from statistical inference of the values of \(\delta B^0_{el}\):

\[
\tau^2_0 = Var[\delta B^0_{el}] = \frac{1}{NE - 1} \sum_{e=1}^{NE} (\delta B^0_{el})^2
\]

(7)

WITHIN-EVENT RESIDUAL AND ITS COMPONENTS

As noted in Equation (3), the within-event residual, \(\delta W_{es}\), is considered to be comprised of two parts. The systematic site-to-site residual can be computed for each site (i.e. strong motion station) as the mean value of \(\delta W_{es}\) from all the events considered:

\[
\delta S2S_s = \frac{1}{NE_s} \sum_{e=1}^{NE_s} \delta W_{es}
\]

(8)
where \( NE_s \) is the number of events at site \( s \). Since not all events are recorded at the same set of locations then \( NE_s \leq NE \), and the value of \( NE_s \) for each site is given in Table 2. Once \( \delta S2S_s \) has been computed, the ‘remaining’ within-event residual can be computed as:

\[
\delta W_{es}^0 = \delta W_{es} - \delta S2S_s
\]  

By definition, since \( \delta S2S_s \) is the mean of \( \delta W_{es} \), \( \delta W_{es}^0 \) has zero mean. In addition to their mean values, both \( \delta S2S_s \) and \( \delta W_{es}^0 \) are uncertain. The uncertainty in \( \delta S2S_s \) results from the fact that it is computed from a finite number of events, and hence its variance can be computed from:

\[
\text{Var}[\delta S2S_s] = \sigma_{S2S_s}^2 = \frac{\sigma^2}{NE_s}
\]  

where \( \sigma^2 \) is the variance in the within-event residual, \( \delta W_{es} \). The variance in the ‘remaining’ within-event residual can be computed simply from statistical inference of the values of \( \delta W_{es}^0 \):

\[
\sigma_0^2 = \text{Var}[\delta W_{es}^0] = \frac{1}{NE_s - 1} \sum_{e=1}^{NE_s} (\delta W_{es}^0)^2
\]  

**NON-ERGODIC PREDICTION**

Having characterized the components of the between- and within-event residuals and their sub-components in the previous section it is now possible to obtain the mean and variance of the non-ergodic GMPE. The mean value of \( \ln SA_{es} \) is given by:

\[
E[\ln SA_{es}] = f_{es} + \delta L2L_t + \delta S2S_s
\]  

since \( E[\delta B_{es}^0] = E[\delta W_{es}^0] = 0 \). Because \( SA_{es} \) has a lognormal distribution then it follows that the median value of \( SA_{es} \) can be obtained as the exponential of the mean of \( \ln SA_{es} \):

\[
\text{Median}[SA_{es}] = \exp[E[\ln SA_{es}]]
\]  

In examining the systematic effects, \( \delta L2L_t \) and \( \delta S2S_s \), it is useful to consider a ‘systematic amplification factor’ which is the ratio of the non-ergodic to ergodic median GMPE predictions. Taking the ratio of Equation (13) and Equation (2) gives:

\[
\frac{\text{Median}[SA_{es}]_{\text{non-ergodic}}}{\text{Median}[SA_{es}]_{\text{ergodic}}} = \exp(\delta L2L_t + \delta S2S_s)
\]
In terms of the prediction variance, making the conventional assumption that the different residuals are uncorrelated, the variance of the non-ergodic prediction can be obtained as:

$$\text{Var}[\ln SA_{ex}] = (\tau_{L2L}^2 + \tau_0^2) + (\sigma_{SS}^2 + \sigma_0^2)$$  \hspace{1cm} (15)

For each of the between-event, within-event, and total residuals, a standard deviation reduction factor can be computed for the ratio of the non-ergodic and ergodic standard deviations:

$$RF_t = \sqrt{\frac{\tau_{L2L}^2 + \tau_0^2}{\tau^2}}$$  \hspace{1cm} (16)

$$RF_a = \sqrt{\frac{\sigma_{SS}^2 + \sigma_0^2}{\sigma^2}}$$  \hspace{1cm} (17)

$$RF_{\sigma_r} = \sqrt{\frac{(\tau_{L2L}^2 + \tau_0^2) + (\sigma_{SS}^2 + \sigma_0^2)}{\tau^2 + \sigma^2}}$$  \hspace{1cm} (18)

OBSERVED SYSTEMATIC EFFECTS FROM THE CANTERBURY EARTHQUAKES

BETWEEN-EVENT RESIDUAL

Figure 4 illustrates the computed values for $\delta B_r$ as a function of SA vibration period for the 10 events considered (specific results for PGA are not shown herein as they are numerically equivalent to those for SA(0.01)). Also shown in Figure 4 is the systematic location-to-location residual, $\delta L2L$. It can be seen that for short vibration periods ($T \sim < 0.3s$) the value of $\delta L2L$ is approximately zero, illustrating that the Bradley (2010) GMPE is, on average, unbiased for these short vibration periods, across the events and strong motion stations considered. However, as the vibration period increases the value of $\delta L2L$ increases. Bradley (2012c) and Bradley and Cubrinovski (2011) have suggested that greater than predicted SA amplitudes at long periods in the 4 September 2010 and 22 February 2011 events, respectively, could be the result of: (i) near-source forward directivity; (ii) nonlinear response of soft surficial soils; (iii) basin-induced surface waves; and (iv) inherent model bias as a result of a limited amount of reliable ground motion records at long vibration periods. While all these points are plausible on a single ground motion observation by observation
basis, the observations in Figure 4 are based on sites in the Canterbury region located at various azimuths from 10 different earthquake events. Firstly, forward directivity rupture effects would not systematically affect sites at the range of azimuths considered, and such effects would not be significant for smaller magnitude events. Secondly, as the majority of the stations considered are located on the Canterbury alluvial deposits, nonlinear response of surficial soils may be of importance, since only ground motions from moderate-to-large magnitude earthquakes at close distances were considered (e.g. the average PGA of the considered motions 0.183g (i.e. Figure 2)), and also basin-induced surface waves are likely of importance. Finally, while inherent model bias is a possibility for very long periods (i.e. $T > 5s$), it is unlikely at shorter periods (i.e. $T=1s$), and therefore this is not considered as a significant factor in the observed departure from zero in Figure 4.

![Figure 4: Computed between-event residuals, $\delta B_e$, for the considered 10 events and the location-to-location residual, $\delta L2L$.](image)

The dependence of the between-event residuals as a function of event magnitude for five different vibration periods is illustrated in Figure 5. It can be seen that there is no apparent (i.e. statistically significant) trends in $\delta B_e$ as a function of event magnitude. The only noteworthy observation is the relatively large $\delta B_e$ values from the $M_w 5.0$ event on 16 April 2011. Without further investigation, the reason for this is not immediately apparent. Hence, for a given vibration period, the assumption that the location-to-location residual is a constant (i.e. is not dependent on event magnitude) is reasonable.
Figure 5: Variation in between-event residuals with magnitude for five different vibration periods.

WITHIN-EVENT RESIDUALS

By considering the within-event residuals at each strong motion station site during the 10 events, systematic site effects can be ascertained via Equation (8). Bradley (2013b, Appendix 1) provides the within-event residuals as a function of vibration period for all 20 strong motion station sites considered, and because of space limitations only a subset are presented here. Figure 6a-c illustrate, for example, the within-event residuals and the systematic site effect at CBGS, LINC, and HVSC, respectively. It can be seen in Figure 6a that the ground motions observed at the CBGS (Canterbury Botanic Gardens) site is on average similar to that predicted by the median of the Bradley (2010) model, with $\delta S_2 S_s$ being close to zero for all vibration periods. Figure 6b illustrates that LINC (Lincoln) generally has ground motion observations which are lower than that predicted by the Bradley (2010) model, with $\delta S_2 S_s$ generally slightly less than zero. Finally, Figure 6c illustrates that HVSC (Heathcote Valley) generally has ground motion observations which are significantly greater than predicted at short periods and less than predicted at long periods, consistent with the identified basin-edge effects at this location (Bradley 2012a).

Figure 6a-c illustrate that different strong motion stations have systematic site specific residuals, $\delta S_2 S_s$, which depart from zero. Figure 6d illustrates these $\delta S_2 S_s$ residuals for all 20 strong motion stations considered, as well as the median, 16th and 84th percentiles of these $\delta S_2 S_s$ values. It can be seen that, on average, the values of $\delta S_2 S_s$ for the 20 strong motion stations considered are very close to zero. This is largely expected, given that the between-
event residual is used to give a within-event residual which is random with approximately zero mean (Lindstrom and Bates 1990). However, Figure 6d serves to clearly illustrate that while the Bradley (2010) model employing the ergodic assumption is generally unbiased, a significant amount of the observed variability is the result of systematic site effects.

![Figure 6: Within-event residuals, and the site specific effect, δS2S, at: CBGS (Christchurch Botanical Gardens); LINC (Lincoln); HVSC (Heathcote Valley); and (d) site specific effect, δS2S, for all stations considered.](image)

The dependence of the within-event residuals on ground motion intensity is illustrated in Figure 7 based on the PGA of each ground motion. The dependence based on spectral acceleration ordinates at each vibration period was also examined, but is not presented here. 18 of the 20 stations considered illustrated trends similar to Figure 7a and Figure 7b, where no apparent trend of $\delta W_{es}$ with PGA is evident. In contrast, Figure 7c and Figure 7d illustrate that at LPCC and KPOC there is a clear trend of high-intensity ground motions resulting in larger $\delta W_{es}$ residuals. For LPCC, this trend is seemingly apparent across all vibration periods, while for KPOC this trend is only apparent at short-to-moderate vibration
periods (i.e. $T \leq 1s$). These results for 18 out of 20 stations (such as shown in Figure 7a Figure 7) demonstrate that, in general, the consideration of nonlinear soil effects in the Bradley (2010) GMPE (which is based on Chiou and Youngs (2008)) is consistent with the observed strong ground motions. Hence, for 18 of the 20 stations considered, the assumption that the within-event residual, $\delta W_{es}$, is constant for a given vibration period is adequate. For the LPCC and KPOC stations $\delta W_{es}$ should be considered explicitly as a function of the ground motion intensity for the purpose of non-ergodic ground motion prediction. It is noted that the intensity-dependence of $\delta W_{es}$ at LPCC and KPOC may be indicative that the site classification of these sites (i.e site classes B and E, as given in Table 2) provides a poor prediction of their true site response for the range of ground motion intensities examined.

Figure 7: Ground motion intensity dependence of within-event residuals at: (a) CACS (Canterbury Aero club); (b) CHHC (Christchurch hospital); (c) LPCC (Lyttelton Port); and (d) KPOC (Kaiapol North school).
SYSTEMATIC MEDIAN AMPLIFICATION FOR SPECIFIC SUB-REGIONS IN CHRISTCHURCH

The location-to-location and site-to-site residuals ($\delta L_2L_1$ and $\delta S2S_2$, respectively) presented in the previous sections, in combination with Equation (12), allows for non-ergodic site-specific prediction of ground motions at the 20 strong motion stations from earthquakes in the Canterbury region. For the purposes of developing design ground motions for the Christchurch region it also desirable to develop predictions which can be utilized over broad sub-regions of Christchurch, rather than only at a specific site location. This section examines the $\delta L_2L_1$ and $\delta S2S_2$ residuals in order to develop such sub-region predictions, while subsequent sections examine the non-ergodic standard deviation.

Systematic site-to-site residuals for various sub-regions of Christchurch

Based on the examination of the $\delta S2S$ residuals as a function of vibration period for the 20 different considered stations it was found possible to group 15 stations into four different subgroups: (i) Central Business District (stations CBGS, CCC, CHHC, REHS; (ii) 'Western suburbs' (stations CACS, TPLC, ROLC, LINC); (iii) 'Eastern suburbs' (stations SHLC, PRPC, HPSC, NNBS, KPOC); and (iv) 'Northern suburbs' (stations PPHS and SMTC). The remaining 5 stations did not exhibit characteristics which allowed them to be grouped easily, and are discussed subsequently.

Figure 8 illustrates the site-specific residuals for various stations by sub-region, as well as the mean residual for each sub-region. Figure 8a illustrates that the four sites in the central business district (CBD) have similar $\delta S2S$ residuals with vibration period, with residuals of approximately zero for vibration periods less than 0.3s, then increasing over the range of $T=0.3-4.0$ s. The value of the residuals over this range are similar for three sites (CBGS, CCC, CHHC), but the residuals for the REHS station between vibration periods of $T=0.4-2.0s$ are notably larger, inferred to be the result of several meters of peat deposits at this location (Canterbury Geotechnical Database 2011).

Figure 8b illustrates the site-specific residuals at four locations on the western extent of the Christchurch urban area, in which the surficial soils are comprised primarily of gravelly deposits (Brown and Weeber 1992, Cubrinovski and McCahon 2011). In contrast to the $\delta S2S$ residuals in the CBD region (Figure 8a) it can be seen that the site-to-site residuals at these locations lie typically in the range of -0.5 to 0.0, except for $T>4s$, where the effects of the deep sedimentary basin generally lead to $\delta S2S_2>0$. 
Figure 8c illustrates the site-to-site residuals in the eastern suburbs of urban Christchurch, as well as Kaiapoi to the north of urban Christchurch (Figure 1). While there is a slightly greater variability among the $\delta S2S$ residuals across these five stations than for the aforementioned results in Figure 8a and Figure 8b, it can be seen that residuals, on average, have a mean value close to zero across the range of vibration periods. The large $\delta S2S$ residual for the KPOC station for $T>4s$ is the result of the deep sediments in the Pegasus basin, which results in strong wave-guide effects, as clearly observed in the 4 September 2010 Darfield earthquake (Bradley 2012c) which contributed to large liquefaction in various areas of Kaiapoi (Cubrinovski et al. 2010) despite a relatively large distance from the rupture source.

Figure 8d illustrates the site-to-site residuals of the PPHS and SMTC stations located in the northern suburbs of urban Christchurch. For $T<0.2s$ it can be seen that these sites exhibit average $\delta S2S$ residuals in the range of -0.5 to -0.3. Between $T=0.2-0.7s$ the residuals, on average, increase from approximately -0.5 to 0.2; and for $T>0.7s$ the residuals vary, on average, between 0.1 and 0.3.

**Locations which don’t conform to sub-region categories**

The RHSC (Riccarton High School) station is a relatively standard site class D site, but with relatively strong SA amplitudes in the range $T = 0.2 - 0.4s$, which is approximately the natural period of the site’s shallow surficial soils. The CMHS (Cashmere High School), HVSC (Heathcote Valley), and LPCC (Lyttelton Port) stations are all affected by the local sub-surface topography of the Banks Peninsula volcanics, which underlie the alluvial deposits at the surface, and respond in ways that cannot be generalized among them. Finally, the NBLC site, while being located in the eastern suburbs, is actually located on a sand dune region, and therefore exhibits a site response more a-kin to a stiff soil site. The systematic $\delta S2S$ residuals for these sites are presented subsequently.
Figure 8: Site-specific residuals for various sub-regions: (a) Central Business District (CBD); (b) ‘Western Suburbs’; (c) ‘Eastern Suburbs’; and (d) ‘Northern Suburbs’.

Comparison of all sub-regions

Figure 9a illustrates the variation in site-to-site residuals, $\delta S2S$, for the four main sub-regions of Christchurch, as well as the HVSC, LPCC, CMHS, RHSC, and NBLC stations which don’t conform to these main four sub-regions. As previously noted with respect to Figure 6d, it can be seen that there is a large variability between the systematic $\delta S2S$ for the different sub-regions and other outlying locations. For short vibration periods ($T < 0.2s$), the HVSC station has the largest positive $\delta S2S$ residual, while the NBLC site has the largest negative $\delta S2S$ residual. The LPCC station also has a notable positive residual, while the North and Western suburbs have notable negative residuals, and all others (i.e. CBD, East, CMHS, and RHSC) have near zero residuals. For moderate-to-long vibration periods there is significant fluctuation of the residuals for each sub-region or location as a function of vibration period.
Figure 9: Comparison of various sub-regions and other locations in Christchurch: (a) site-to-site residuals; and (b) systematic median amplification factors from systematic location-to-location and site-to-site effects.

Figure 9b illustrates the systematic amplification factor which would be applied to the median ground motion prediction (i.e. Equation (14)). Relative to Figure 9a, the most notable feature is the contribution of the $\delta L2L_4$ residual for long vibration periods (i.e. Figure 2), which causes all sub-regions and other locations to have amplification factors greater than 1.0 for long vibration periods. For the Christchurch CBD, in particular, it can be seen that the amplification factor is approximately 1.0 for $T<0.2$, then increases to a value of approximately 1.8 from $T = 0.5 - 2s$, contains a minor localized increase for $T = 2 - 4s$, and then gradually increases up to values 2.2 for $T = 10s$. Parametric forms of these systematic effects for the four sub-regions (i.e. CBD, and Western, Eastern, Northern
suburbs) are developed in Bradley (2013b) for use in Canterbury-specific seismic hazard analysis for the ongoing Christchurch rebuild. The implications of these amplification factors for ergodic and non-ergodic prediction are examined in a subsequent section as compared to ground motion observations.

NON-ERGODIC STANDARD DEVIATIONS

Between-event standard deviations

In order to fully incorporate non-ergodic aspects into a GMPE it is necessary to modify both its median and standard deviation prediction. Figure 10a presents the ergodic between-event standard deviation of the Bradley (2010) model, \( \tau \), in comparison with the standard deviation in the location-to-location residual, \( \tau_{L2L} \); that of the ‘remaining’ between-event residual, \( \tau_0 \); and their SRSS combination. It can be seen that, even with only 10 events, the value of \( \tau_{L2L} \) is notably smaller than \( \tau_0 \) (i.e. \( \sqrt{10} \) times smaller, as per Equation (6)). This is also true for the within-event residuals, and therefore herein only the combined standard deviation for the between- and within-event residuals is examined. For \( T \leq 0.3s \) the non-ergodic between-event standard deviation is on the order of 85\% of the ergodic standard deviation (i.e. \( RF_T = 0.85 \) in Equation (16)), while for \( T > 1s \) it becomes an increasing smaller proportion of the ergodic value, taking a value as small as 60\% for \( T = 10s \). Hence, it can be seen that the non-ergodic between-event standard deviation is relatively constant for the range of vibration periods considered, in contrast to the ergodic standard deviation which increases notably for long vibration periods. Since long-period ground motion is dominated by large scale features of the seismic source, and propagation through the earth’s crust and local sedimentary basins, it is not surprising that gross differences in tectonics and crustal/basin structure from worldwide events (used in the ergodic model) lead to a notably larger long period between-event standard deviation than those specifically for the Canterbury region and Canterbury earthquake sequence (used in the non-ergodic model).
Within-event standard deviations

The within-event standard deviations of the ergodic Bradley (2010) model are a function of event magnitude and the level of nonlinearity in the site response portion of the model. Because the variation in the within-event standard deviation due to these factors is notably larger than the variation in the between-event standard deviation, it is more instructive to examine the non-ergodic within-event standard deviation on the basis of its ratio to the ergodic within-event standard deviation (i.e. $RF_\sigma$ given in Equation (17)).

Figure 11 presents the reduction in the within-event standard deviation as a function of vibration period for the 20 different strong motion stations which were considered. On average across the 20 stations, it can be seen that the within-event standard deviation ranges from $RF_\sigma = 0.65 - 0.85$ over the spectrum of vibration periods, with smallest values at long vibration periods, and largest values at moderate periods ($T = 0.2 - 0.6s$). Despite the fact that, on average, the non-ergodic standard deviation is less than the ergodic standard deviation, it can be seen that in several instances (both several sites and several vibration periods) $RF_\sigma > 1$, indicating that the non-ergodic standard deviation is greater than the ergodic standard deviation.
Figure 11: Reduction in the within-event standard deviation of the non-ergodic model as compared to the ergodic model for all stations, the subset of CBD stations, and the proposed design parameterization.

**Total standard deviation**

Similar to Figure 11, Figure 12 presents the reduction in the non-ergodic vs. ergodic total standard deviation as a function of vibration period for the 20 different strong motion stations which were considered. It can be seen that the reduction factor values for the total standard deviation are very similar to the within-event standard deviation – a consequence of the fact that the within-event standard deviation is notable larger than the between-event standard deviation, and hence it is the dominant contributor to the total standard deviation.

Figure 12: Reduction in the ‘total’ standard deviation of the non-ergodic model as compared to the ergodic model for all stations, the subset of CBD stations, and the proposed design parameterization.
COMPARISON OF NON-ERGODIC STANDARD DEVIATION REDUCTION WITH PREVIOUS STUDIES

Figure 13 provides a comparison of the non-ergodic total standard deviation reduction obtained in this study with those of previous studies, which are classified into two types. The first are so-called ‘single station’ non-ergodic studies in that only the site-to-site systematic effect (i.e. $\delta S_{S_1}$) is considered, such as Lin et al. (2011) (L11), Chen and Tasi (2002) (CT02), and Atkinson (2006) (A06). The second are so-called ‘single-path single-station’ non-ergodic studies that consider systematic site, path and source effects, such as L11, A06 and Morikawa et al. (2008) (M08). Firstly, it can be seen that the ‘single station’ studies (i.e. L11, CT02, and A06) result in a mean reduction in total standard deviation which is less than the mean obtained in the present study. This is consistent with the notion that this study also considered the location-to-location systematic effect, $\delta L_2 L_1$. Secondly, it can be seen that ‘single-path single-station’ studies result in a mean reduction in total standard deviation which is greater than the mean obtained in this study. This can be attributed to the additional path-effect considered in such studies, which was ignored here. It may also be a result of the fact that the aforementioned studies have generally used small amplitude ground motions, so that additional uncertainty resulting from nonlinear site effects is not scrutinized.

![Figure 13: Comparison of the reduction in the total standard deviation from non-ergodic consideration in this study compared with previous studies. PGA results from other studies plotted at T=0.01s.](image)

In addition to the above comments, Figure 13 also serves to re-iterate the significance of the variation in the reduction of the total standard which can occur on a site-by-site basis. Figure 13 illustrates, for example, that while the mean reduction factor is on the order of 0.80
for short periods, the 16\textsuperscript{th}-84\textsuperscript{th} percentile confidence interval is on the range of nearly 0.60-1.0. Therefore, it is clear that caution should be exercised when considering the use of mean reduction factors for application to cases which do not have site-specific data.

**COMPARISON OF ERGODIC AND NON-ERGODIC PREDICTIONS**

Having developed non-ergodic modification factors for the Bradley (2010) GMPE, it is useful to directly examine the relative comparison between these two predictions and observations from notable scenario ground motions from the Canterbury earthquake sequence. Figure 14 compares the difference in predictions from the ergodic and non-ergodic Bradley (2010) model with two ground motions for each of the 4 September 2010 and 13 June 2011 (2:20pm) events. Non-ergodic predictions at the CCCC and REHS stations in Figure 14a and Figure 14b are based on the ‘CBD’ specific modification; at TPLC based on the ‘Western suburbs’ modification; and at HVSC based on the HVSC-specific modification (i.e. Figure 9b). Examination of Figure 14 illustrates that, on average, the non-ergodic prediction provides a closer comparison to the observations than the ergodic prediction. This is expected given that the non-ergodic prediction is basically ‘calibrated’ using these same observations. However, it is also important to note that the non-ergodic prediction does not always provide an improved prediction on a case-by-case basis. This is illustrated in Figure 14a for $T = 0.3 - 1.5s$, for example, where the geometric mean of the observation is closer to the median of the ergodic prediction than the non-ergodic median prediction.

**CONCLUSIONS**

This study has examined ground motion observations from the most significant 10 events in the 2010-2011 Canterbury earthquake sequence at near-source sites to scrutinize the New Zealand (NZ)-specific Bradley (2010) GMPE and develop region-specific modification factors based on relaxing the conventional ergodic assumption. It was observed that the location-to-location residual (i.e. systematic feature of the between-event residuals) had values close to zero for short-to-moderate vibration periods, but became increasingly positive for $T > 1s$, likely indicative of important Canterbury-specific sedimentary basin and near-surface soil effects which are not adequately accounted for in the Bradley (2010) GMPE. In general, the site-to-site residuals were found to be independent of ground motion intensity, however for the LPCC (Lyttelton Port) and KPOC (North Kairnapo) stations there was an apparent dependence on ground motion intensity; likely indicating that the nonlinear
Figure 14: Comparison of the ergodic and non-ergodic forms of the Bradley (2010) GMPE prediction compared to observations: (a) CCCC (Christchurch Cathedral College); (b) REHS (Resthaven); (c) HVSC (Heathcote Valley); and (d) TPLC (Templeton). Note that both horizontal components and the geometric mean of the observed response spectra are illustrated.

The site response at these sites is not well characterized by their designated site classes (B:rock and E:very soft soil, respectively). On the basis of the similar site-to-site residuals, 15 of the 20 stations were adequately grouped into four sub-regions, while the remaining 5 stations did not fit any of these general sub-regions. The grouping of sites into sub-regions allows the possibility of non-ergodic ground motion prediction over sub-regions of Canterbury, rather than site-specific predictions only at strong motion stations. Examination of the standard deviations in the residuals illustrates that, on average, the non-ergodic standard deviation is 65-85% of the ergodic standard deviation of the Bradley (2010) model. However, on a site-by-site basis this percentage can easily vary by ±20%.

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